Borrowing Stigma and Lending-of-Last-Resort Policies

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Abstract

How should the lender of last resort provide liquidity to banks during periods of financial distress? During the 2008-2010 crisis, banks avoided borrowing from the Fed's long-standing discount window (DW), but actively participated in its special monetary program, the Term Auction Facility (TAF), although both programs had the same borrowing requirements. Using an adverse selection model with endogenous borrowing decisions, we explain why two programs suffer from different stigma and how the introduction of TAF incentivized banks' borrowing. We synthesize several data sources to confirm our main theoretical predictions.

Keywords: lending of last resort, discount window stigma, Term Auction Facility, adverse selection

JEL: G01, E52, D44, E58

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"[Banks] deliberately did not ask for the liquidity they needed for fear of damaging their reputation—the "stigma" problem... I do not think we were conscious of this before the crisis started and I do not think central banks have a convincing answer to it... This is, I think, still a challenge in how to manage the process of central bank provision of liquidity support. This is one of the big intellectual issues that has not been fully resolved."

— Governor Mervyn King (Bank of England, 2016)

"For various reasons, including the competitive format of the auctions, [Term Auction Facility] has not suffered the stigma of conventional discount window lending and has proved effective for injecting liquidity into the financial system... Another possible reason that [Term Auction Facility] has not suffered from stigma is that auctions are not settled for several days, which signals to the market that auction participants do not face an immediate shortage of funds."

— Ben Bernanke (2010) testimony to US House of Representatives

1 Introduction

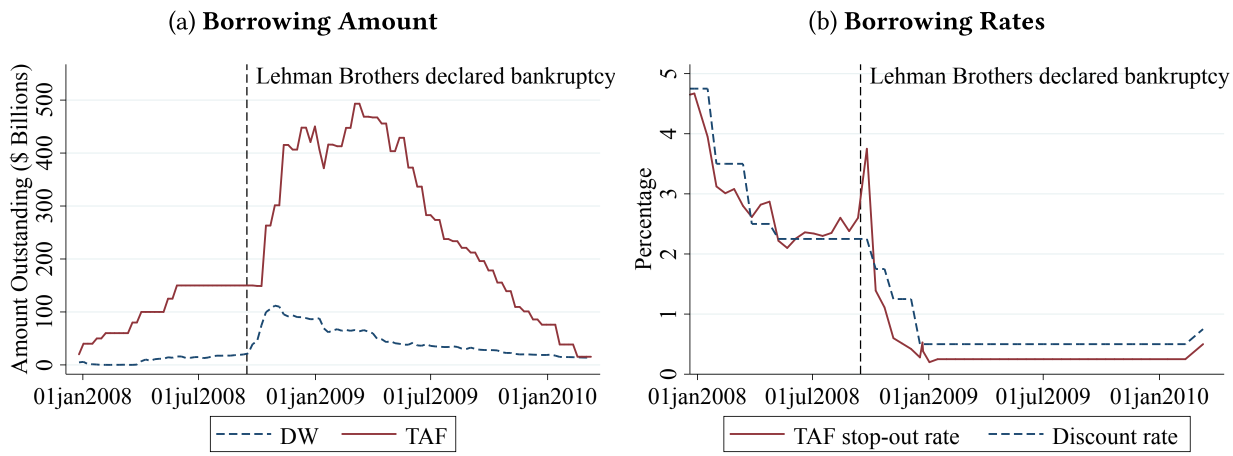
Financial crises are typically accompanied by liquidity shortages in the entire banking sector, in which case the central bank should act as the lender of the last resort (LOLR) (Bagehot, 1873). How should the lender of the last resort lend to depository institutions and provide liquidity during such episodes? The answer is not obvious. The discount window (DW) has been the primary lending facility used by the Federal Reserve, but it was severely underutilized when the interbank market froze at the beginning of the financial crisis in late 2007. A main reason for the underutilization is believed to be the stigma associated with DW borrowing: Tapping the discount window conveys a negative signal about borrowers' financial condition to their counterparties, competitors, regulators, and the public.[1]

In response to the credit crunch and banks' reluctance to borrow from DW, the Fed created a temporary program, the Term Auction Facility (TAF), in December 2007. TAF held an auction every other week and provided a preannounced amount of loans with *identical* loan maturity, collateral margins, and eligibility criteria to those of the DW.

[1] Banks have regularly paid more for loans from the interbank market than for loans they could readily get from DW (Peristiani, 1998; Furfine, 2001, 2003, 2005). Although the Fed does not publicly disclose which institutions have received loans from DW, the Board of Governors publishes weekly the total amount of DW lending by each of the 12 Federal Reserve Districts. Therefore, a surge in total DW borrowing could send the market scrambling to identify the loan recipients. Because of the interconnectedness of the interbank lending market, it is not impossible for other banks to infer which institutions went to the discount window. Market participants and social media could also infer from other activities.

Surprisingly, TAF provided much more liquidity than DW: Figure 1a shows that the outstanding balance in TAF far exceeded that in DW during 2007–2010; the outstanding balance in DW made up at most 33.4 percent of the total outstanding balance at any moment between 2007 and 2010. Even more surprisingly, banks consistently paid a higher interest rate to obtain liquidity through the TAF auction: Figure 1b shows that the *stop-out rate*—the rate that clears the auction—was higher than the concurrent *discount rate*—the rate readily available in DW—in 21 of the 60 auctions, especially from March to September 2008, the peak of the financial crisis.[2]

Figure 1: Borrowing Amounts and Rates in DW and TAF from 2008 to 2010



This episode suggests the importance of the design of emergency lending programs to effectively cope with liquidity shortages. More specifically, it raises a series of questions about lending-of-last-resort policies. Why could TAF overcome the stigma and generate more borrowing than DW? Shouldn't the same stigma also prevent banks from participating in TAF? How did banks decide to borrow from DW and/or TAF? Was there any systematic difference between the banks that borrowed from the two facilities? How could the program be further improved? There is no consensus to the answers of the questions (Armantier and Sporn, 2013; Bernanke, 2015).

This paper provides a comprehensive analysis of lending of last resort in the presence of borrowing stigma. We introduce a model in which banks have private information about their financial condition. Weaker banks have more urgent liquidity needs and enjoy higher borrowing benefits. Two lending facilities are available. An auction is held to allocate a set amount of liquidity, and DW is always available—before, during, and after the auction. Importantly, TAF delays its release of funds. Borrowing from each facility incurs a stigma cost, which is endogenously

[2]The stop-out rate ranged from 1.5 percentage points above (on September 25, 2008) to 0.83 percentage points below (on December 4, 2008) the concurrent discount rate. The stop-out rate was above the concurrent discount rate for almost all auctions between March 2008 (when Bear Stearns filed for bankruptcy) and September 2008 (when Lehman Brothers filed for bankruptcy).

determined by the financial condition of participating banks.

In equilibrium, banks self-select into different programs. The weakest banks borrow immediately from the DW because they are desperate for liquidity and cannot afford to wait. Stronger banks, in contrast, are lured to participate in the auction because the potential of borrowing cheap renders the auction more attractive than DW. Their liquidity needs are not as imperative, and they value the lower expected price in the auction more than weaker banks do. Of the banks that participate in TAF, some may bid higher than the discount rate because they would like to avoid the discount window stigma brought by pooled with the weakest banks. As a result, the clearing price in the auction may exceed the discount rate. Of the banks that have lost in TAF, relatively weaker ones might still borrow from DW.

We demonstrate that TAF, used in accordance with DW, could increase liquidity provision through three channels. First, by setting a low reserve price in the auction, TAF attracted moderately weak banks (that would have borrowed from DW without TAF) to participate and take their chances on borrowing cheap. Second, participating banks can submit bids to internalize any stigma cost associated with TAF, so TAF also attracted moderately strong banks (that would not have borrowed at all without TAF) to participate. Finally, due to the selection by stronger banks into the auction, the auction stigma is endogenously lower than the discount window stigma, which further encourages stronger banks to participate in TAF. Hence, the combination of TAF and DW expands the set of banks who try to, and may obtain, liquidity, thus increasing the overall supply of short-term credit to the economy.

We collect granular data on DW and TAF borrowing during the crisis and match them with several commonly used data sources, including the regulatory Y-9C data and Markit CDS spreads. From several aspects, our analysis shows that financially weaker banks borrowed relatively more from the DW than TAF, compared with their stronger peers. First, using quarterly-frequency data, we show that compared with TAF banks, DW banks have less core deposit, higher leverage, lower tier-1 capital ratio, more unused loan commitment and relies more on short-term wholesale funding, all signs of financial weakness. The decisions to borrow from DW relative to TAF could in turn predict future deteriorating financial conditions as well as bank failures, beyond other observable characteristics from the regulatory reports. Second, we take advantage of the international aspects of the data and explore the staggeredly-implemented credit guarantee programs implemented in various countries in October 2008. Our difference-in-difference analysis shows that following these policies' operation, banks in those countries reduces the share of their borrowings from the DW and increases the share or their borrowings from the TAF. Finally, we exploit the daily-frequency data on DW and TAF borrowing and show that prior to the borrowing dates, DW banks had persistently higher credit default swap spreads—higher risks of default—than TAF banks, and TAF borrowing could reduce the spreads by a larger amount.

Literature

The paper contributes to the theoretical and empirical literature on the lending-of-last-resort policies, starting from Bagehot (1873). Freixas et al. (1999) offers an earlier review to this literature. Theoretically, our paper discusses how to design LOLR facilities to mitigate the participation stigma. Philippon and Skreta (2012) and Tirole (2012) use a mechanism-design approach to study government intervention into markets plagued by adverse selection. In the dynamic context, Fuchs and Skrzypacz (2015) show trading restrictions and subsidizations could be optimal. Our paper contributes to this literature by allowing for multiple and dynamic policy intervention programs, which have the potential of separating heterogeneous participants. We show how one program could have a higher stigma cost than the other, despite that both have *identical* requirements. More relatedly, our paper contributes to the theoretical understanding of LOLR (Rochet and Vives, 2004) and the associated stigma (Ennis and Weinberg, 2013; Lowery, 2014; Ennis, 2017). La'O (2014) also explains how TAF may alleviate discount window stigma from the perspective of predatory trading. The explanation focuses on the signaling perspective of TAF borrowing. We offer a complementary explanation on how delayed funding settlement creates separation, which according to Bernanke (2015), is crucial to the design of TAF. Moreover, La'O (2014) predicts that in equilibrium, banks always pay a premium for TAF loans over the discount rate, which is at odds with the empirical observation. Che et al. (2020) show that a stigma could have a salutary effect: refusing bailouts could be a useful signal that firms send to their market participants. Gorton and Ordoñez (2020) also study central bank liquidity provision and show that stigma is desirable to implement opacity. Our paper rationalizes the borrowing behavior in the last financial crisis and improves the understanding of appropriate interventions during a financial crisis.

Empirically, our paper contributes to the literature on government bailout during financial crisis. Acharya and Mora (2015) show that the aggregate banking sector suffered from liquidity shortage at the onset of the crisis, and government-sponsored facilities enabled banks to continue to provide liquidity during the crisis. Acharya et al. (2017) further show that dealers with lower equity returns and greater leverage were more likely to participate in the Securities Lending Facility (TSLF) and bid higher (and thus borrow more) in the Primary Dealer Credit Facility (PDCF). Using data during the European Sovereign Debt Crisis, Drechsler et al. (2016) show that weakly-capitalized banks borrowed more from LOLR and subsequently invested in risky assets. Cassola et al. (2013) use bidding data in European Central Bank Auctions for Short-Term Funds and find that banks' bid strategically, and their bids reflect their funding cost in the interbank market. All these studies do not account for the widely acknowledged stigma associated with LOLR borrowing. More relatedly, Peristiani (1998) and Furfine (2001, 2003, 2005) offer evidence that banks prefer the federal funds market to DW. Armantier et al. (2015) use TAF as a laboratory to show the existence of discount window stigma, estimate its magnitude, and examine the relationship with various bank characteristics. Armantier and Holt (2020) use lab experiment to test policies that have been proposed to mitigate the stigma and find that random DW borrowing could be helpful. The TAF was shown to be effective in reducing liquidity concerns (Wu, 2011), lowering LIBOR (McAndrews et al., 2017), and conferred a benefit on the real economy (Berger et al., 2017; Moore, 2017). While these papers largely focus on measuring either DW stigma or the subsequent economic effects of DW/TAF borrowing, our paper focuses on the interplay between DW and TAF borrowing and offers a theoretical explanation to why TAF was able to mitigate the discount window stigma. We provide conclusive evidence that the participating banks of the two programs have different financial strengths that should be directly related to the microfoundation for the stigma.[3]

The rest of the paper is organized as follows. Section 2 describes lending-of-last-resort facilities during the financial crisis. Section 3 sets up the model. Section 4 characterizes the equilibrium of the model and discusses liquidity provision under different settings. Section C presents empirical evidence consistent with the predictions of the model. Section 5 concludes. The appendix contains omitted proofs, figures, and tables.

2 Background

Stress in the interbank lending market began to loom in the summer of 2007. In June, two of Bear Stearns' mortgage-heavy hedge funds reported large losses. On July 31, they declared bankruptcy. On August 9, BNP Paribas, France's largest bank, barred investors from withdrawing money from investments backed by US subprime mortgages, citing evaporated liquidity as the main reason. Subsequently, many other banks and financial institutions experienced liquidity dry-ups in wholesale funding (in the form of asset-based commercial paper or repurchase agreements).

With the growing scarcity of short-term funding, banks were supposed to borrow from the lender of last resort (LOLR).[4] In the US, the role of LOLR has largely been fulfilled by the discount window, which allows eligible institutions—mostly commercial banks—to borrow money from the Federal Reserve on a short-term basis to meet temporary shortages of liquidity caused by internal and external disruptions.[5] Discount window loans were extended to sound institutions with good collateral. Since its founding in 1913, the Fed has never lost a penny on a discount

[3]Berger et al. (2017) also show that small banks and big banks borrowed differently.

[4]The Federal Home Loan Bank (FHLB) system also helped reduce financial stress at the onset of the crisis. However, Ashcraft et al. (2010) show that the FHLB system was not enough to ease liquidity stress by the end of 2007. Also, many institutions such as foreign banks and primary dealers were ineligible for FHLB membership. For example, Dexia group, the bank that borrowed the most from DW, was not a member of FHLB. A list of FHLB-member banks is available at https://www.fhfa.gov/DataTools/Downloads/Pages/Federal-Home-Loan-Bank-Member-Data.aspx.

[5]The discount window was once an actual teller window staffed by a lending officer.

window loan. However, banks were reluctant to use the discount window, due to the widely held perception that a stigma was associated with borrowing from the Fed. As advised by Bagehot (1873), a penalty—1 percentage point above the target federal funds rate—was charged on discount window loans, with the goal of encouraging banks to look first to private markets for funding. However, this penalty generated a side effect for banks: Banks would look weak if it became known that they had borrowed from the Fed.

Individual banks' discount window borrowing was kept confidential.[6] However, banks were nervous that investors, in particular money market participants, could guess when they had come to the window by observing banks' behavior and carefully analyzing the Fed's balance sheet figures.[7] For example, the Fed had to disclose weekly the level of discount window borrowing at both the aggregate and district level.[8]

The Fed subsequently made a few changes to discount window policies. In particular, on August 16, 2007, it halved the interest rate penalty on discount window loans. The maturity of loans was also extended from overnight to up to 30 days with an implicit promise of further renewal. Moreover, the Fed tried to persuade some leading banks to borrow at the window, thereby suggesting that borrowing did not equal weakness. On August 17, Timothy Geithner and Donald Kohn hosted a conference call with the Clearing House Association, claiming that the Fed would consider borrowing at the discount window "a sign of strength." Following the call, on August 22, Citi announced that it was borrowing $500 million for 30 days. JPMorgan Chase, Bank of America, and Wachovia subsequently announced that they had borrowed the same amount, increasing the total amount borrowed at the discount window by $2 billion. However, the four big banks—with the borrowing stigma in mind—made it clear in their announcements that they

[6] The Dodd-Frank Act required the disclosure of details of discount window loans after July 2010 on a 2-year lag from the date on which the loan is made.

[7] According to Bernanke (2015), Ron Logue, CEO of State Street, approached the Boston Fed and asked whether the weekly district-by-district reporting of loan totals could be eliminated. The request was turned down for legal reasons and concerns about market-wide confidence.

[8] The stigma associated with borrowing from the government was also significant in the UK. In August 2007, Barclays twice tapped the emergency lending facility offered by the Bank of England. The news came out on Thursday, August 30, when the Bank of England said it had supplied almost 1.6 billion pounds as a lender of last resort, without naming the borrower(s). Journalists and the market scrambled to find out. Barclays declined to confirm that it had used the central bank's standing borrowing facility, but later, it cited a technical breakdown in the clearing system as the reason for the large pile of cash. In its statement, Barclays said, "The Bank of England sterling standby facility is there to facilitate market operations in such circumstances. Had there not been a technical breakdown, this situation would not have occurred." Its share fell 2.5 pounds immediately after the statement, which cast doubt on its 45 billion pound bid to take over the Dutch bank ABN Amro.

Shin (2009) described the bank run on Northern Rock, UK's fifth-largest mortgage lender. In the UK, there was no government deposit insurance, and banks relied on an industry-funded program that only partially protected depositors. On September 13, 2007, the BBC broke the news that Northern Rock had sought the Bank of England's support. The next morning, the Bank of England announced that it would provide emergency liquidity support. It was only *after* the announcement—that is, after the central bank had announced its intervention to support the bank—that retail depositors started queuing outside the branch offices.

did not need the money. Thirty days later, the discount window borrowing fell back to $207 million.[9] On December 11, 2007, the Fed lowered its discount rate to 4.75%, but the attempt was unsuccessful in injecting liquidity to the financial system.

