Deriving Valuations from Bidding Behaviors: British Third-Generation Auction in 2000

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Abstract

The fundamental principles of bidders' behaviors and preferences in the simultaneous ascending auction are identified and justified empirically. With the restrictions imposed on the bidders' behaviors, an econometric model of disequilibrium structural equations constructed by Plott and Salmon (2004) is implemented to estimate parameters that determine bidders' values of various items being auctioned. After bidders' values are identified, winners, final winning prices, and number of rounds can be forecast. This set of information can be obtained not only after the auction is over, but also during the auction. Using only information from the earlier rounds results in predictive power of the model. The information obtained is useful for participating bidders in modifying their strategies as the auction progresses. Compared to the results from Plott and Salmon (2004), the model predicts more accurately winners of the UK Third-Generation Auction in 2000 by using partial data set. Estimated bidders' values can be improved when combined with outside information.

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1 Introduction

Auction theory is one of the most successful applications of game theory. Auctions in drilling rights, timbers, and artworks, for example, have been applauded by the media, government, and private sectors alike. Auctions are popular because they generate great revenues for sellers, achieve efficient allocations of resources for social planners, and elicit bidders' true valuations with appropriate mechanism design.

In this paper, we focus on applications of auctions to sales of spectrum rights. In the United States, before Federal Communications Commission (FCC), under guidance of economists McAfee, Milgrom and Wilson, introduced auctions for sales of Personal Communications Services (PCS) licenses in 1994, outcomes of allocations have ranged from unsatisfactory to bizarre. For example, the rights of using certain regional frequencies were appropriated to six dentists by lottery (Milgrom 2004). The externalities associated with delays to desirable allocation, including transaction costs and litigation fees, were enormous and unnecessary.

Since 1994, simultaneous ascending auction (SAA) has been the most popular format for sale of the spectrum licenses (Cramton 2002). Due to its complex and dynamic nature in contrast to the simple and standard Dutch or first-price auctions, interest arises for both theoretical and empirical purposes. Though theory of auctions has been fairly developed beyond simple models in classic works like Milgrom and Weber (1982) and Myerson (1981), SAA is not as developed[1].

Main features of the SAA include simultaneous bids of multiple distinctive objects, multiple rounds, and ceasing auction only when no bid is submitted to any item. SAA is widely used because it is more likely to achieve efficiency than simultaneous sealed-bid auctions. In sealed-bid auctions, the bidder with higher value may bid lower than his value to generate some positive profit from obtaining the item, thus giving inferior bidder a chance to win, resulting in inefficient allocation. Since it achieves good results for bidders who value the item at higher price, SAA may deter entries from smaller bidders. However, the disadvantage is not significant empirically (Klemperer 2004).

The 2000-2001 European auctions of third-generation (3G) mobile telecommunication (UTMS) licenses were some of the largest in history, and wide range of results from different countries are of interest and importance. Revenues differed from 20 euros per capita in Switzerland to 650 euros per capita in the United Kingdom, though the values per capita should be similar given the similarities in geographic location, social status, and economic well-being. Poor auction designs and unfortunate timing at the dawn of technology and

[1]For a theoretical treatment, refer to Milgrom (2000).

telecommunication bubble, however, resulted in the enormous differences. In 2000, UK, Netherlands, Italy, and Switzerland conducted similar simple ascending auctions but except UK, the other three ranged from bad (Italy, Netherlands) to worse (Switzerland) judged by Klemperer (2004). Similarities in geographical, chronological, and socioeconomic settings but differences in the results make the data suitable for empirical research of factors that affect outcomes and values of the auctions. The UK data is particularly suitable as described in Section 3.

Ability to obtain information during the bidding period of a SAA which lasts at least a few weeks is crucial. If bidders can infer about the possible strategies or values of competitors, then they can change their strategies or quit participating in a hopeless auction to save time and reduce cost. Plott and Salmon (2004) developed a model of the bidders' behaviors in simultaneous ascending auctions based on two principles: principles of surplus maximization and of bid minimization. These principles, tested from field experiments, led to "models of both price dynamics and equilibration, leading to disequilibrium structural equations that can be used for estimating bidder values". When bidders' values are estimated, winners of the auction, winning prices, and number of rounds can be forecast. The values of bidders can be obtained only using partial data. The partial information is particularly helpful for firms and governments.