To further relieve stress in the short-term lending market, the Fed established the Term Auction Facility in December 2007. The rule of the auction was as follows. On Monday, banks phoned their local Fed regional banks to submit their bids specifying their interest rate (and loan amount) and to post collaterals. On Tuesday, the Fed secretly informed the winners and publicly announced the stop-out rate (as well as the number of banks receiving loans), determined by the highest losing bid (or the minimum reserve price if the auction was undersubscribed). On Thursday, the Fed released the loans to the banks. Throughout the whole auction process, banks were free to borrow from DW. The following Monday, each regional Fed published total lending from last week; banks may be inferred from these summaries or other channels. The first auction, held on December 17, released $20 billion in the form of 28-day loans. The participation requirement was the same as for DW. The Fed received over $61 billion in bids and released the full $20 billion to 93 institutions. In February 2008, Dick Fuld, CEO of Lehman Brothers, urged the Fed to include Wall Street investment banks in auctions, which would require invoking Section 13(3) to allow the Fed to have authority to lend to non-bank institutions, but the Fed refused. From March to September 2008, the stop-out rate in TAF consistently exceeded the concurrent discount rate. The final auction was held on March 8, 2010, as the auctions had been consistently undersubscribed since 2009.

As shown in Figure 1, TAF was clearly more successful than DW in providing liquidity, and banks were also willing to pay a higher interest rate in TAF than the concurrent discount rate in DW. As Bernanke (2015) acknowledged, before implementing TAF, policy makers were also concerned that the stigma that had kept banks away from the discount window would also be attached to the auctions. The program was implemented as "give it a try and see what happens," but turned out to be quite successful.

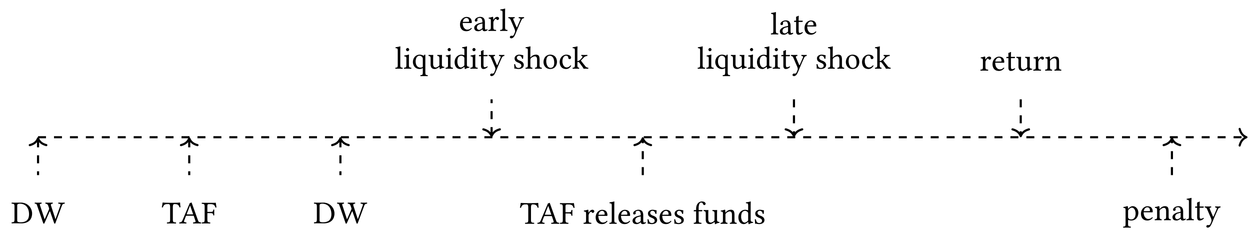
3 The Model

There are n banks. Each bank is endowed with one unit of an illiquid asset. Before the asset pays off, each bank faces a liquidity shock. The probability that a bank may be affected by the liquidity shock is privately known. Banks can borrow from two facilities: the discount window (DW), which provides liquidity before any liquidity shock hits, and the Term Auction Facility (TAF), which provides liquidity after an early liquidity shock may have hit the bank. Borrowing

[9] Records released later show that JPMorgan and Wachovia returned most of the money the next day, whereas Bank of America and Citi—already showing signs of problems—kept the money for a month.

banks may incur a penalty if detected borrowing. The penalty depends on the facility one borrows from and the average financial condition of the other banks that borrow from the same facility. Figure 2 sketches the sequence of events, which we will describe in detail next.

Figure 2: Timeline of the Model



3.1 Preferences, Technology, and Shocks

All banks are risk neutral and do not discount future cash flows. Each bank has one unit of long-term, illiquid assets that will mature at the end of the game. The asset generates cash flows R upon maturity, but nothing if liquidated early. Each bank may be hit with a liquidity shock à la Holmström and Tirole (1988). Let $1-\theta_i \in [0,1]$ be the probability that the liquidity shock affects bank i, where θ_i follows the independently and identically distributed cumulative distribution function (cdf) F with associated probability density function (pdf) f on the support $[0,1]$. Assume that F is log-concave. This assumption is not restrictive, as many standard distributions satisfy it; it is imposed to guarantee equilibrium uniqueness.[10] Throughout the paper, we assume that θ_i is private information and only known by the bank itself. We drop subscript i whenever no confusion arises. Type θ is also referred to as a bank's financial strength.[11] We sometimes refer to a type-θ bank as bank θ.

Conditional on a liquidity shock hitting, let $1-\delta$ be the probability of the shock being early and δ be the probability of the shock being late. Before the potential of an early liquidity shock, each bank has opportunities to borrow. Receiving a loan with interest rate r will help the bank defray the liquidity shock and bring a net benefit of $(1-\theta)R$ at the cost of interest rate r. Bank θ's expected payoff from borrowing a rate-r loan is $(1-\theta)R-r$ if it receives the loan immediately,

[10] Distributions with a log-concave pdf, which implies a log-concave cdf, include normal; exponential; uniform over any convex set; logistic; extreme value; Laplace; chi; Dirichlet if all parameters are no less than 1; gamma if the shape parameter is no less than 1; beta if both shape parameters are no less than 1; Weibull if the shape parameter is no less than 1; and chi-square if the number of degrees of freedom is no less than 2. Distributions with a log-concave cdf but non-log-concave pdf include log-normal; Pareto; Weibull if the shape parameter is smaller than 1; and gamma if the shape parameter is smaller than 1. Student's t, Cauchy, and F distributions are not log-concave for all parameters (Bagnoli and Bergstrom, 2005).

[11] In reality, one can proxy a bank's strength θ by either its reserve of liquid assets or the level of its demandable liabilities that can evaporate in a flash.

and $\delta(1-\theta)R-r$ if an early liquidity shock hits with probability $1-\delta$ before it receives the loan.[12] As it will become clear later on, the specific functional form of the borrowing benefit does not matter for any of our results. What matters is that the benefit is lower if the bank is stronger and if the interest rate is higher.

We describe the two lending facilities in the next subsection.

3.2 Lending Facilities

Any bank is able to borrow from either the discount window or the Term Auction Facility.[13]

3.2.1 Discount Window

The discount window is a facility that offers loans at a fixed interest rate r_D, which is commonly referred to as the discount rate and is exogenously set by the Federal Reserve. Since a bank can always borrow from the discount window with certainty, the net borrowing benefit is $(1-\theta)R-r_D$.

3.2.2 Term Auction Facility

The Term Auction Facility allocates preannounced m units of liquidity through an auction. In the auction, banks that decide to participate simultaneously submit their sealed bids, which are required to be higher than the preannounced minimum bid r_A. After receiving all of the bids, the auctioneer ranks them from highest to lowest. The auction takes a uniform-price format: All winners pay the same interest rate, which is referred to as the stop-out rate s, and losers do not pay anything. If there are fewer bids than the units of liquidity provided, each bidder receives a loan and pays r_A. If there are more bidders than the total liquidity, each of the m highest bidders receives one unit of liquidity by paying the highest *losing* bid. Formally, suppose there are l bidders in total. If $l \leq m$, each bidding bank receives a loan by paying $s = r_A$. If $l > m$, each of the m highest bidding banks receives one unit of liquidity by paying the $m+1^{\text{st}}$ highest bid. The remaining $l-m$ banks do not pay anything and, of course, do not receive any liquidity.

We have modeled the TAF auction as an extended second-price auction: All winning parties pay the highest losing bid. In reality, TAF is closer to an extended first-price auction: All winning banks pay the lowest winning bid. The two auctions generate the same revenue for the auction and the same expected payoffs for the bidders, by the revenue equivalence theorem (Myerson,

[12] According to Bernanke (2015), one main reason to implement the Term Auction Facility was that it would take time to conduct an auction and determine the winning bids, so that borrowers would receive funds with a delay, and thus signal that they were not desperate for cash.

[13] The interbank market essentially froze during the 2007-2008 financial crisis. We will describe in Appendix A.1.10 an extension in which the interbank market is well-functioning and demonstrate that no results change.

1981), and consequently make the same borrowing decisions. We present the analysis with the extended second-price auction because it is notationally simpler, as it is a weakly dominant strategy for each bank to simply bid the maximum interest rate it is willing to pay (Vickrey, 1961).[14]

In reality, winners receive their TAF funds three days after the auction. Recall that there is a probability, $1-\delta$, that an early liquidity shock hits each bank before it receives the funds.[15] Hence, the expected net borrowing benefit of a winner who pays stop-out rate s is $\delta(1-\theta)R-s$. Losers, upon learning the result of the auction, may borrow from the discount window if needed.

3.3 Borrowing Stigma Costs

Banks are assumed to incur a facility-dependent stigma cost. We have argued that a key reason that banks were reluctant to borrow from the lender of last resort is stigma cost. Detected borrowing may signal financial weakness to counterparties, investors, and regulators. Although θ is private information, the public can still draw inferences based on whether the bank has borrowed or which facility the bank has used if it has borrowed. We assume that upon detection, the public can perfectly tell whether the borrowing has been achieved through the discount window or the auction.

We capture the notion of stigma cost in a parsimonious way. We assume that after all of the borrowing is complete, banks that have successfully borrowed may be detected independently. Denote the probability of a bank's being detected borrowing from a particular facility to be p. This penalty can be understood as the combined cost in the bank's deteriorated reputation, a reduced chance to find counterparties, or the cost of a heightened chance of runs and increasing withdrawals by creditors. Let G_D and G_A be the type distributions of the banks that have borrowed from DW and from TAF, respectively. Let the stigma cost depend on the expected financial condition of the bank. For simplicity, we assume linear dependence. That is, for any detected borrowing decision $\omega \in \{D, A\}$,

$$k_\omega \equiv k(G_\omega) = K - \kappa \int_0^1 \theta dG_\omega(\theta).$$

If the dependence is non-linear, our model will in general have multiple equilibrium, but the qualitative features remain unchanged. For the same reason, we assume the degree of stigma is

[14]In contrast, in the first-price auction, banks shade their bids, which depend on the liquidity supply and other participating banks.

[15]Note that if the early liquidity shock has occurred but yet a TAF winning bank is still waiting for funds to be settled, it cannot borrow from the DW. In reality, both DW and TAF loans are collateralized. Thus, if a bank has already pledged its collaterals to TAF, it could no longer borrow from the DW had a liquidity shock hit. This assumption is consistent with the narratives in Bernanke (2015) which emphasizes that winning in TAF is a signal that the bank is likely to survive at least during the three-day settlement period.

low relative to the borrowing benefits: $\kappa \le \min\left\{\frac{\delta R}{p}, \frac{(1-\delta)R}{p}\right\}$. For the rest of the paper, we normalize the stigma cost of a bank believed to have an unconditional average condition to be 0, $k_{\emptyset} \equiv 0$. [16]

3.4 Definition of Equilibrium

In summary, the setting is summarized by the return R, type distribution F of banks, discount rate r_D in DW, number m of units of liquidity auctioned, minimum bid r_A in TAF, and the penalty function $k : G \mapsto \mathbb{R}_+$ attached to different belief distributions of bank's type.

Without loss of generality, we restrict each bank's strategy to be type-symmetric. Each bank θ's strategy can be succinctly described by $\sigma(\theta) = (\sigma_{D1}(\theta), \sigma_A(\theta), \beta(\theta), \sigma_{D2}(\theta))$, where $\sigma_\omega(\theta)$ is the probability of borrowing from $\omega \in \{D1, A, D2\}$, and $\beta(\theta)$ is its bid if it participates in the auction. $D1$ and $D2$ refer to borrowing from the DW before and after the TAF auction, respectively. Given strategies σ, beliefs about the financial situation can be inferred by Bayes' rule; in this case, we say that aggregate strategies σ generate posterior belief system $G = (G_A, G_D)$.

Definition 1. *Borrowing and bidding strategies σ^* and belief system G^* form an equilibrium if (i) each type-θ bank's strategy $\sigma^*(\theta)$ maximizes its expected payoff given belief system G^*, and (ii) the belief system G^* is consistent with banks' aggregate strategies σ^*.*

Clearly, the best (i.e., type-1) bank has no intention of borrowing at all, because it would pay a price, incur a stigma cost, and receive no benefit from borrowing. We assume that the borrowing benefit of the worst (i.e., type-0) bank is sufficiently high that it has a strict incentive to borrow even given the most pessimistic belief about banks that borrow: $R - r_D - k\left(\underline{G}\right) > 0$, where $\underline{G}(\theta) = 1$ for all $\theta > 0$.

3.5 Discussion of Modeling

Our model has built in the three-day delayed settlement feature of TAF. As we will show below, this feature is crucial to generate endogenous separation in banks borrowing from DW and TAF and therefore explain why TAF did not suffer as much stigma as the DW. Indeed, anecdotal evidence is consistent with this argument. For example, on page 157, Bernanke (2015) wrote, "because it takes time to conduct an auction and determine the winning bids, borrowers would receive their funds with a delay, making clear that they were not desperate for cash."

Moreover, Carlson and Rose (2017) mention a similar point. Specifically, the authors wrote, "The TAF had several features designed to minimize stigma. The TAF featured delayed settlement, with funds generally being delivered two days after the auction, so use of the facility would not

[16] This implies $K \equiv \kappa \int_0^1 \theta dF(\theta)$.

signal that the bank had an immediate funding need. The rate at which institutions could borrow at the TAF was determined by auction, so that it was market-determined." Courtois and Ennis (2010) from the Richmond Fed held similar views. In an economic brief, they wrote "A three-day settlement period between the close of the auction and disbursement of funds may have reduced the appearance of a desperate need for cash and thus financial distress."

On the other hand, we would like to stress that the same endogenous separation in bank borrowing from DW and TAF can be generated even without the delayed settlement. In appendix B, we present such a model, in which TAF is only held once every other week, whereas DW is always immediately available. The only qualitative difference between the two models is, whether bids in TAF are monotonic in the bank's type. We decide to use the current model because the remarks by policy makers and bank regulators seem to highlight the particular feature of the three-day delay in settlement.

4 Theoretical Analysis

We present the solution of the benchmark design (only DW) and the solution of the actual design (DW and TAF with a delayed release of funds). Then we discuss four alternative designs (only TAF, DW and TAF with immediate release of funds, two DWs with different releases of funds, and two DWs with different interest rates), and argue why they do not improve liquidity provision. Finally, since it remains unclear how banks' borrowing decisions are detected by the public, we discuss our results under alternative detection technologies.

4.1 Only DW

We start by examining the equilibrium when the government only sets up the discount window. The optimal borrowing decision can be characterized by one threshold: Weaker banks borrow from the discount window, and stronger banks do not borrow at all.

Note (again) that the best bank never borrows, because it knows that a liquidity shock could never affect it and therefore it never needs the liquidity; instead, borrowing incurs an interest cost as well as a stigma cost. The larger is the probability a liquidity shock affects the bank, the more incentive the bank has to borrow. If the assumption $r_D < R - k(\underline{G})$ holds, the worst bank has a strict incentive to borrow from the discount window.

Furthermore, there is a unique equilibrium, which is guaranteed by the assumption of a log-concave cdf F.

Theorem 1 (Equilibrium with only DW). *Suppose only DW is available, i.e., $m = 0$. There exists a unique equilibrium characterizable by a threshold $\theta^{DW} > 0$: Banks $\theta \in [0, \theta^{DW}]$ borrow from DW,* *and banks $\theta \in (\theta^{DW}, 1]$ do not borrow. The equilibrium discount window stigma is*

$$k^{DW}(\theta^{DW}) = K - \kappa \int_0^{\theta^{DW}} \theta dF(\theta)/F(\theta^{DW}),$$

where the threshold θ^{DW} satisfies

$$(1-\theta^{DW})R - r_D - pk^{DW}(\theta^{DW}) = 0. \tag{DW}$$

The discount window provides liquidity to all banks worse than θ^{DW}, but banks better than θ^{DW} do not borrow, because the real economic benefits of borrowing to save the unrealized assets are dwarfed by the interest cost and the stigma cost. The change in the returns, interest rate, and stigma costs will affect liquidity provision as follows.

Proposition 1 (Liquidity Provision with only DW). *The expected total liquidity to be provided with only DW, L^{DW}, is $nF(\theta^{DW})$. It increases as (i) the return R increases, (ii) the discount rate r_D decreases, (iii) the probability of detection p decreases, and (iv) the stigma severity κ decreases.*

How total liquidity depends on the change in the distribution of banks' types is interesting, though: It may decrease when banks face higher liquidity risks overall.

Proposition 2 (Market Condition and Liquidity Provision with only DW). *Total liquidity with only DW, L^{DW} changes ambiguously when the type distribution F shifts in a FOSD way.*

To understand this result, note that there are two effects. First, when the distribution of banks becomes worse, holding the stigma cost unchanged, more banks would choose to borrow from the DW, increasing total liquidity provision. However, there is a second, counterveiling force. When banks worse than θ^{DW} face even higher liquidity risks than before, banks that borrow from DW are perceived to be of even lower quality than before. As a result, the stigma cost rises, and bank θ^{DW}, which was indifferent between borrowing from DW and not, is no longer interested in borrowing. In other words, *the worsened conditions of infra-marginal borrowing banks adversely affect the borrowing decision of the marginal borrowing bank.* As a result of the stigma cost, the discount window may not be effectively providing liquidity when the worst banks become worse.

This result implies that when all banks face higher liquidity risks, banks might borrow less from the DW, because the heightened stigma cost may dominate the increased liquidity demand. The fact that banks were initially reluctant to borrow from DW before the introduction of TAF suggests that the worst banks in the economy were facing higher liquidity risks.

4.2 DW and TAF

We now solve for the equilibrium when both the discount window and Term Auction Facility with delayed release of funds are available. We will first describe a bank's bidding strategy in TAF, followed by its incentives in choosing between DW and TAF. We show that among the banks willing to borrow from DW, stronger ones have more incentives to bid in TAF than to borrow immediately from DW, which is the key force behind the separation of banks into the two facilities in equilibrium.

Lemma 1. *Only banks $\theta \leq \theta_D$ would borrow from the discount window if they have lost in the auction, where $\theta_D = 1 - (r_D + pk_D)/R$, where k_D is the equilibrium stigma cost from discount window borrowing.*

Lemma 2. *Banks $\theta \in (\theta_1, \theta_A]$ participate in the auction, where*

$$\theta_1 = 1 - \frac{r_D - r_A + pk_D - pk_A}{(1-\delta)R}, \quad \theta_A = 1 - \frac{r_A + pk_A}{\delta R}.$$

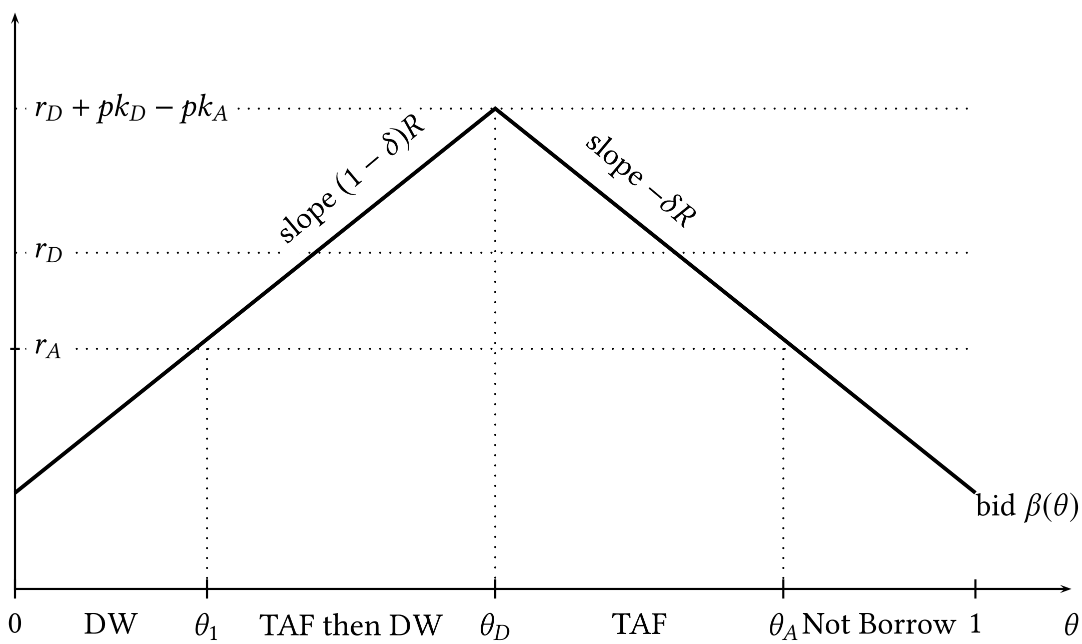
and bid

$$\beta(\theta) = \begin{cases} r_D + pk_D - pk_A - (1-\delta)R(1-\theta) & \text{if } \theta < \theta_D \\ \delta R(1-\theta) - pk_A & \text{if } \theta \geq \theta_D \end{cases}.$$

Note that bids are increasing in θ when $\theta < \theta_D$ and decreasing in θ when $\theta \geq \theta_D$. Intuitively, banks $\theta < \theta_D$ will always borrow; they will still tap the DW after losing in the TAF auction. However, if they win in the auction, chances are that they could be hit by the early liquidity shock before the liquidity funds get settled. In this case, the bank will have to fail. Therefore, delayed settlement is more costly for worse banks that are more likely to be hit by the early liquidity shock. As a result, they bid less. In fact, the bids increase at the rate of $(1-\delta)R$ for banks worse than θ_D. On the other hand, banks $\theta > \theta_D$ will choose not to borrow at all after losing in the TAF auction. Among them, worse banks will bid more since they are more likely to be hit by the late liquidity shock. In fact, the bids decrease at the rate of δR for banks better than θ_D. Therefore, bank θ_D has the highest willingness to pay, and banks further away from θ_D have lower willingness to pay. Winners in the auction are going to be the banks that are the closest to θ_D. For any bank, as long as its willingness to pay is above r_A, it will participate in the auction by submitting a bid higher than r_A. Figure 3 shows the willingness to pay (i.e., bid) in TAF and the optimal facility choice of different banks.

The difference in the stigma cost between the two borrowing facilities could lead to banks bidding more than the discount rate r_D. Specifically, bank θ_D is willing to bid up to $r_D + pk_D - pk_A$ to avoid the stigma cost. As we will show later, $k_D > k_A$ in equilibrium, so that bank θ_D always bids more than r_D. In general, if the realized bank distribution is concentrated around θ_D, the stop-out rate in the TAF auction will be above the concurrent discount rate r_D.

Figure 3: Facility Choice and TAF Bids in the DW-and-TAF Design



Lemma 3 (Equilibrium with Both DW and TAF: High Chance of Early Liquidity Shock). *Suppose DW and TAF are both available, and there is a sufficiently high chance of an early liquidity shock:* $m > 0$, $r_D < R - k(\underline{G})$, *and* $\delta \leq \left[r_A + k(\theta^{DW})\right] / \left[r_D + pk^{DW}(\theta^{DW})\right]$. *In the unique equilibrium, banks* $\theta \in [0, \theta^{DW}]$ *borrow from DW, and banks* $\theta \in (\theta^{DW}, 1]$ *do not borrow.*

Therefore, delaying the release of the funds from TAF for too long will render the program ineffective.

Theorem 2 (Equilibrium with Both DW and TAF: Low Chance of Early Liquidity Shock). *Suppose DW and TAF are both available, and there is a sufficiently low chance of an early liquidity shock:* $m > 0$, $r_D < R - k(\underline{G})$, *and* $\delta > \left[r_A + k(\theta^{DW})\right] / \left[r_D + pk^{DW}(\theta^{DW})\right]$. *Suppose* $\delta R \geq p\kappa$ *and* $(1-\delta)R \geq p\kappa$. *In the unique equilibrium, there exist three thresholds* θ_1, θ_D, *and* θ_A *such that (i) banks* $\theta \in [0, \theta_1]$ *borrow from the discount window before the auction; (ii) banks* $\theta \in (\theta_1, \theta_D]$ *bid in the auction and borrow from the discount window if they lose in the auction; (iii) banks* $\theta \in (\theta_D, \theta_A]$ *bid in the auction and do not borrow if they lose in the auction; and (iv) banks* $\theta \in (\theta_A, 1]$ *neither borrow from the discount window nor participate in the auction.*

Theorem 2 immediately implies:

Corollary 1. *In equilibrium, discount window stigma k_D^* is larger than auction stigma k_A^*.*

Three forces separate banks borrowing in DW and those in TAF. First, the possibility of early liquidation as a result of the delayed release of funds in TAF forces the worst banks to borrow from DW, and deters them from participating in TAF. Second, the exclusion of the worst banks from the auction increases the discount window stigma and decreases the auction stigma, thus further attracting more banks to borrow from TAF. Finally, the competitive nature of the auction attracts banks that would not have borrowed with only DW by offering them a chance to borrow cheaper than the discount rate. TAF serves as a substitute for DW for banks that are close to and worse than θ^{DW}. They substitute into borrowing in the auction from borrowing in the discount window. TAF serves as a complement for DW in terms of total lending. Banks that are close to and better than θ^{DW} substitute into borrowing in the auction from not borrowing.

Liquidity Provision. For total liquidity, consider the expected marginal borrower. The expected marginal borrower is better than θ^{DW}, because they borrow from the auction, and the distribution of the types of banks participating in the auction in the DW and TAF setting first-order stochastically dominates the distribution of the types of banks borrowing from DW.

Proposition 3 (Liquidity Provision with Both DW and TAF). *The combination of TAF and DW provides more total liquidity in expectation than does DW alone: $L^* > L^{DW}$. The liquidity provided by DW decreases when TAF is introduced.*

Even though the combination of TAF and DW provides more liquidity in expectation, it is still possible that the combination of the two facilities can lead to less liquidity provision. In particular, if many realized banks' types are slightly below θ_D, then they will bid in the TAF auction, hoping to take advantage of the low reserve price. The losing banks, which would have borrowed from the discount window if TAF were not available, would choose not to borrow at all.

4.3 Alternative Designs

Instead of the combination of a periodic TAF and the always available DW, could the Fed have improved liquidity provision with only TAF or the addition of (i) only a TAF, (ii) a TAF with immediate release of funds, (iii) a DW with delayed release of funds, or (iv) a DW with a different interest rate? We explore these possibilities next.

4.3.1 Only TAF

Next, we examine the equilibrium when the government only sets up the auction. The equilibrium can also be characterized by one threshold: Weaker banks bid their willingness to pay in the auction, and stronger banks do not participate in the auction and do not borrow at all.

Proposition 4 (Equilibrium with only TAF). *Suppose only TAF is available, i.e., $m > 0$ and $r_D \geq R - k(\underline{G})$. Furthermore, suppose $\delta R - pk(0) > r_A$. There exists a unique equilibrium characterized by a threshold θ^{TAF}: (i) banks $\theta \in \left[0, \theta^{TAF}\right]$ bid $\beta^{TAF}(\theta) = \delta(1-\theta)R - pk_A$ in TAF, and (ii) banks $\theta \in \left(\theta^{TAF}, 1\right]$ do not bid. Equilibrium auction stigma is*

$$k^{TAF}(\theta^{TAF}) = K - \kappa \int_0^{\theta^{TAF}} \int_0^{\theta_s} \frac{\theta dF(\theta)}{F(\theta_s)} h(\theta_s) d\theta_s - \kappa \int_{\theta^{TAF}}^1 \int_0^{\theta^{TAF}} \frac{\theta dF(\theta)}{F(\theta^{TAF})} h(\theta_s) d\theta_s,$$

where $h(\theta_s) = \binom{n}{m} F^{m-1}(\theta_s) f(\theta_s)(1 - F(\theta_s))^{n-m}$ is the pdf of the m^{th} weakest bank, and the threshold θ^{TAF} satisfies

$$\delta R(1 - \theta^{TAF}) - r_A - pk_A^{TAF}(\theta^{TAF}) = 0. \tag{TAF}$$

TAF alone is not necessarily more effective than DW in providing liquidity. If the facilities are used alone, it is unclear which one will provide more liquidity. Therefore, the combination of DW and TAF is needed to increase liquidity provision compared with the DW-only design.

4.3.2 DW and Immediate TAF

Suppose TAF immediately releases funds to winners, and DW is always available. This is essentially a special case of the DW-and-TAF design above, with probability $1 - \delta = 0$ of encountering a liquidity shock between winning the auction and receiving the loan. Essentially, TAF becomes a free option. DW no longer possesses an immediacy advantage, so all of the weakest banks bid in the auction first. All of the banks that would borrow from DW after losing in the auction—banks $\theta \leq \theta_D'$—bid the same rate $r_D + pk_D - pk_A$, and all of the banks that would not borrow from DW after losing in the auction—banks $\theta > \theta_D'$—bid lower rates. In summary, as Figure A1 illustrates, banks $\theta \in [0, \theta_A']$ participate in the auction. Winners receive loans from TAF, and losers with sufficiently weak financial conditions—banks $\theta \leq \theta_D'$—borrow from DW afterward.

Proposition 5 (Equilibrium with DW and Immediate TAF). *Suppose TAF releases funds immediately and DW is always available. In the unique equilibrium, there exist two thresholds θ_D and θ_A' such that banks $\theta \in [0, \theta_D']$ bid in TAF and borrow from DW if they lose in TAF, and banks $\theta \in (\theta_D', \theta_A')$ bid in TAF and do not borrow if they lose in TAF.*

Compared with the original design, this design could provide less liquidity for three reasons. First, the weakest banks—banks $\theta \leq \theta_1$—no longer immediately borrow from DW but participate in the auction, so they take away liquidity from stronger banks that would not have borrowed from DW if they lose in the auction, i.e., banks $\theta \in [\theta_D', \theta_A']$. Second, stronger banks that would not have borrowed from DW are less incentivized to participate in the auction, as their willingness to pay in the auction is lower. Third, there is less separation between DW and TAF banks, so the additional rate that banks are willing to pay in TAF is lower. There is a countervailing force whereby banks borrowing from DW are stronger than banks borrowing from DW in the original case—thus expanding the number of banks willing to borrow from DW.

4.3.3 DW or Immediate TAF

Suppose DW and TAF are simultaneously offered, and banks can *only* choose to borrow from one facility. Then in equilibrium there continues to be a separation between TAF and DW borrowing.

Proposition 6 (Equilibrium with Simultaneous DW and Immediate TAF). *Suppose DW and TAF are simultaneously offered. In the unique equilibrium, there exist two thresholds θ_D and θ_A such that banks $\theta \in [0, \theta_D]$ bid borrow from DW, and banks $\theta \in (\theta_D, \theta_A)$ bid in TAF and do not borrow if they lose in TAF.*

This hypothetical situation highlights the importance of the competitive nature of the auction in the separation of banks, in addition to the channel of delayed release of fund. Intuitively, auction introduces uncertainty in terms of whether a bidding bank is able to borrow at a low rate, lower than its willingness to pay, at the cost of potentially failing to borrow. This cost of not borrowing is lower for stronger banks because their borrowing benefits are lower. Therefore, they are more inclined to participate in the auction and take advantage of the opportunity to borrow when rates are sufficiently low. In this case, borrowing is able to borrowers into two groups, by the so-called "single crossing" condition. It is worthwhile to point out that our result on separation does not depend on the assumption that delaying cost is bigger than weaker banks. To see this, note that a bank's overall payoff has three components that vary with θ. First, a stronger bank has lower borrowing benefits. Second, in equilibrium, a stronger bank submits lower bid and is less likely to win in the auction. However, third, conditional on winning in the auction, it pays less in expectation. When a bank bids optimally, it is indifferent between raising the bid to increase the winning probability and paying more conditional on winning. Therefore, the last two effects cancel out. As a result, the overall effect is the decreasing benefits of borrowing times the probability of winning in the auction, which is increasing in bank's financial weakness.

4.3.4 DWs with Immediate and Delayed Release of Funds

If the delay in releasing funds is important, why doesn't the Fed simply set up a separate discount window D' that releases funds later? The main problem with this separate discount window is that banks are separated into the two facilities only for certain combinations of discount rate r_D and discount factor δ. Let's explore this possibility and see how this design does not inject liquidity as desired. Suppose DW D' charges the interest rate $r_{D'}$.

Proposition 7 (Equilibrium with Two Differentially Timed DWs). *Suppose there are two DWs: D releases funds immediately and D' releases funds with a delay. Suppose $\delta R \geq p\kappa$ and $(1-\delta) \geq p\kappa$. In the unique equilibrium, there exist two thresholds θ_1 and θ_2 such that banks $\theta \in [0, \theta_1]$ borrow from D, and banks $\theta \in [0, \theta_2]$ borrow from D' if they do not borrow from D.*

To guarantee the separation of banks into two facilities, the conditional probability of the early liquidity shock $1-\delta$ can be neither too large nor too small.[17] Otherwise, all banks borrow early (when the chance of an early liquidity shock is high) or borrow late (when the chance of an early liquidity shock is low). The possible inability to separate banks into two facilities may render the design less useful, as the main purpose of such a design is to separate banks to inject liquidity to stronger banks with a delay. The DW-and-TAF design circumvents this potential problem by setting a relatively low minimum required bid to attract banks to participate in the auction and to allow individual bids, so that those willing to pay the most emerge as winners and separate themselves from other banks.

4.3.5 Cheap and Expensive DWs

Setting up two DWs with different interest rates does not provide more liquidity. It provides less liquidity than simply setting up the cheaper DW.

Proposition 8 (Equilibrium with Two Differentially Priced DWs). *Suppose there are two DWs: D charges interest rate r_D and DW D' charges interest rate $r_{D'} > r_D$. In equilibrium, banks are indifferent between the two DWs. The design offers less liquidity than setting up only the cheaper DW D.*

Bank θ gets $(1-\theta)R - r_D - pk_D$ from D, and gets $(1-\theta)R - r_{D'} - pk_{D'}$ from D'. All banks are indifferent between the two facilities if $r_D + pk_D^* = r_{D'} + pk_{D'}^*$. Therefore, the average bank borrowing from D is worse than the average bank borrowing from D', and consequently the average bank of all borrowing banks is better than the average bank borrowing from D. In equilibrium, it must be that all banks are indifferent between the two DWs; otherwise, they would borrow from the one with strictly lower total costs, including borrowing and the stigma cost.

4.4 Alternative Detection Technologies

In this subsection, we discuss how alternative assumptions on detection technology could affect our equilibrium results.

[17] The specific expression is $\frac{r_D + pk_D^*}{R}\left[1 - \frac{r_{D'} + pk_{D'}^*}{r_D + pk_D^*}\right] < 1 - \delta < 1 - \frac{r_{D'} + pk_{D'}^*}{r_D + pk_D^*}$.

Pooled DW and TAF Detection. Suppose borrowing from DW faces the same stigma cost and the same probability of detection as borrowing from TAF. In other words, the public can only tell whether a bank has borrowed from the Fed but not whether the borrowing was from DW or TAF. The equilibrium borrowing behavior is qualitatively the same as characterized in Section 4.2: Weaker banks immediately borrow from the DW, and stronger banks first bid in the auction. However, no bank would be willing to bid more than the discount rate, because the auction would not have a lower stigma cost than the discount window, as the borrowing cannot be distinguished. This predicted borrowing behavior—bids being capped at the concurrent discount rate—is against the observed pattern that in more than a third of the auctions each winning bank was paying more than the discount rate and in more than two thirds of the auctions some banks were bidding more than the discount rate.

Separate Early and Late DW Detection. Suppose non-auction-week DW and auction-week DW borrowing can be separately detected, as the Fed publishes its balance sheets weekly. Such finer detection technology could further deter banks from borrowing immediately from the early (i.e., non-auction-week) DW, as the stigma cost of early DW increases. It would encourage more banks to bid in the auction, as the auction is a substitute to the early DW. It would also encourage more banks to borrow from the late DW, because the weakest banks that borrow in the early DW are not associated with the late DW stigma anymore. A consequence of a lower late DW stigma cost is lower bids submitted by banks in the TAF; nonetheless, the late DW stigma cost is still higher than the TAF stigma cost, so some banks still bid higher than the concurrent discount rate.

Separate TAF Participation and Borrowing Stigma. Suppose participating in but not borrowing from the TAF also incurs a stigma cost. This additional stigma cost would decrease the participation in the auction—as some stronger banks choose not to try in the auction—and as a consequence may reduce aggregate borrowing, as the auction may end up undersubscribed. Safeguarding and not disclosing the participation list would encourage borrowing.

Public Stop-Out Rate. In reality the Fed announces the stop-out rate after each auction. However, whether or not the actual market-clearing borrowing rate is announced does not affect banks' bidding decisions ex ante. Banks rationally and correctly expect the distribution of stop-out rates in equilibrium, and make appropriate borrowing and bidding decisions accordingly. The late DW borrowing decision may be affected by the disclosed stop-out rate, as opposed to an expected stop-out rate when it is not publicly announced. The actual borrowing from the post-auction DW may change due to the disclosure policy, but the expected aggregate borrowing is unaffected by the disclosure policy.

Different Detection Probabilities. Suppose the probability of being detected borrowing in the DW is different from being detected in the TAF. For example, the equilibrium probability of being detected can depend on the number of banks that actually participate into the liquidity provision programs. It is straightforward to show that Theorem 2 continues to hold. Mathematically, the terms involving stigma costs all cancel out in the single-crossing conditions. Intuitively, heterogeneous detection probability does not affect the relative trade-off between using DW and TAF across banks with different financial strength θ.

5 Conclusion

In this paper, we investigate how the Term Auction Facility mitigates the stigma associated with borrowing from the discount window when it was used in accordance with the discount window. We constructed an auction model with endogenous participation and showed that optimal auction bidding strategies that internalized any stigma associated with the auction increased participation and consequently mitigated the borrowing stigma.

The theoretical predictions we derived from the model are consistent with empirical observations. First, banks with strong financial health are reluctant to borrow from the discount window due to their reluctance to associate themselves with banks worse than them (Akerlof, 1970). Second, when both DW and TAF are available, weaker banks borrow from DW, and stronger ones participate in TAF. Of those that lose in the auction, weaker ones borrow from DW. Third, we show that the introduction of TAF may or may not expand the set of banks that obtain liquidity; it is the combination of TAF and DW that mitigates borrowing stigma and increases liquidity provision. Lastly, the stop-out rate of TAF may be higher or lower than the discount rate.

Our theoretical and empirical analyses together provide a better understanding of the role of a special monetary program, the Term Auction Facility, played during the financial crisis, and suggest how to better design lending-of-last-resort programs in the future. After comparing with several alternative designs, we conclude that the Fed's design of DW and delayed-funds-release TAF achieved its intended goal of lowering the borrowing stigma by separating the banks into distinct groups, encouraging participation by stronger banks, and providing more liquidity to the economy. The improvement over the current design is a quantitative matter of setting the more appropriate discount rate, minimum bid, and number of days to delay the release of funds. We leave this important quantitative exercise to future researches.

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A Appendix

A.1 Omitted Proofs

A.1.1 Proof of Theorem 1

Bank θ prefers borrowing from DW over not borrowing if and only if

$$u_D(\theta) = (1-\theta)R - r_D - pk_D - (1-p)k_\emptyset \geq 0.$$

Since we normalize $k_\emptyset$ to be 0, we can simplify the condition to

$$(1-\theta)R - r_D - pk_D \geq 0.$$

Clearly, the gain from borrowing from DW is strictly decreasing in θ. Therefore, for any given k_D, bank θ borrows from the discount window if and only if

$$\theta \leq 1 - \frac{r_D + pk_D}{R}.$$

Therefore, there exists a threshold—let's denote it by θ^{DW}—such that bank θ^{DW} is indifferent between borrowing from DW and not borrowing; banks worse than θ^{DW} borrow from DW; and banks better than θ^{DW} do not borrow. In equilibrium, k_D depends on θ^{DW}:

$$k_D = K - \kappa \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F\left(\theta^{DW}\right)}.$$

Plugging equilibrium k_D into the equilibrium condition above, we see that θ^{DW} is determined by

$$\left(1-\theta^{DW}\right)R - r_D - p\left[K - \kappa \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F\left(\theta^{DW}\right)}\right] = 0,$$

which is rearranged as

$$R - r_D - \theta^{DW}R + p\kappa \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F\left(\theta^{DW}\right)} = 0. \tag{DW}$$

The terms involving θ^{DW} can be rearranged as

$$-\theta^{DW}(R - p\kappa) - p\kappa\left[\theta^{DW} - \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F\left(\theta^{DW}\right)}\right].$$

The first term, $-\theta^{DW}(R-p\kappa)$, is decreasing in θ^{DW}, because $R>1>p\kappa$. For the second term, $-p\kappa\left[\theta^{DW}-\int_0^{\theta^{DW}}\frac{\theta dF(\theta)}{F(\theta^{DW})}\right]$, the expression in the square brackets is *mean advantage over inferiors*, as Bagnoli and Bergstrom (2005) name it. Because the distribution is assumed to be log-concave, by Bagnoli and Bergstrom (2005, Theorem 5), the term in the square brackets is weakly increasing in θ^{DW}, so the second term is weakly decreasing in θ^{DW}.In summary, the left-hand side of Equation (DW) is strictly decreasing in θ^{DW}.

To show the existence of a unique solution to Equation (DW), it remains to show that its left-hand side is positive for $\theta^{DW}=0$ and negative for $\theta^{DW}=1$. When $\theta^{DW}=0$, the left-hand side is

$$R-r_D-p\kappa\int_0^1\theta dF(\theta)=R-r_D-pK>0,$$

where the equality follows from the normalization of $K=\kappa\int_0^1\theta dF(\theta)$, and the inequality comes from the assumption that $R>r_D+pK$. When $\theta^{DW}=1$, the left-hand side is

$$-r_D+p\kappa\int_0^1\theta dF(\theta)=-r_D+pK<0,$$

where the inequality follows from $r_D>1>pK$. Hence, there is a unique equilibrium. □

A.1.2 Proof of Proposition 1

The left-hand side of Equation (DW) strictly shifts up when (i) R increases, (ii) r_D decreases, (iii) p increases, or (iv) κ increases. Since the left-hand side of Equation (DW) is strictly decreasing in θ^{DW}, the equilibrium θ^{DW} increases as a result of any of the changes (i)-(iv). □

A.1.3 Proof of Proposition 2

The left-hand side of Equation (DW) strictly shifts down when F for $\theta<\theta^{DW}$ shifts in a first-order stochastically dominated way, because the only term affected by the change, $\int_0^{\theta_{DW}}\frac{\theta dF(\theta)}{F(\theta^{DW})}$, strictly decreases. Hence, the new threshold $\widetilde{\theta}^{DW}$ is strictly smaller than θ^{DW}. Total liquidity expected to be provided, $\widetilde{L}^{DW}=nF(\widetilde{\theta}^{DW})$, is also smaller than $L^{DW}=nF(\theta^{DW})$. □

A.1.4 Proof of Theorem 4

Bank θ bids (gross) interest rate $\beta(\theta)$ such that its payoff from winning in the auction with this rate is the same as the payoff from not borrowing,

$$\delta(1-\theta)R-\beta(\theta)-pk_A=0.$$

In other words, the bid is the bank's maximum willingness to pay (WTP) for the loan:

$$\beta(\theta) = \delta(1-\theta)R - (pk_A).$$

Note that the bid is strictly decreasing in θ. Therefore, worse banks are willing to bid higher interest rates. Consequently, given any stigma cost k_A, there exists a threshold bank θ^{TAF} such that banks worse than θ^{TAF} are willing to bid more than the minimum bid r_A, and all banks better than θ^{TAF} are not willing to bid more than r_A. Bank θ^{TAF} bids exactly the prespecified minimum bid r_A:

$$\beta(\theta^{TAF}) = r_A \Rightarrow \theta^{TAF} = 1 - \frac{pk_A + r_A}{\delta R}.$$

Now, consider the equilibrium stigma cost:

$$k_A(\theta^{TAF}) = K - \kappa \int_0^{\theta^{TAF}} \int_0^{\theta_s} \frac{\theta dF(\theta)}{F(\theta_s)} dH(\theta_s) - \kappa \int_{\theta^{TAF}}^1 \int_0^{\theta^{TAF}} \frac{\theta dF(\theta)}{F(\theta^{TAF})} dH(\theta_s),$$

where $H(\theta_s)$ is the distribution of the m^{th} weakest bank of all; that is, $H(\theta_s) = \int_0^{\theta_s} h(\theta)d\theta$, where

$$h(\theta) = \binom{n}{m} F^{m-1}(\theta) f(\theta)(1-F(\theta))^{n-m}.$$

Rearranging the expression for θ^{TAF}, we have

$$[\delta R - r_A] - [\delta R \theta^{TAF} + pk_A(\theta^{TAF})] = 0. \tag{TAF}$$

The terms in the first pair of square brackets do not depend on θ^{TAF}. The terms in the second pair of square brackets can be expanded and rearranged as

$$(\delta R - p\kappa)\theta^{TAF} + pK + p\kappa \int_0^{\theta^{TAF}} \int_{\theta_s}^{\theta^{TAF}} \frac{\theta dF(\theta)}{F(\theta_s)} dH(\theta_s) + p\kappa \left[\theta^{TAF} - \int_0^{\theta^{TAF}} \frac{\theta dF(\theta)}{F(\theta^{TAF})} dH(\theta_s) \right].$$

The square bracket in the integral is increasing in θ^{TAF}, and the second term is also increasing in θ because each term in the integral (Bagnoli and Bergstrom, 2005, mean advantage over inferiors) is positive, as long as $\delta R > p\kappa$. The term in the third pair of square brackets in Equation (TAF) is decreasing in θ^{TAF}. Therefore, the left-hand side of Equation (TAF) is strictly decreasing in θ^{TAF}.

To show the existence of a unique solution to Equation TAF, it remains to show that its left-hand side is positive for $\theta^{TAF} = 0$ and negative for $\theta^{TAF} = 1$. When $\theta^{TAF} = 0$, its left-hand side is $\delta R - r_A - pk(0) > 0$, and when $\theta^{TAF} = 1$, its left-hand side equals $-r_A < 0$. Hence, there is a unique equilibrium. □

A.1.5 Proof of Lemma 1

Bank θ would borrow in DW if and only if $(1-\theta)R - r_D - pk_D \geq 0$, which simplifies to $\theta \geq \theta_D \equiv 1 - (r_D + pk_D)/R$. □

A.1.6 Proof of Lemma 2

Banks that could still get a positive payoff from borrowing in the discount window if they lose in the auction are willing to pay up to $\beta^D(\theta)$:

$$R(1-\theta) - r_D - pk_D = \delta R(1-\theta) - c - \beta^D(\theta) - pk_A.$$

Rearrange:

$$\beta^D(\theta) = r_D + pk_D - pk_A - (1-\delta)R(1-\theta) - c.$$

Note that the bid is increasing in θ, for $\theta < \theta_D$.

On the other hand, for banks that could not get a positive payoff from borrowing in the discount window, they are willing to pay up to $\beta^N(\theta)$:

$$0 = \delta R(1-\theta) - c - \beta^N(\theta) - pk_A.$$

Rearrange:

$$\beta^N(\theta) = \delta R(1-\theta) - c - pk_A.$$

Note that the bid is decreasing in θ, for $\theta > \theta_D$.

Altogether, the maximum WTP in the auction is

$$\beta(\theta) = \begin{cases} \beta^D(\theta) = r_D + pk_D - pk_A - (1-\delta)R(1-\theta) - c & \text{if } \theta < \theta_D, \\ \beta^N(\theta) = \delta R(1-\theta) - c - pk_A & \text{if } \theta \geq \theta_D. \end{cases}$$

Bank θ participates in the auction if its maximum WTP in the auction is greater than the minimum required bid r_A—that is, if the bank's type is between θ_1 and θ_A, where $\beta^D(\theta_1) = r_A$ and $\beta^N(\theta_A) = r_A$. Solving for those conditions and simplifying, we get

$$\theta_1 = 1 - \frac{r_D - r_A + pk_D - pk_A - c}{(1-\delta)R}, \quad \text{and} \quad \theta_A = 1 - \frac{r_A + c + pk_A}{\delta R}.$$

□

A.1.7 Proof of Lemma 3

By Lemma 1, banks borrow from the discount window if and only if

$$\theta \leq \theta_D = 1 - \frac{r_D + pk_D}{R}.$$

Of these banks, some are willing to wait for the auction if and only if

$$\theta > \theta_1 = 1 - \frac{r_D - r_A + pk_D - pk_A}{(1-\delta)R}.$$

Banks that borrow from the discount window would not participate in the auction if and only if $\theta_1 \geq \theta_D$, which is

$$1 - \frac{r_D - r_A + pk_D - pk_A}{(1-\delta)R} \geq 1 - \frac{r_D + pk_D}{R}.$$

The inequality can be simplified to

$$r_D + pk_D \geq \frac{r_D - r_A + pk_D - pk_A}{1-\delta},$$

which further simplifies to

$$r_D + pk_D - \delta(r_D + pk_D) \geq r_D + pk_D - r_A - pk_A,$$

which can be further simplified to $\delta \leq (r_A + pk_A)/(r_D + pk_D)$. Hence, in equilibrium, if $\delta \leq r_A/(r_D + pk_D^*)$, banks that would borrow from the discount window if they lost in the auction would not participate in the auction in the first place.

Knowing the condition derived above, we can directly verify that banks $\theta \in [0, \theta^{DW}]$ borrowing from the discount window immediately is part of an equilibrium. When banks $\theta \in [0, \theta^{DW}]$ borrow from the discount window, the equilibrium discount window stigma is $k_D^* = k_D^{DW}(\theta^{DW})$, and since we have the assumption $\delta \leq r_A/\left[r_D + pk_D^{DW}(\theta^{DW})\right]$, by the condition derived above, we have that no discount window bank would be willing to participate in the auction. Furthermore, since bank θ^{DW}, which should have the highest WTP in the auction, is not willing to participate in the auction, no bank will participate in the auction. □

A.1.8 Proof of Theorem 2

An equilibrium is determined by three thresholds, θ_1, θ_D, and θ_A, where

$$\theta_D = 1 - \frac{r_D + pk_D}{R},$$

$$\theta_1 = 1 - \frac{r_D + pk_D - r_A - pk_A}{(1-\delta)R},$$

$$\theta_A = 1 - \frac{r_A + pk_A}{\delta R}.$$

Rearranging the three equations, we have

$$(1-\theta_D)R - r_D - pk_D = 0, \tag{DW2}$$

$$(1-\theta_1)(1-\delta)R - r_D - pk_D = r_A + pk_A, \tag{DW1}$$

$$(1-\theta_A)\delta R = r_A + pk_A. \tag{A}$$

The stigma costs are

$$k_D(\theta_D, \theta_1, \theta_A) = K - \kappa \frac{\int_0^{\theta_1} \theta dF(\theta) + \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{\theta dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A)}{F(\theta_1) + \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A)},$$

and

$$k_A(\theta_1, \theta_A) = K - \kappa \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_{s2}(s)} \frac{\theta dF(\theta)}{F(\theta_{s2}(s)) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A),$$

where $[\theta_{s1}(s), \theta_{s2}(s)]$ is the interval of types of banks winning the auction when s is the stop-out rate, and $H(s|\theta_1, \theta_2)$ is the distribution of the stop-out rate.

Plugging $k_A(\theta_1, \theta_A)$ into Equation (A), we have

$$\delta R - r_A - pK - (\delta R - p\kappa)\theta_A - p\kappa \left[\theta_A - \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_{s2}(s)} \frac{\theta dF(\theta)}{F(\theta_{s2}(s)) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A) \right] = 0.$$

The expression in the square brackets is mean advantage over inferiors for an order statistics distribution. Then by Chen et al. (2009), the order statistics distribution is log-concave. Hence, by Bagnoli and Bergstrom (2005, Theorem 5), the expression in the square brackets is increasing in θ_A. If $\delta R > p\kappa$, then the left-hand side of the equation above is strictly decreasing in θ_A. For each fixed θ_1, there is a unique θ_A that satisfies the equation. Let $\widetilde{\theta}_A(\theta_1)$ represent this function, and note that $\widetilde{\theta}_A(\theta_1)$ is strictly increasing in θ_1.

Plugging k_D into Equation (DW2) and rearranging, we have

$$R - r_D - pK - \theta_D R + p\kappa \frac{1}{\Delta} \int_0^{\theta_1} \theta dF(\theta) - p\kappa \frac{1}{\Delta} \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{\theta dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \widetilde{\theta}_A(\theta_1)) = 0,$$

where $\Delta = F(\theta_1) + \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A)$ represents the denominator in the fractional part of the expression of k_D. The terms that include θ_D can be rearranged as

$$-\theta_D(R-p\kappa)-p\kappa\left[\theta_D-\frac{\int_0^{\theta_1}\theta dF(\theta)+\int_{r_A}^{\infty}\int_{\theta_{s1}(s)}^{\theta_D}\frac{\theta dF(\theta)}{F(\theta_D)-F(\theta_{s1}(s))}dH(s|\theta_1,\widetilde{\theta}_A(\theta_1))}{F(\theta_1)+\int_{r_A}^{\infty}\int_{\theta_{s1}(s)}^{\theta_D}\frac{dF(\theta)}{F(\theta_D)-F(\theta_{s1}(s))}dH(s|\theta_1,\widetilde{\theta}_A(\theta_1))}\right].$$

Again, the expression in the square brackets is mean advantage over inferiors for a truncated order statistics distribution, which continues to be log-concave, so it is increasing in θ_D. Therefore, for each θ_1, there is a unique θ_D that satisfies Equation (DW2). Let $\widetilde{\theta}_D(\theta_1)$ represent this function.

Plugging $\widetilde{\theta}_D(\theta_1)$, $\widetilde{\theta}_A(\theta_1)$, k_D, and k_A into Equation (DW1), we have

$$-r_D-r_A+(1-\delta)R-\theta_1(1-\delta)R-pk_D(\theta_1,\widetilde{\theta}_D(\theta_1),\widetilde{\theta}_A(\theta_1))-pk_A(\theta_1,\widetilde{\theta}_A(\theta_1))=0.$$

Using the same trick as before, we extract and rearrange all the terms that include θ_1:

$$\begin{aligned}&-\theta_1[(1-\delta)R-p\kappa]-pk_A(\theta_1,\widetilde{\theta}_A(\theta_1))\\&-p\kappa\left[\theta_1-\frac{\int_0^{\theta_1}\theta dF(\theta)+\int_{r_A}^{\infty}\int_{\theta_{s1}(s)}^{\theta_D}\frac{\theta dF(\theta)}{F(\theta_D)-F(\theta_{s1}(s))}dH(s|\theta_1,\theta_A)}{F(\theta_1)+\int_{r_A}^{\infty}\int_{\theta_{s1}(s)}^{\theta_D}\frac{dF(\theta)}{F(\theta_D)-F(\theta_{s1}(s))}dH(s|\theta_1,\theta_A)}\right].\end{aligned}$$

The expression is strictly decreasing for the same reason as in the previous argument, as long as $(1-\delta)R>p\kappa$. Therefore, there is a unique θ_1. □

A.1.9 Proof of Proposition 3

From the previous proof we see that the equilibrium condition for the banks that borrow from DW in the DW-and-TAF setting is

$$(1-\theta_D^*)R-r_D-pk_D^*=0.$$

Compare this condition to the equilibrium condition for banks that borrow from DW in the DW-only setting:

$$(1-\theta^{DW})R-r_D-pk^{DW}=0.$$

As long as $k^{DW}<k_D^*$, fewer banks are willing to borrow from DW in the DW-and-TAF setting. This condition indeed holds, because the strongest banks of the banks worse than θ_D win in the auction.

For total liquidity, consider the expected marginal borrower. The expected marginal borrower is better than θ^{DW}, because they borrow from the auction, and in the DW-and-TAF setting, the type distribution of banks winning in TAF first-order stochastically dominates that of banks borrowing form DW. □

A.1.10 Interbank Market

Suppose banks can borrow from the interbank market at a rate of $r > r_D$. The borrowing benefit is then $(1-\theta)R - r - p_I k_I$, where now I denotes borrowing from the interbank market. A bank borrows from the interbank market if and only if $(1-\theta)R - r > (1-\theta)R - r_D - pk_D$. The condition is simplified to $r < r_D + pk_D$. Hence, banks are willing to pay a higher interest rate in the interbank market to avoid the discount window, consistent with the empirical evidence. But for sufficiently large r, interbank market borrowing is not optimal even if it is available. □

A.1.11 Proof of Proposition 5

Banks $\theta \le \theta_D$ prefer borrowing from DW to not borrowing, where $\theta_D = 1 - (r_D + pk_D)/R$, as characterized in the proof of Proposition 1. Banks $\theta \le \theta_D$ bid $\beta(\theta) = r_D + pk_D - pk_A$, which follows from $(1-\theta)R - r_D - pk_D = (1-\theta)R - \beta(\theta) - pk_A$. If they participate in the auction, banks $\theta > \theta_D$ would bid $\beta(\theta) = (1-\theta)R - pk_A$, which follows from $(1-\theta)R - \beta(\theta) - pk_A = 0$. Only banks θ such that $\beta(\theta) \ge r_A$ participate in the auction. That is, only banks $\theta \le \theta_A$ participate in the auction, where $\theta_A = 1 - (r_A + pk_A)/R$ is derived from $(1-\theta_A)R - pk_A = r_A$.

Fix cutoffs θ_D and θ_A. The stigma cost of borrowing from DW is

$$k_D(\theta_D) = K - p\kappa \int_0^{\theta_D} \theta \frac{dF(\theta)}{F(\theta_D)}.$$

The stigma cost $k_A(\theta_D, \theta_A)$ of borrowing from TAF is lower, as some banks stronger than θ_D may obtain liquidity from TAF:

$$K - p\kappa \left[\int_0^{\theta_D} \int_0^{\theta_D} \frac{\theta dF(\theta)}{F(\theta_D)} dH(\theta_s) + \int_{\theta_D}^{\theta_A} \int_0^{\theta'} \frac{\theta dF(\theta)}{F(\theta')} dH(\theta_s) + \int_{\theta_A}^{1} \int_0^{\theta_A} \frac{\theta dF(\theta)}{F(\theta_A)} dH(\theta_s) \right],$$

where $H(\theta_s)$ is the distribution of the m^{th} weakest bank, that is, $H(\theta_s) = \int_0^{\theta_s} h(\theta) d\theta$, where

$$h(\theta_s) = \binom{n}{m} F^{m-1}(\theta_s) f(\theta_s) [1 - F(\theta_s)]^{n-m}.$$

In equilibrium, θ_D^* is uniquely pinned down by $R(1-\theta) - r_D - pk_D(\theta_D) = 0$, and θ_A^* is uniquely pinned down by $R(1-\theta_A) - r_A - pk_A(\theta_A, \theta_D^*) = 0$. The uniqueness follows from the monotonicity of the left-hand side of the two equations, which is argued in previous proofs. □

A.1.12 Proof of Proposition 6

A type-θ bank who would participate in the auction would bid $\beta(\theta) = (1-\theta)R - pk_A$, which is a decreasing function of θ; that is, worse banks would bid higher. Hence, the probability of winning, $w(\theta)$, is decreasing in θ; that is, worse banks are more likely to win in the auction. The payoff of bank θ in the auction would be $u_A(\theta) = \int_s^{\beta(\theta)} ((1-\theta)R - s - pk_A)h(s)ds$, where s is the realized stop-out rate and $h(s)$ is the probability density of s. Alternatively, bank θ would get a payoff of $u_D(\theta) = (1-\theta)R - r_D - pk_D$ from borrowing in DW. The slope of $u_D(\theta)$ with respect to θ is $-R$, and the slope of $u_A(\theta)$ is $-R\int_s^{\beta(\theta)} h(s)ds$, negative but greater than $-R$. Hence, there is a single crossing in $u_D(\theta)$ and $u_A(\theta)$ such that there exists θ_D such that for any $\theta \leq \theta_D$, $u_D(\theta) \geq u_A(\theta)$, and for any $\theta > \theta_D$, $u_D(\theta) < u_A(\theta)$. Banks $\theta < \theta_A$ would be willing to participate in the auction, where $(1-\theta_A)R - pk_A = 0$, which simplifies to $\theta_A = 1 - pk_A/R$.

A.1.13 Proof of Proposition 7

Bank θ, by borrowing in DW D, gets $u_D(\theta) = (1-\theta)R - r_D - pk_D$, and by borrowing in DW D' gets $u_{D'}(\theta) = \delta(1-\theta)R - r_{D'} - pk_{D'}$. Therefore, bank θ prefers borrowing from D to borrowing from D' if and only if

$$u_D(\theta) = (1-\theta)R - r_D - pk_D \geq u_{D'}(\theta) = \delta(1-\theta)R - r_{D'} - pk_{D'},$$

which is rearranged as

$$(1-\delta)(1-\theta)R - (r_D - r_{D'}) - (pk_D - pk_{D'}) \geq 0.$$

Hence, banks $\theta \leq \theta_1$ borrow from DW D, where

$$\theta_1 = 1 - \frac{(r_D - r_{D'}) + (pk_D - pk_{D'})}{(1-\delta)R}.$$

Furthermore, bank θ prefers borrowing from DW D' to not borrowing if and only if

$$u_{D'}(\theta) = \delta(1-\theta)R - r_{D'} - pk_{D'} \geq 0,$$

which is rearranged as

$$\theta \leq \theta_2 \equiv 1 - \frac{r_{D'} + pk_{D'}}{\delta R}.$$

To have banks borrowing from DW D', we must have $\theta_2 > \theta_1$, that is,

$$1 - \frac{r_{D'} + pk_{D'}}{\delta R} > 1 - \frac{(r_D - r_{D'}) + (pk_D - pk_{D'})}{(1-\delta)R},$$

$$\frac{r_{D'}+pk_{D'}}{\delta} < \frac{(r_D - r_{D'}) + (pk_D - pk_{D'})}{(1-\delta)},$$

which is rearranged as

$$\delta(r_D + pk_D) > r_{D'} + pk_{D'}.$$

Since banks $\theta \in [0, \theta_1]$ borrow from DW D, and banks $\theta \in (\theta_1, \theta_2]$ borrow from DW D', the stigma costs are

$$k_D(\theta_1) = K - \kappa \int_0^{\theta_1} \frac{\theta dF(\theta)}{F(\theta_1)} \quad \text{and} \quad k_{D'}(\theta_1, \theta_2) = K - \kappa \int_{\theta_1}^{\theta_2} \frac{\theta dF(\theta)}{F(\theta_2) - F(\theta_1)}.$$

Equilibrium θ_1 and θ_2 satisfy

$$(1-\delta)(1-\theta_1)R - (r_D - r_{D'}) - pk_D(\theta_1) + pk_{D'}(\theta_1, \theta_2) = 0 \quad \text{and} \tag{D1}$$

$$\delta(1-\theta_2)R - r_{D'} - pk_{D'}(\theta_1, \theta_2) = 0. \tag{D2}$$

Plug $k_D(\theta_1)$ into and rearrange the left-hand side of Equation (D1):

$$(1-\delta)R - (r_D - r_{D'}) - pK - [(1-\delta)R - p\kappa]\theta_1 - p\kappa\left[\theta_1 - \int_0^{\theta_1} \frac{\theta dF(\theta)}{F(\theta_1)}\right] + pk_{D'}(\theta_1, \theta_2).$$

The expression is strictly decreasing in θ_1 as long as $(1-\delta R) > p\kappa$. In addition, the expression is strictly decreasing in θ_2. Therefore, given any θ_2, there is a unique $\theta_1(\theta_2)$ that satisfies Equation (D1), and $\theta_1(\theta_2)$ is strictly decreasing in θ_2. Plug $k_{D'}(\theta_1, \theta_2)$ into and rearrange Equation (D2):

$$\delta R - r_{D'} - pK - (\delta R - p\kappa)\theta_2 - p\kappa\left[\theta_2 - \int_{\theta_1(\theta_2)}^{\theta_2} \frac{\theta dF(\theta)}{F(\theta_2) - F(\theta_1(\theta_2))}\right] = 0. \tag{D2'}$$

Consider the derivative of $\theta_2 - \int_{\theta_1(\theta_2)}^{\theta_2} \frac{\theta dF(\theta)}{F(\theta_2) - F(\theta_1(\theta_2))}$ with respect to θ_2. Fixing $\theta_1(\theta_2)$, the derivative is positive, because the expression is a mean advantage over inferiors for the truncated cdf $F(\theta)$ between $\theta_1(\theta_2)$ and θ_2. The derivative with respect to $\theta_1(\theta_2)$ is decreasing, but $\theta_1'(\theta_2) < 0$. Hence, the derivative overall is increasing. Therefore, the left-hand side of Equation (D2) is strictly decreasing in θ_2, as long as $\delta R > p\kappa$, and there is a unique θ_2 that satisfies Equation (D2'). □

A.1.14 Proof of Proposition 8

Bank θ gets $(1-\theta)R - r_D - pk_D$ from D, and gets $(1-\theta)R - r_{D'} - pk_{D'}$ from D'. All banks are indifferent between the two facilities if $r_D + pk_D^* = r_{D'} + pk_{D'}^*$. Therefore, the average bank borrowing from D is worse than the average bank borrowing from D', and consequently the average bank of all borrowing banks is better than the average bank borrowing from D. The marginal bank θ^* satisfies $(1-\theta^*)R - r_D - pk_D^* = 0$. However, if the average bank of all banks $\theta \in [0, \theta^*]$ is better than the average bank borrowing from D, $(1-\theta^*)R - r_D - p\left[K - \kappa \int_0^{\theta^*} \frac{\theta dF(\theta)}{F(\theta^*)}\right] > 0$. Some banks $\theta > \theta^*$ would have borrowed if only D with interest rate $r_D < r_{D'}$ were offered. □

Figure A1: Facility Choice and TAF Bids in the DW-and-Immediate-TAF Design

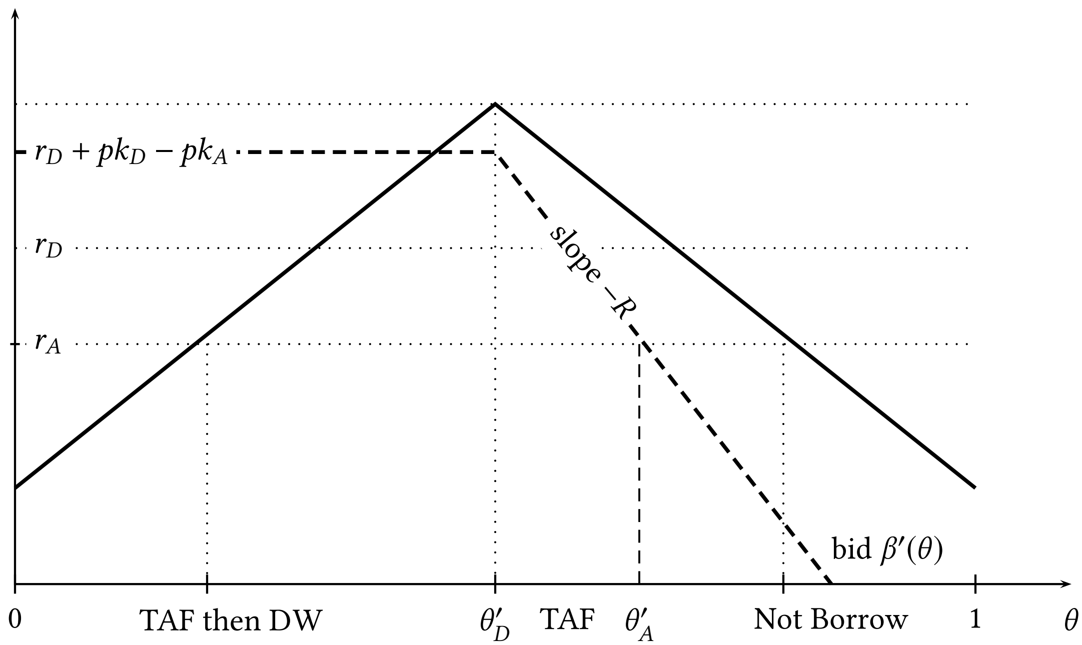

Table 1: Announcement and Operation Dates of Credit Guarantee Programs

This table lists the announcement and operational dates of all the credit guarantee programs carried out by G7 countries and others that followed. The data are collected by Yale Program on Financial Stability (2019).

Country	Announcement Date	Operational Date
Australia	10/12/2008	11/28/2008
Austria	10/27/2008	10/27/2008
Belgium	10/15/2008	10/15/2008
UK	10/8/2008	10/13/2008
Canada	10/23/2008	2/25/2009
Denmark	10/10/2008	10/11/2008
France	10/12/2008	10/17/2008
Germany	10/13/2008	10/27/2008
Ireland	11/20/2009	12/9/2009
Italy	10/13/2008	12/4/2008
Netherlands	10/13/2008	10/23/2008
Portugal	10/12/2008	10/29/2008
South Korea	10/19/2008	10/20/2008
Spain	10/13/2008	11/21/2008
Sweden	10/20/2008	10/29/2008
US	10/14/2008	10/14/2008

B A model without the delay of funds

This appendix shows that the separation of weaker banks to DW and stronger banks to TAF continues to hold without the delayed release of funds (Proposition 1 below). It demonstrates that the competitive nature of the auction and the delayed release of funds from the auction can drive the separation of banks' borrowing behavior in borrowing from different facilities. In the language of the model below, Proposition 1 below holds when $\delta = 1$ and/or $c = 0$.

B.1 Model

We introduce a two-period, n-bank model. The timeline of the model is as follows. Each bank is endowed with an illiquid asset that pays off after the second week. Before the asset pays off, a liquidity shock may hit a bank with a probability that is privately known by the bank; the shock may arrive in the first week or the second week. Before the shock, each bank can borrow from DW and TAF. Borrowing banks may incur a penalty if detected of borrowing. Figure B1 sketches the timing and sequence of events, which we will describe in detail next.

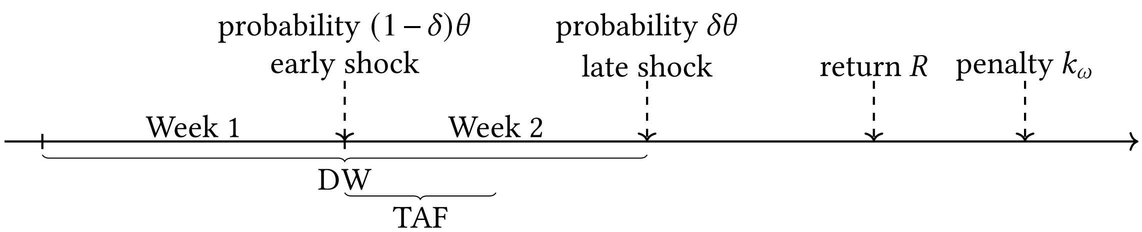

Figure B1: Timeline of the model

Technology, Preferences, Shocks. All parties are risk neutral and do not discount future cash flows. At the beginning of the first week, each bank has one unit of long-term, illiquid assets that will mature at the end of the second week. The asset generates cash flows R upon maturity but nothing if liquidated early. Shortly before the end of the second week, each bank may be hit with a liquidity shock. The size of the shock is normalized as one unit. Let $1-\theta_i \in [0,1]$ be the probability that the liquidity shock hits bank i, where θ_i follows the independent and identically distributed cdf F and associated pdf f on the support $[0,1]$. We assume that θ_i is private information and only known by bank i itself. Without loss of generality, we drop the subscript i subsequently.

A loan in the first week will help the bank defray the liquidity shock and therefore brings net benefits $(1-\theta)R$ at the cost of interest rate r. Finally, to capture the idea that earlier liquidity may be more valuable, we assume that the liquidity shock may arrive in the first week with probability $1-\delta$ and in the second week with probability δ, conditional on the shock arriving. To capture the same idea, there can be an additive delayed cost of $c \geq 0$, which can be interpreted as the cost incurred when banks sell illiquid assets at fire-sale prices in order to satisfy immediate liquidity needs. To summarize, a type-θ bank's payoff is $\pi_1(\theta, r) = (1-\theta)R - r$ if it borrows in week 1, and is $\pi_2(\theta, r) = \delta(1-\theta)R - r - c$ if it borrows in week 2.

Borrowing. A bank can borrow from DW or TAF. DW offers loans at a fixed interest rate r_D. TAF allocates pre-announced m units of liquidity through an auction. In the auction, banks who decide to participate simultaneously submit their sealed bids. Bid β_i specifies the maximum interest rate bank i is willing to pay. The bid needs to be higher than the reserve interest rate r_A. After receiving all the bids, the auctioneer ranks them from the highest to the lowest. All winners pay the same interest rate while losers do not pay anything. If there are fewer bids than the units of liquidity provided, each bidder receives a loan and pays r_A. If there are more bidders than the total offering liquidity, each of the m highest bidders receives one unit of liquidity by paying the highest *losing* bid. In this case, the highest losing bid is also called the stop-out rate s, which is the clearing price at which aggregate demand in the auction matches the aggregate supply. Let $w(\theta, \beta)$ denote the (equilibrium) probability that bank θ can win the auction by bidding β. We will focus on symmetric strategies in bidding and as a result can write $w(\theta, \beta(\theta))$ as $w(\theta)$ without loss of generality.

Stigma. Denote the probability of being detected of borrowing from DW, borrowing from TAF, and the probability of verifying that a bank has not borrowed to be p_D, p_A, and p_N, respectively. Let G_D, G_A, and G_N be the type distributions of the banks that have borrowed from the DW, from the TAF, and have not borrowed, respectively. We capture the notion of stigma in a parsimonious way. Specifically, we assume that after all the borrowings are accomplished, the banks that have successfully borrowed may be detected independently, after which a penalty will be imposed. This penalty can be understood as a cost in bank's deteriorated reputation, a cost in a reduced chance to find counterparties, or a cost from a heightened chance of runs and increasing withdrawals by creditors. Let the stigma cost be $k(\theta, G_\omega)$, where $\omega \in \{D, A, N\}$. The stigma cost is naturally assumed to be higher when the borrowing banks are worse. Formally, $k(\theta, G) > k(\theta, G')$ if G is strictly first-order stochastically dominated by G'. In the baseline model, we eliminate the dependence of stigma cost on a bank's private type and instead assume that it only depends on the borrowing facility $\omega \in \{D, A, N\}$. In other words, $k(\theta, G_\omega) = k(G_\omega) \equiv k_\omega$. For simplicity, we normalize k_N to be 0.

Equilibrium. In summary, the setting is summarized by the return R, probability δ of late shock, type distribution F of banks, discount rate r_D in the DW, number m of units of liquidity auctioned, minimum bid r_A in the TAF, and the penalty function $k(G)$ attached to different belief distributions of bank's type. A type-θ bank's strategy can be succinctly described by $\sigma(\theta) = (\sigma_D(\theta), (\sigma_A(\theta), \beta(\theta)))$, where $\sigma_\omega(\theta)$ is the probability of participating in $\omega \in \{D, A\}$ and $\beta(\theta)$ is its bid if it participates in the auction. Given strategies σ, beliefs about the financial situation can be inferred by the Bayes' Rule. In this case, we say aggregate strategies $\sigma(\cdot)$ generate posterior belief system $G = (G_A, G_D, G_N)$. Note that we have restricted each bank's strategy to be symmetric so that $\sigma(\cdot)$ only depends on θ. Strategies σ^* and beliefs G^* form an equilibrium if (i) each type-θ bank's strategy $\sigma^*(\theta)$ maximizes its expected payoff given belief system G^*, and (ii) the belief system G^* is consistent with banks' aggregate strategies σ^*. Clearly, the best (i.e., type-1) bank has no intention to borrow at all, because it would only pay a price and stigma cost but has no benefit from borrowing. We assume that the borrowing benefit of the worst (i.e., type-0) bank is so high that it has a strict incentive to borrow even given the most pessimistic belief about the banks who borrow: $\delta R - r_D - k(G) > 0$ when $G(\theta) = 1$ for all $\theta > 0$.

B.2 Equilibrium Characterization

We now solve for the equilibrium when both DW and TAF are available. We first describe a bank's bidding strategy in TAF, followed by its incentives in choosing between DW and TAF. Our result shows that relatively stronger banks have more incentives to bid in TAF rather than borrow immediately from DW, which is the key force behind the separation of types in equilibrium.

Let's start by describing a bank's bid in the auction. In general, a bank's bidding strategy depends on its plan after losing in the auction: It can either borrow from the DW in the second period or not to borrow at all. Clearly in this case, the incentive to borrow declines with a bank's financial strength.

Lemma 1. *Only banks $\theta \leq \theta_2$ will borrow from DW in the second week if they have not borrowed.*

Let $\beta^D(\theta)$ be a type-θ bank's bid if it plans to borrow from discount window after losing the auction. Let $\beta^N(\theta)$ be its bid if it doesn't plan to borrow after losing the auction. Given that a bank's bid does not (directly) affect its payment conditional on winning the auction, a bank bid its own willingness to pay (WTP), as follows.

Lemma 2. *Bank θ who borrows from the discount window after losing in the auction bids*

$$\beta^D(\theta) \quad = \quad r_D + k_D - k_A. \tag{1}$$

Bank θ who does not borrow from the discount window after losing in the auction bids

$$\beta^N(\theta) \quad = \quad \delta(1-\theta)R - k_A. \tag{2}$$

Note that $\beta^D(\theta)$ does not depend on θ. In other words, any bank who plans to go to the discount window bids up to the same amount, which equals the sum of r_D, the discount rate, and $k_D - k_A$, the net stigma cost of discount window relative to TAF. Intuitively, these banks will always borrow in equilibrium, from either the DW or the TAF. Therefore, since the discount window charges the same rate to all borrowers and the stigma cost is also homogeneous across all borrowers from the same facility, their willingnesses to pay are also the same. On the other hand, $\beta^N(\theta)$, however, does depend on θ. Among these banks, weaker ones have higher willingnesses to pay because they have stronger demand for liquidity but will not borrow if they lose in TAF.

Proposition 1 is our main result. It describes the incentive to borrow from DW1 against participating in the auction. In particular, it shows the skimming property that stronger banks are more willing to wait for the TAF.

Proposition 1 (Skimming property). *Let $u_1(\theta)$ be bank θ's expected equilibrium payoff if it borrows from the discount window in period 1, and $u_A(\theta)$ its expected payoff if it bids in the auction. In any equilibrium, $u_1(\theta) - u_A(\theta)$ is decreasing in θ.*

Intuitively, auction introduces uncertainty in terms of whether a bidding bank is able to borrow and if so at what price. Specifically, it introduces one mechanism that enables a bank to borrow at a low rate, lower than its own willingness to pay, at the cost of potentially failing to borrow (for banks $\theta \in [\theta_2, 1]$) or delaying to borrow (for banks $\theta \in [0, \theta_2]$). This cost of not borrowing (or delayed borrowing) is lower for stronger banks because their borrowing benefits are lower. Therefore, they are more inclined to participate in the auction and take advantage of the opportunity to borrow when rates are sufficiently low. In this case, auction is able to separate borrowers into two groups, the so-called "single-crossing" condition. Mathematically, a bank $\theta \in [0, \theta_2]$ will always borrow even if it chooses to participate in the TAF: it will turn to the discount window in week 2 in the event of losing in the TAF, in which case the cost of delay is $(1-\delta)(1-\theta)R$, decreasing in θ. Bank $\theta \in [\theta_2, 1]$ no longer borrows if it loses in the auction, with the cost of failing to borrowing being $(1-\theta)R$.

Our result on separation does not depend on the assumption that delaying cost is bigger for weaker banks; that is, the result continues to hold when $c = 0$ and/or $\delta = 1$. We would like to emphasize that not any mechanism that offers a trade-off between probability of winning and price paid can separate borrower. To see this, note that a bank's overall payoff has three components that vary with θ. First, a stronger bank has lower borrowing benefits. Second, in equilibrium, a stronger bank is less likely to win in the auction. However, conditional on winning in the auction, it pays less in expectation. When a bank bids optimally, it is indifferent between raising the bid to increase the winning probability and paying more conditional on winning. Therefore, the last two effects exactly cancel out. As a result, the overall effect is simply the decreasing benefits of borrowing times the probability of winning in the auction: $-R[1-H(\theta)]$. Next, let us consider a mechanism $(w(\theta), b(\theta))$ where $w(\theta)$ is the probability of receiving one unit of liquidity and $b(\theta)$ is the price paid. Let $u_\omega(\theta)$ be bank θ's payoff in this mechanism.

$$u_1(\theta) - u_M(\theta) = w(\theta)\left[b(\theta) + k_\omega + c - r_D - k_D\right] + \left[1 - w(\theta)\right]\left[(1-\theta)R - r_D - k_D\right].$$

By taking derivatives with respect to θ, we can see clearly that the overall effect is ambiguous.

Given Proposition 1, in any equilibrium, weaker banks choose to borrow from the discount window in week 1, and stronger banks bid in the auction. Among the banks who lose in the auction, relatively stronger ones (if any) will still go to the auction.

Theorem 1. *There exists an equilibrium. Equilibrium borrowing decision is characterized by three thresholds, θ_1, θ_2, and θ_A: (i) Banks $\theta \in [0, \theta_1]$ borrow directly from week 1's DW; (ii) Banks $\theta \in (\theta_1, \theta_A]$ participate in the auction; (iii) Banks $\theta \in [\theta_2, \theta_A]$ borrow in week 2's auction if they lose in the auction; and (iv) Banks $\theta \in (\theta_A, 1]$ do not borrow at all.*

B.3 Proofs

Proof of Lemma 1. The payoff of not borrowing is $u_N(\theta) = 0$, and the payoff of borrowing from DW in the second week is $u_2(\theta) = \delta(1-\theta)R - r_D - k_D - c$. Bank θ borrows from DW in week 2 if and only if $u_2(\theta) \geq u_N(\theta)$, which is rearranged as $\theta \leq 1 - (r_D + k_D + c)/(\delta R) \equiv \theta_2$. □

Proof of Lemma 2. In the auction, the winning bank pays the highest bid among the losers. Therefore, its own bid does not affect its equilibrium payment but only its chance of winning the auction. Therefore, it is its dominant strategy to bid its own willingness to pay. Bank θ's willingness to pay $\beta(\theta)$ satisfies $\delta(1-\theta)R - \beta(\theta) - k_A - c = \max\{\delta(1-\theta)R - r_D - k_D - c, 0\}$. If $\delta(1-\theta)R - r_D - k_D \geq 0$ so that the losing bank will go to the discount window, $\beta(\theta) = r_D + k_D - k_A$. Otherwise, $\beta(\theta) = \delta(1-\theta)R - k_A - c$. □

Proof of Proposition 1. The benefit of borrowing in week 1's DW is $u_1(\theta) = (1-\theta)R - r_D - k_D$. Let $\tau \in [0,1]$ be the highest losing bank and $H(\tau)$ its distribution. First consider $u_A(\theta)$ for $\theta < \theta_2$. If $\tau < \theta_2$, bank θ's payoff from winning the auction is $\delta(1-\theta)R - \beta^D(\theta) - k_A - c$, which simplifies to $\delta(1-\theta)R - r_D - k_D - c$. If it loses, it turns to DW and receives the same payoff. If $\tau \geq \theta_2$, bank $\theta < \theta_2$ wins the auction for sure and receives payoff $\delta(1-\theta)R - \beta^N(\tau) - k_A - c$, which simplifies to $\delta(\tau-\theta)R$. Therefore, $u_A(\theta) = \delta(1-\theta)R - (r_D + k_D + c)H(\theta_2) - \int_{\theta_2}^1 [\delta(1-\tau)R]\,dH(\tau)$ if $\theta < \theta_2$. Next, consider $u_A(\theta)$ for $\theta \geq \theta_2$. In this case, bank θ receives $\delta(\tau-\theta)R - c$ if it wins in the auction. Therefore, $u_A(\theta) = \int_\theta^1 [\delta(\tau-\theta)R - c]\,dH(\tau)$ if $\theta \geq \theta_2$. Taking the difference, we have

$$u_1(\theta) - u_A(\theta) = \begin{cases} (1-\delta)(1-\theta)R - H(\theta_2)c - \int_{\theta_2}^1 [\delta(1-\tau)R + r_D + k_D]\,dH(\tau) & \text{if } \theta < \theta_2 \\ (1-\theta)R - r_D - k_D - \int_\theta^1 [\delta(\tau-\theta)R + c]\,dH(\tau) & \text{if } \theta \geq \theta_2. \end{cases}$$

Clearly, $u_1(\theta) - u_A(\theta)$ is continuous and decreasing at the rate of $(1-\delta)R$ when $\theta < \theta_2$. When $\theta > \theta_2$, $\frac{d(u_1(\theta)-u_A(\theta))}{d\theta} = [-1+\delta(1-H(\theta))]R < 0$. □

Proof of Theorem 1. Denote the three thresholds by θ_1, θ_2, and θ_A. Let $u_\omega(\theta|\theta_1,\theta_2,\theta_A)$, $\omega \in \{1,2,A\}$, denote bank θ's expected payoff of participating in mechanism ω. The three equilibrium thresholds are determined by three conditions: $u_1(\theta_D|\theta_1,\theta_2,\theta_A) = u_A(\theta_D|\theta_1,\theta_2,\theta_A)$, $u_2(\theta_2|\theta_1,\theta_2,\theta_A) = 0$, and $u_A(\theta_A|\theta_1,\theta_2,\theta_A) = 0$. Let $h_m^n(x) \equiv \binom{n}{m} x^m (1-x)^{n-m}$. Define three correspondences:

$$\phi_1(\theta_1,\theta_2,\theta_A) = \left\{\theta : u_1(\theta|\theta_1,\theta_2,\theta_A) - \max\{u_A(\theta|\theta_1,\theta_2,\theta_A), u_N(\theta|\theta_1,\theta_2,\theta_A)\} \geq 0\right\} \cup \{0\},$$

$$\phi_2(\theta_1,\theta_2,\theta_A) = \left\{\theta : u_2(\theta|\theta_1,\theta_2,\theta_A) - u_N(\theta|\theta_1,\theta_2,\theta_A) \geq 0\right\} \cup \{0\},$$

and

$$\phi_A(\theta_1,\theta_2,\theta_A) = \left\{\theta : u_A(\theta|\theta_1,\theta_2,\theta_A) - u_N(\theta|\theta_1,\theta_2,\theta_A) \geq 0\right\} \cup \{0\}.$$

Economically, if it is believed that (i) $[0,\theta_1]$ is the set of banks willing to borrow from discount window 1, (ii) $[0,\theta_A]$ is the set of banks willing to bid if it has not borrowed from discount window 1, and (iii) $[0,\theta_2]$ is the set of banks willing to borrow from discount window 2 if it has not borrowed after auction, then optimally, (i) $\phi_1(\theta_1,\theta_2,\theta_A)$ is the set of banks willing to borrow from discount window 1, (ii) $\phi_A(\theta_1,\theta_2,\theta_A)$ is the set of banks willing to bid in the auction if it has not borrowed from discount window 1, and (iii) $\phi_A(\theta_1,\theta_2,\theta_A)$ is the set of banks willing to borrow from discount window 2 if it has not borrowed after auction. We have an equilibrium if the belief is consistent with the optimal action: $[0,\theta_1] = \phi_1(\theta_1,\theta_2,\theta_A)$, $[0,\theta_2] = \phi_2(\theta_1,\theta_2,\theta_A)$, and $[0,\theta_A] = \phi_A(\theta_1,\theta_2,\theta_A)$; or more simply, if $(\theta_1,\theta_2,\theta_A) \in \phi(\theta_1,\theta_2,\theta_A) \equiv (\phi_1(\theta_1,\theta_2,\theta_A), \phi_2(\theta_1,\theta_2,\theta_A), \phi_A(\theta_1,\theta_2,\theta_A))$. Hence, to prove the existence of an equilibrium, it suffices to show that the correspondence $\phi \equiv (\phi_1,\phi_2,\phi_A)$ has a fixed point. Each of the three correspondences is well-defined on $X \equiv [0,1]^3 \cap \{(\theta_1,\theta_2,\theta_A) : \theta_1 \leq \theta_A\}$, a non-empty, compact, and convex subset of the Euclidean space $\mathbb{R}^3$, and is upperhemicontinuous with the property that $\phi_\omega(x)$ for each $\omega \in \{1,2,A\}$ is non-empty, closed, and convex for all $x \in X$. By Kakutani's fixed point theorem, $\phi : X \to 2^X$ has a fixed point $x \in X$. □

C Online Appendix: Empirical Implications

A main prediction of our theory is, the banks that borrowed more from DW over time were fundamentally weaker than the banks that borrowed more from TAF. In this section, we examine this hypothesis using data from various sources, including banks' regulatory reporting, subsequent failure, and credit default swap (CDS) spread. Throughout this section, all analysis is conducted at the bank holding company (BHC) level, so our sample is restricted to large banks. Although under Section 23A of the Federal Reserve Act, it is illegal for a member bank to channel funds borrowed from LOLR to other affiliates within the same BHC, temporary exemptions of Section 23A were granted in late 2007 (Bernanke, 2015). Therefore, by conducting our analysis at the BHC level, we implicitly assume an efficient internal capital market within a BHC, which is consistent with the evidence in Cetorelli and Goldberg (2012) and Ben-David et al. (2017).

C.1 Descriptive Statistics of DW and TAF Borrowing

Let us start by describing the BHCs' borrowing behaviors from DW and TAF. The main dataset we use is obtained through Bloomberg and includes 407 institutions that borrowed from the Fed between August 1, 2007 and April 30, 2010. These data were released by the Fed on March 31, 2011, under a court order, after Bloomberg filed a lawsuit against the Fed.[18] The data contain information on each institution's daily outstanding balance of its borrowing from DW, TAF, and five other related programs. We will merge this dataset with the banks' regulatory database and CDS spreads to study how financial conditions affected the BHCs' borrowing decisions.

Since the Bloomberg dataset was collected by scraping over 29,000 pages of PDF files released from the Fed, data processing could be compromised. To evaluate the data's quality, we calculate the aggregate weekly outstanding balance in DW and TAF programs from the Bloomberg dataset and compare these numbers with the official ones released by Board of Governors of the Federal Reserve System (2019). Figure B2 shows the comparison. Clearly, the Bloomberg data managed to capture the vast majority of borrowing in both DW and TAF.

Table B1 provides the summary statistics of the BHCs' borrowing behavior during the crisis. Approximately 73 percent of borrowing institutions (313 out of 407) are banks, together with diversified financial services (mostly asset management firms), insurance companies, savings and loans, and other financial service firms. Foreign banks that borrowed through their U.S. subsidiaries were also included. Banks' choices of borrowing facilities were heterogeneous: 260 borrowing institutions tapped both facilities, 18 used only TAF, and 86 used only DW. Borrowing frequencies in both programs exhibit large skewness. While the median bank tapped the discount window twice, the Alaska USA Federal Credit Union used it 242 times. Similarly, for the 60 TAF

[18] For details, see Torres (2011). In May 2008, Bloomberg News reporter Mark Pittman filed a FOIA request with the Fed, requesting data about details of discount window lending and collateral. Unsurprisingly, it was stonewalled by the Fed. In November 2008, Bloomberg LP's Bloomberg News filed a lawsuit challenging the Fed, with the Fox News Network later filing a similar lawsuit. Other news organizations also showed support by filing legal briefs. In March 2011, the US Supreme Court ruled that the Fed must release information on the discount window loans in response to the lawsuits. Later that month, the Fed released the data, in the form of 894 PDF files with more than 29,000 pages on two CD-ROMS. Bloomberg News later published an exhaustive analysis that included the detailed data.

auctions, while the median bank borrowed only three times, Mitsubishi UFJ Financial Group borrowed 28 times. On average, TAF lent more liquidity ($3,174 million) than DW ($1,529 million) to an average bank, consistent with the evidence in Figure 1a that TAF was more successful in providing liquidity. However, the Dexia Group—the BHC that borrowed the most from DW—borrowed approximately $190 billion over the 3-year period, far exceeding $100 billion from the largest borrower in TAF (Bank of America Corporation). This evidence suggests that DW banks were in need of larger amount of liquidity than TAF banks.

C.2 Evidence from Banks' Fundamentals

C.2.1 Domestic Banks

We link the Bloomberg data to FR Y-9C reports, the Consolidated Financial Statements for Holding Companies. The Y-9C reports collect financial-statement data from BHCs on a quarterly basis, which are then published in the Federal Reserve Bulletin. All domestic BHCs are required to submit these reports within 40 or 45 calendar days following the end of a quarter. While this merge allows us to use proxies for banks' financial condition, it excludes all foreign banks from the borrowing sample, which took out about 60% of total TAF loans (Benmelech, 2012). Among the 289 U.S.-based banks that borrowed from either DW or TAF, we managed to merge Y-9C reports to 135 of them. These banks account for 42.2% of all American banks' loans from DW, and 81.8% from TAF. Given the reasons for missing matches, our subsequent analysis essentially compare the relatively healthier subsample among DW-borrowing banks with (almost) the whole sample among U.S. TAF-borrowing banks.[19] Therefore, the later results that DW-borrowing banks are on average weaker than TAF-borrowing banks would go through if we could have found all the matches for DW-borrowing banks.

Did bank fundamentals predict LOLR borrowing decisions? To explore how the BHCs' financial condition affects their borrowing from DW and TAF, we estimate the following specification:

$$\frac{DW_{it}}{DW_{it}+TAF_{it}} = \alpha + \beta_1 \cdot x_{it} + \Gamma \cdot [\text{Size}_{it}, ROA_{it}] + \gamma_i + Q_t + \varepsilon_{it}, \tag{3}$$

[19]There are several reasons behind the missing matches. First, many borrowers were credit unions or savings and loans holding companies that did not file Y-9C reports. For example, US Central Federal Credit Union took out $39,101 million in loans from the two facilities. Another example is Washington Mutual Inc. Even though it had an RSSD 2550581, it was an S&L holding company instead of a bank holding company. Therefore, it was regulated by the Office of Thrift Supervision and did not file a Y-9C report. Second, there are certain thresholds for reporting Y-9C. For example, banks with assets below $1 billion did not have to report. Finally, there were several mergers and acquisitions during the crisis period. For example, Wachovia borrowed $34,460 million from DW from 2007 Q3 to 2008 Q4, with the majority ($29,000 million) borrowed in 2008 Q4. However, Wachovia was acquired by Wells Fargo in 2008 Q4, and thus did not file a Y-9C report that quarter.

where DW_{it} and TAF_{it} are bank i's average daily outstanding balance from DW and TAF in quarter t. The left-hand side of Equation (3) therefore measures the use of the DW relative to the TAF. On the right-hand side, x_{it} is one of the proxies for BHC i's financial condition in quarter t, including its core deposit to assets ratio, book leverage, tier-1 capital to risk-weighted asset ratio (T1RWA), unused commitment to assets, and short-term wholesale funding to assets. These variables are defined following Ellul and Yerramilli (2013). In all regressions, γ_i is the bank fixed effect to take into account time-invariant conditions in the bank's fundamentals, and Q_t is the quarter fixed effect to incorporate variations in aggregate economic conditions. We include bank size and return to assets (ROA) as additional controls. Note that we use the *contemporaneous* measurement of banks' financial condition, for two reasons. First, the results are qualitatively unchanged if we control for lagged measurements $x_{i,t-1}$. Second, since these risk measurements were not available until at least 30 days after the quarter ended, we interpret the contemporaneous risk measurements as the part of banks' fundamentals that are not entirely observed by the public yet.

Table B2 reports the results if the above-mentioned bank fundamental measurements are included one by one.[20] Columns titles indicate the measurement used for bank fundamentals. Column (1) and (2) show that once a bank's core deposits to assets ratio goes up by 1%, the same bank borrows relatively 1% less from the DW. The results are economically and statistically significant and also not driven by either time-varying aggregate conditions or the bank's time-invariant variables. Clearly, banks with more stable funding tried to avoid borrowing from the DW. Column (3), (4), (5) and (6) confirm similar results if we measure a bank's fundamental through its capital adequacy. Banks with higher book leverage and lower tier-1 capital to risk-weighted assets tend to borrow more from the DW. Moreover, Ivashina and Scharfstein (2010) show that borrowers heavily drew down their credit lines during the crisis, implying that banks with more unused loan commitments were more vulnerable and therefore had more urgent liquidity demand. Column (7) and (8) show that indeed, these banks tend to borrow relatively more from DW. Finally, it is widely acknowledged that the 2008 crisis was a run by short-term wholesale creditors (Shin, 2009). Our results in Column (9) and (10) show that banks relied more on short-term wholesale funding also borrowed relatively more from the DW as well. Table B3 reports the regression results when we simultaneously control for all these bank fundamental measurements.[21] Clearly, book leverage and tier-1 capital ratio still stand out as important predictors on a bank's relative use of the DW.[22]

[20] We use robust standard errors in all the regressions.

[21] Since tier-1 capital ratio and book leverage are highly correlated (correlation≈ -0.7), we don't control for both in the same regression.

[22] We have run additional robustness checks. In particular, the results are largely unchanged if 1) we only use the subsample before 2008 Q3; 2) if we eliminate banks that exclusively borrow from the DW throughout the crisis; 3) if we use the lagged bank fundamental measurement x. Moreover, note that we have used the share of outstanding

Did LOLR borrowing decisions predict future bank fundamentals? Did DW and TAF loans capture potentially unobservable risks in banks' fundamentals? In particular, did these loans predict changes in banks' fundamentals? To answer this question, we estimate the following specification:

$$x_{i,t+1} = \alpha + \beta_1 \cdot x_{it} + \beta_2 \cdot \frac{DW_{it}}{DW_{it} + TAF_{it}} + \Gamma \cdot [\text{Size}_{it}, ROA_{it}] + \gamma_i + Q_t + \varepsilon_{it}, \tag{4}$$

where $x_{i,t+1}$ is one of the previous proxies for BHC i's financial condition in quarter $t+1$. We control for the one-quarter lagged financial condition, size, ROA, as well as bank and quarter fixed effects.

Table B4 reports the results. Across all columns, the results show that the relative borrowing from DW could have additional predictive power regarding a bank's core deposits, book leverage, tier-1 capital ratio, unused loan commitment, and reliance on short-term whole sale funding in the next quarter. In particular, if a bank borrows relatively more from the DW, all these measurements will imply that the bank becomes less healthy in the next quarter. In other words, the relative borrowing from the DW can predict deterioration in a bank's future financial condition, controlling for the relevant financial condition this quarter. Therefore, a bank's reliance on DW captures certain financial condition that is not publicly observable.

The results also have strong economic significance. For example, if a bank switches from 0% to 100% DW borrowing (which is not rare in the sample), its book leverage increases by somewhere between 0.2% and 0.3% after controlling for either the quarter-specific fixed effects or the bank-specific fixed effects. Meanwhile, the unconditional standard deviation of the book leverage is merely 0.01% in our sample. Similarly, the standard deviation of core deposits over assets is 0.06%, whereas a bank that switched from 0% to 100% DW borrowing would reduce its core deposits to assets ratio by somewhere between 0.4% and 1.4%. In terms of the remaining proxies for financial strength, T1RWA has a standard deviation of 0.02%, unused commitment/assets 0.05%, and STWF/Assets 0.04%. All of them are small relative to the magnitude reported in Table B4.

C.2.2 International Evidence

Specification (3) suffers from potential endogeneity issues. In particular, it does not control unobserved time-varying bank fundamental conditions. To address concerns about these omitted variables, we further employ a difference in differences (DID) approach and explore the international aspects of borrowing banks. In October 2008, leaders from the G7 countries met and estab-

balance from the DW as the left-hand-side variable. The results also stay unchanged if instead we use the share of new borrowing loans from the DW.

lished a plan of actions that aimed to stabilize financial markets, restore the flow of credit, and support global economic growth. Following the meeting, all of the G7 countries except Japan immediately announced to launch credit guarantee programs that effectively reduced the liquidity risk faced by domestic financial institutions (Yale Program on Financial Stability, 2019). Later on, many other countries also undertook similar credit guarantee programs to combat the potential crisis.[23] The operation dates of country-specific policies were staggered, however, as these policies could be largely driven by political obstacles through bargaining and renegotiation.[24] The staggered structure offers us an ideal setup to study the difference in these countries' banks' decisions to borrow from the lender of last resort in the US.[25] Specifically, we compare the decisions to borrow from DW and TAF by banks from different countries before and after their country-specific credit guarantee programs. In particular, we focus on the TAF auction held on October 20, 2008 and examine whether implementing (and also announcing) a credit guarantee program prior to that date affects banks' decisions to borrow from DW or TAF. The following equation is estimated a biweekly basis using data from 2008 Q3:

$$\frac{DW_{iw}}{DW_{iw} + TAF_{iw}} = \alpha + T_i + \lambda_w + \delta \cdot (T_i \times \lambda_w) + \varepsilon_{iw}, \tag{5}$$

where DW_{iw} and TAF_{iw} are bank i's outstanding balance from DW and TAF in the w's bi-week, respectively. In the specification, T_i is a dummy variable for the treated group, which takes a value of 1 if the country's operation (announcement) date happens before October 20, 2008. The control group therefore includes countries with policies implemented (announced) after October 20, 2008, as well as countries that did not announce any policy. λ_w is the time trend, which equals one after October 20, 2008. We are mainly interested in the coefficient δ before the interaction term, which estimates the difference in differences (DID) effect.

We plot the the dependent variable $\frac{DW_{iw}}{DW_{iw}+TAF_{iw}}$ in Figure B3. The two dashed vertical lines mark the two TAF auctions held on October 6 and October 20. Clearly, there was a sharp decline by the treatment group on the relative usage of the DW. Prior to Oct 6 and post Oct 20, 2008, the two groups have parallel trend in terms of the relative borrowing from DW and TAF.[26] A t-test on the difference in the growth rate of the dependent variable across the two groups shows the t statistic is only 0.1070 prior to October 6. By contrast, the same t-test during the post-treatment period has a t-statistic 1.7839. Table B5 presents the results to specification (5). Column (1) shows that after the policy shock, banks from treated countries, i.e., those countries with credit

[23] The details of these programs are available at https://newbagehot.yale.edu/find/all/credit-guarantee.

[24] Table 1 lists announcement and operation dates.

[25] Buch et al. (2018) show that access to TAF eased German banks financial stress.

[26] Note that the treatment group experiences a small upward jump in the week of Sep 23. This jump is statistically insignificant and possibly driven by the collapse of Lehman (15) and AIG (Sep 16).

guarantee programs started before October 20, 2008 borrow about 11% less from the DW. Note that these banks originally borrowed more from the DW, compared with banks from the control groups. Column (2) conducts the same analysis, but using the announcement date of the credit guarantee program as the quasi-experiment. The results stay largely unchanged. Note that we do not further explore countries whose policies were implemented between the auctions held on Oct 20 and Nov 3, because only very few banks fall into the treatment group in this case.[27]

C.3 Evidence from Bank Failure

Next, we study whether banks that borrowed more from DW were also more likely to fail subsequently. To do so, we manually collect data on whether a bank failed, was acquired, or got nationalized by the government by December 31, 2011. Our results are robust to the choice of this ending date. In the borrowing sample, 36 financial institutions failed by December 31, 2011. Of these, 11 failed in 2008, eight in 2009, seven in 2010, and 10 in 2011. We study whether banks that borrowed more from DW were more likely to fail from the following linear-probability specification.

$$\mathbb{1}\{\text{bank } i \text{ fails in } t\} = \alpha + \beta_1 \cdot \frac{DW_{it}}{DW_{it} + TAF_{it}} + \gamma_i + Q_t + \varepsilon_{it}, \tag{6}$$

where $\mathbb{1}\{\text{bank } i \text{ fails in } \tau\}$ is an indicator function on whether bank i failed in quarter t, and $\frac{DW_{it}}{DW_{it}+TAF_{it}}$ is the fraction of DW outstanding balance. We will also run the unconditional regression where the left-hand side variable is whether the bank failed during the crisis, and the right-hand side includes the aggregate borrowing during the entire sample period (2007Q3 to 2010Q2).

Table B6 reports the results. Column (1) shows that compared with a bank that only borrowed from TAF, a bank that solely borrowed from DW was more likely to fail within the same quarter by an additional probability of 1.1%. Column (2) confirms the result if we control for aggregate conditions by adding quarter fixed effects. Column (3) controls for bank fixed effects, where the result is no longer statistically significant. Finally, column(4) shows that if a bank borrows more from the DW during the entire sample period, the chance that is fails during the crisis increases by 12.8%. Therefore, the borrowing from DW relative to TAF is associated with more bank failure, so that there are systematic differences between DW-borrowing banks and TAF-borrowing banks.

[27] Indeed, Table 1 shows only Austria, Germany, The Netherlands, Portugal, and Sweden fall into this treatment group. Among banks from these countries, only a total of eight banks were borrowing from both DW and TAF during the crisis.

C.4 Evidence from CDS Spreads

In this subsection, we take advantage of the high frequency of the Bloomberg data and match borrowing banks with their CDS spreads in the Markit database. Since only very large banks have CDS contracts outstanding, we could match 70 of them, which accounts for 24.8% of DW borrowing and 79.4% of TAF borrowing.

Figure B4 plots the level of 5-year CDS spreads around borrowing dates, after removing fixed effects of BHC, month, and CDS rating. Two observations are prominent. First, prior to the borrowing event, DW banks have persistently higher CDS spreads than TAF banks. The difference (about 0.05) is significant relative to the standard deviation (less than 0.002), implying that prior to the borrowing, DW banks have a higher probability of default as acknowledged by the CDS price. Second, following both borrowing events, BHCs' CDS spreads drop within the next 5 days, even though it seems that TAF banks drop slightly more than DW banks. Two reasons can potentially explain the difference in drop. First, TAF banks in general take out larger loans, and therefore their funding constraint is more relaxed. Second, if borrowing from DW and TAF has an identical probability of being detected, TAF borrowing suffers a lower level of stigma cost.

Formally, we estimate the following specification(s) at the daily frequency:

$$y_{it} = \alpha + \beta \cdot \text{CDS}_{i,t-1} + \gamma \cdot \text{CDS rating}_{i,t-1} + Q_m + \gamma_i + \varepsilon_{it}, \tag{7}$$

where y_{it} is a dummy variable for if a bank borrows from DW or TAF on date t. Table B7 reports the results. In Column (1), y_{it} equals 1 if BHC i borrows from DW on date t and 0 if BHC i borrows from TAF on date t. The coefficient shows that if the BHC's 5-year CDS spreads on date $t-1$ increases by 100 basis points, its probability to borrow from DW, as opposed to TAF, increases by 0.1%. In Columns (2) and (3), $y_{it} = 0$ if BHC i does not borrow from either DW or TAF. In Column (2), $y_{it} = 1$ if it borrows from DW, whereas in Column (3), $y_{it} = 1$ if it borrows from TAF. Clearly, the results show that lagged CDS can predict DW borrowing but not TAF borrowing.

Figure B2: Comparison of Bloomberg Data and Fed Data

This figure plots the total weakly borrowing amount from the DW (left panel) and from the TAF (right panel), aggregated from the Bloomberg data (red solid line) and reported from the Federal Reserve (blue dashed).

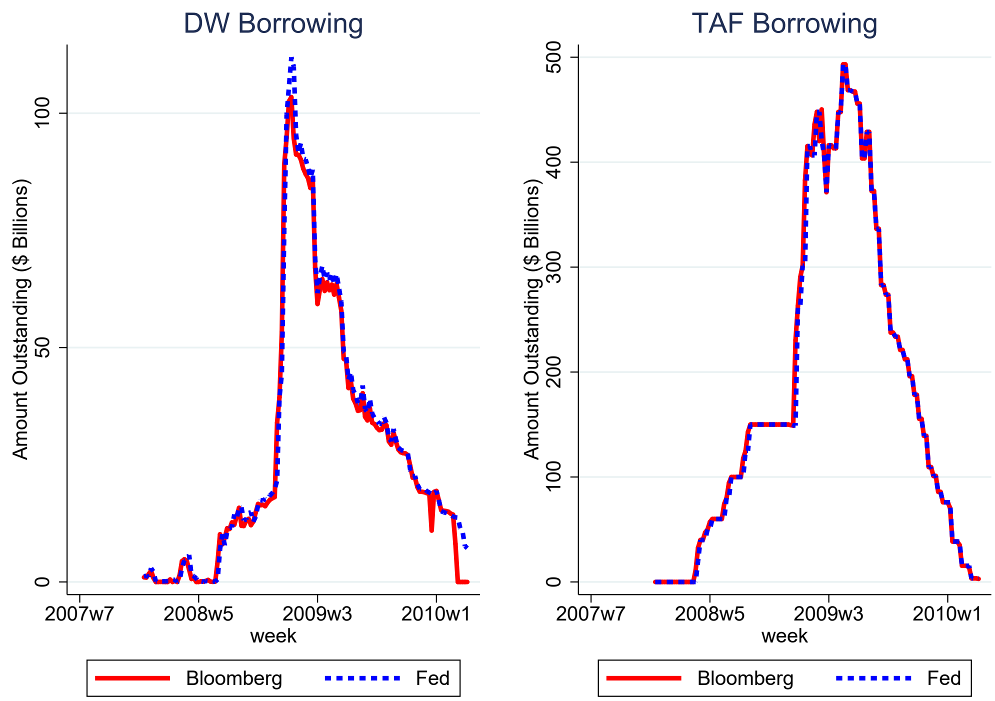

Figure B3: Share of Discount Window Borrowing in 2008 Q3 and Q4

This figure plots the average of the variable $\frac{DW_{iw}}{DW_{iw}+TAF_{iw}}$ across different groups, where DW_{iw} and TAF_{iw} are bank i's outstanding balance from DW and TAF in the two week indexed by w, respectively. The red solid line shows the average across all banks in the treated groups, i.e., banks from countries whose credit guarantee programs operating before October 20, 2008. The blue dashed line shows the average across all the banks in the remaining countries, i.e., the control group. The two dashed vertical lines mark the two subsequent TAF auctions held on October 6 and October 20.

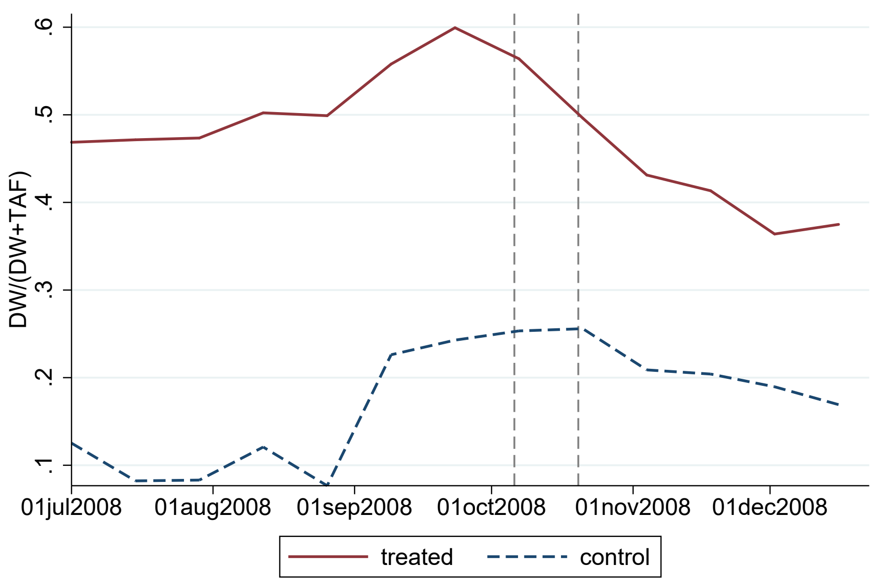

Figure B4: CDS Spreads around Borrowing Events

This figure plots an average bank's 5-year CDS spreads on date -5 and 5 surrounding a borrowing event, after removing BHC, month, and CDS-rating fixed effects. The blue solid line shows the spreads if a bank borrows from DW on date 0, whereas the red dashed line shows the spreads if it borrows from TAF on date 0.

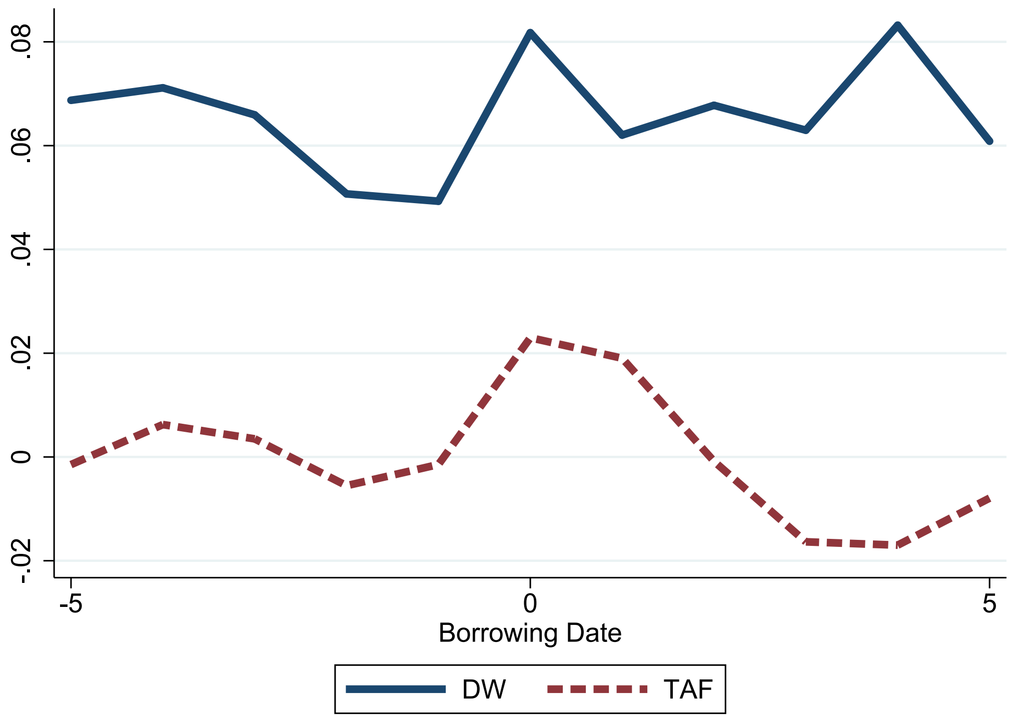

Table B1: Summary Statistics of Bloomberg Data

This table reports the summary statistics of borrowers in the Bloomberg data. The data cover institutions that borrowed from the Fed between August 1, 2007 and April 30, 2010. These data were released by the Federal Reserve on March 31, 2011 and subsequently collected by Bloomberg.

	N	Mean	Max	Min	SD	10^{th}	50^{th}	90^{th}
Borrowers	407							
Banks	313							
Diversified Financial Services	24							
Insurance Companies	12							
Savings and Loans	30							
Market Cap on 8/1/07 (MM)		28525	399089	11	49876.8	107	7331	81813
Foreign Banks	92							
DW-only banks	18							
TAF-only banks	86							
borrow both	260							
Total DW events		12	242	0	28.7	0	2	35
Total TAF events		5	28	0	5.1	0	3	13
Total DW amount (MM)		1529	190155	0	10393.8	0	20	1809
Total TAF amount (MM)		3174	100167	0	10727.5	0	58	7250
Number of days in debt to Fed		323	814	28	196.8	85	306	606

Table B2: LOLR Borrowing and Univariate Bank Fundamentals

This table reports OLS and fixed-effect regression results in specification (3), where we put univariate proxy for financial health. The sample contains all BHCs (bank holding companies) that have borrowed in the Bloomberg sample and filed FR Y-9C reports. The columns differ in the measurement of financial strength: (1) and (2) use core deposits over assets; (3) and (4) use book leverage; (5) and (6) use tier-1 capital to risk-weighted asset ratio (T1RWA), (7) and (8) use unused commitment to assets, (9) and (10) use short-term wholesale funding to assets. All the regressions control for bank size and return to assets (ROA). Standard errors in the parentheses are robust standard errors.

	Core Deposits/Assets		Book Lev		Tier-1 Capital/RWA		Unused Commit/Assets		STWF/Assets	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
x	-0.098	-1.105***	2.111***	4.179***	-1.982***	-4.736***	0.066	2.730***	0.227	0.889**
	(0.116)	(0.363)	(0.650)	(1.277)	(0.676)	(0.970)	(0.245)	(0.565)	(0.189)	(0.347)
ROA	0.443	17.959***	2.653	20.599***	2.740	17.195***	0.355	13.573***	0.298	18.121***
	(3.387)	(4.287)	(3.437)	(4.225)	(3.467)	(4.220)	(3.437)	(4.613)	(3.377)	(4.308)
log(Size)	-0.045***	-0.752***	-0.037***	-0.770***	-0.042***	-0.668***	-0.045***	-0.508***	-0.041***	-0.775***
	(0.008)	(0.160)	(0.007)	(0.160)	(0.007)	(0.159)	(0.008)	(0.172)	(0.007)	(0.160)
Constant	1.050***	12.665***	-1.039*	8.573***	1.170***	11.271***	0.997***	7.816***	0.897***	12.254***
	(0.167)	(2.517)	(0.622)	(2.725)	(0.136)	(2.485)	(0.118)	(2.745)	(0.121)	(2.516)
Fixed Effects	Quarter	BHC	Quarter	BHC	Quarter	BHC	Quarter	BHC	Quarter	BHC
N	731	731	731	731	731	731	674	674	731	731
R^2	0.19	0.53	0.20	0.53	0.19	0.54	0.19	0.54	0.19	0.53

Table B3: LOLR Borrowing and Multivariate Bank Fundamentals

This table reports OLS and fixed-effect regression results in the specification (3), where we include multiple proxies for financial health. Due to collinearity, we do not simultaneously include book leverage and tier-1 capital to risk-weighted asset ratio (T1RWA). The sample contains all BHCs (bank holding companies) that have borrowed in the Bloomberg sample and filed FR Y-9C reports. All the regressions control for bank size and return to assets (ROA). Standard errors in the parentheses are robust standard errors.

	(1)	(2)	(3)	(4)
Core Deposits/Assets	-0.107	-1.325**	-0.013	-1.404***
	(0.155)	(0.533)	(0.155)	(0.536)
Tier 1 Capital/Risk-Weighted Assets	-2.212***	-4.246***		
	(0.740)	(1.150)		
Book Leverage			2.005***	4.751***
			(0.714)	(1.449)
Unused Commitments/assets	0.117	2.087***	0.181	2.651***
	(0.267)	(0.587)	(0.268)	(0.581)
Short-Term Wholesale Fund/Assets	0.158	-0.683	0.107	-0.663
	(0.234)	(0.520)	(0.235)	(0.525)
ROA	3.256	12.300***	2.593	14.004***
	(3.549)	(4.558)	(3.511)	(4.591)
log(Size)	-0.051***	-0.473***	-0.043***	-0.521***
	(0.011)	(0.171)	(0.011)	(0.171)
Constant	1.331***	8.660***	-0.893	4.590
	(0.252)	(2.717)	(0.720)	(2.988)
Fixed Effects	Quarter	BHC	Quarter	BHC
N	674	674	674	674
R^2	0.20	0.55	0.20	0.55

Table B4: LOLR Borrowing and Future Bank Fundamentals

This table reports OLS and fixed-effect regression results in specification (4), where we put into proxies for future financial health one by one. The sample contains all BHCs (bank holding companies) that have borrowed in the Bloomberg sample and filed FR Y-9C reports. The columns differ in the measurement of financial strength: (1) and (2) use core deposits over assets; (3) and (4) use book leverage; (5) and (6) use tier-1 capital to risk-weighted asset ratio (T1RWA), (7) and (8) use unused commitment to assets, (9) and (10) use short-term wholesale funding to assets. All the regressions control for bank size and return to assets (ROA). Standard errors in the parentheses are robust standard errors.

	Core deposits/assets		Book Lev		Tier-1 Capital/RWA		Unused commit/assets		STWF/assets	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
DW/(DW+TAF)	-0.004	-0.014***	0.002***	0.003**	-0.003**	-0.003**	0.001	0.004**	0.008***	0.012***
	(0.003)	(0.004)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.003)	(0.004)
x_{it}	0.984***	0.716***	0.950***	0.626***	0.905***	0.670***	0.960***	0.658***	0.913***	0.692***
	(0.009)	(0.033)	(0.016)	(0.035)	(0.019)	(0.032)	(0.012)	(0.030)	(0.015)	(0.031)
log(Size)	0.000	0.061***	-0.000**	-0.010**	0.000	0.013**	0.001***	0.002	-0.001	-0.040***
	(0.001)	(0.015)	(0.000)	(0.004)	(0.000)	(0.005)	(0.000)	(0.009)	(0.001)	(0.015)
ROA	0.080	-0.713*	-0.250***	-0.040	0.204**	-0.225	0.353**	1.019***	0.490*	1.154***
	(0.260)	(0.390)	(0.085)	(0.116)	(0.097)	(0.137)	(0.178)	(0.242)	(0.266)	(0.394)
Constant	0.012	-0.804***	0.051***	0.503***	0.007*	-0.162**	-0.019***	0.018	0.023**	0.677***
	(0.013)	(0.229)	(0.015)	(0.074)	(0.004)	(0.081)	(0.006)	(0.143)	(0.010)	(0.230)
Fixed Effects	Quarter	BHC	Quarter	BHC	Quarter	BHC	Quarter	BHC	Quarter	BHC
N	726	726	726	726	726	726	597	597	726	726
R^2	0.96	0.97	0.85	0.89	0.81	0.86	0.94	0.96	0.85	0.89

Table B5: Credit Guarantee Programs and LOLR Borrowing

This table reports difference-in-differences regression results in the specification (5). The sample contains all international BHCs (bank holding companies) that have borrowed in the Bloomberg sample between July 1, 2008 and Dec 31, 2008. All borrowings are aggregated at the bi-weekly frequency. The treament group includes BHCs from countries whose credit guarantee program happens before October 20, 2008. The control group includes countries with programs after October 20, 2008, as well as countries that did not announce any policy. Column (1) uses the operational dates for the credit guarantee programs at the cutoff, whereas column (2) uses announcement dates. Table 1 lists announcement and operation dates, collected by Yale Program on Financial Stability (2019). Standard errors in the parentheses are robust standard errors.

	Operational Dates	Announcement Dates
Treated × after 10/20/2008	-0.112***	-0.172***
	(0.043)	(0.050)
Treated countries	0.364***	0.373***
	(0.032)	(0.035)
After 10/20/2008	0.029	0.095**
	(0.034)	(0.044)
Constant	0.150***	0.104***
	(0.025)	(0.030)
Observations	1844	1844
Adjusted R^2	0.076	0.042

Table B6: LOLR Borrowing and Bank Failure

Column (1)-(3) report the regression results in the specification (6) with and without BHC/quarter fixed effects. In Column (4), we report the results from the unconditional version of (6), where the dependent variable is whether a bank fails by the end of 2011, and the variable DW_{it} and TAF_{it} are respectively replaced by the aggregate borrowing DW and TAF between 2007Q3 and 2010Q2. The sample contains all U.S.-based and international BHCs (bank holding companies) that have borrowed in the Bloomberg sample between 2007Q3 and 2010Q2. We manually collect data on whether a bank failed, was acquired, or got nationalized by the government by December 31, 2011. Standard errors in the parentheses are robust standard errors.

	Fail this quarter			Fail during Crisis
DW/(DW+TAF)	0.011***	0.009**	0.006	0.128**
	(0.004)	(0.004)	(0.005)	(0.064)
Constant	0.002	0.002	0.003	0.070***
	(0.002)	(0.002)	(0.002)	(0.016)
Fixed Effects	No	Quarter	BHC	No
N	2025	2025	2025	364
R^2	0.00	0.01	0.19	0.02

Table B7: CDS Spreads and Borrowing Events

This table reports the regression results in the specification (7). The three columns differ in the dummy variable on the left-hand side of the specification. In column (1), the variable takes 1 if a bank borrows from DW on date t and 0 if it borrows from TAF on date t. In (2), the variable takes 1 if it borrows from DW on date t and 0 if it does not borrow on date t. Finally, in (3), the variable takes 1 if it borrows from TAF on date t and 0 if it does not borrow on date t. The sample contains BHCs (bank holding companies) with outstanding CDS data available in the Markit database. Standard errors in the parentheses are robust standard errors.

	(1) DW/TAF	(2) DW/None	(3) TAF/None
Lagged 5y CDS spread	0.129**	0.004***	0.001
	(0.058)	(0.001)	(0.002)
Constant	1.257***	0.030***	0.004
	(0.279)	(0.007)	(0.010)
N	707	33440	33617
R^2	0.466	0.043	0.016