Valuations of bidders in SAA are rare in literature though empirical models of auctions are developing quickly and investigated extensively by Athey and Haile (2006). Hong and Shum (2003) and Plott and Salmon (2004) are the only two I can find. Hong and Shum (2003), however, has a set of restrictions inapplicable to the estimation of UK3G auction. I therefore adopt Plott and Salmon's estimation model with a different set of parameters to represent the the values of the bidders by differentiating paired and unpaired spectra.

I proceed as follows. First, the basic rules of SAA are introduced to facilitate reading. In Section 3, I describe in detail the British 3G auction, including procedures, bidding patterns, and notations. In Section 4, features of the SAA are elaborated, and the underlying economic theory and principles follow naturally. Econometric models are described in order to estimate the values of the parameters that lead to estimations of bidders' values. Results follow and discussions on validity and improvement of the model conclude.

2 Rules of the Simultaneous Ascending Auction

By the name of the auction format, it can be inferred immediately that it consists of multiple rounds and considerable duration with several objects sold at the same time. Each auction, SAA in particular, has its own variation to achieve satisfactory goals such as efficiency, transparency, entry, and competition (Klemperer 2004). A variety of auctions can be classified into one category based on several fundamental properties. In this section, I outline the few rules that distinguish SAA (Cramton 2002; Plott and Salmon 2004).

- **Objects auctioned**. The objects are usually those with highly interdependent values and functions. For example, the radio frequencies are used for the same purposes of telecommunication by mobile phones, with licenses of similar sizes valuing similarly.
- **Progress of auction**. Some SAA are conducted by continuous time process, with bids submitted at any time. The British 3G auction is a discrete version. Bids by all bidders are submitted to the agency in a fixed period of time, and then winners of the round for each object are simultaneously announced.
- **Minimum bid requirement**. Since all bids are denoted in money value, for large values of objects in millions of dollars, bidding one more cent in each new round would be a slow and ridiculous process. Therefore, usually a percent increment from the previous bids with a rounding to ten thousands or million dollars is implemented to speed up the auction. For instance, the British 3G auction sets an initial percent increment to be 5%, and gradually decreased to 1.5% by the end of auction. Setting a reasonable percent increment is important because a small one makes the progress slow, but a large one deters entry and causes inefficiency.
- **Restrictions on Biddings**. Restrictions, including how many or which items each bidder can bid, are placed on bidders. In the British 3G auction, each bidder can place a bid on at most one item in each round, and incumbent firms cannot bid on item A.
- **Activity Rules**. Actions are needed from each bidder in order to spur activity and diminish bidders' incentive to wait until the end to bid low. Therefore, in the UK 3G auction, unless each bidder is either a leader, or requests waiver, it must bid on one of the five items. Otherwise, the bidder drops out of the race immediately. In the continuous time auctions, actions are prompted to take action by the possibility of the ending of the auction if no one else bids.
- **Stopping Rule**. In a continuous time process, a clock is used to countdown to, for example, five minutes, that is reset after a new bid is placed. In the UK auction, the entire auction ends when no new bids are placed on ANY of the items. This stopping rule is important to enable arbitrage among substitutes, because a bidder may become interested in bidding on one license only after the price of a substitute has risen significantly.

3 UK3G Auction Data Description

The first in a wave of 3G spectrum auctions, the British auction in spring 2000 achieved glamorous success and optimism for subsequent ones. It was heralded by the British media as the "biggest auction ever", generating 23 billion pounds from selling five licenses of 140 MHz of radio wave length in total.

The auction is the most suitable among all for four reasons according Börgers and Dustmann (2005). First, because it was an auction with gigantic money values involved, firms are serious about each bid they make. Second, all information of bidders and bids is publicized after each round, including each bidder's action, current leader, current price, and minimum bid. Third, each firm is restricted to bid and win only one license, so the problems like synergies (package valuations) and collusions are not present, so that simple model can be used to have great predictive power (Ausubel et al. 1997; Cramton and Schwartz 2000; Cramton and Schwartz 2002). Finally, the licenses, though few in number, were not identical. This property makes possible the key econometric identity by bounding parameter estimates. Without differences of types of licenses, analysis that is based on the switching behavior both deterministically and probabilistically is impossible.

UK3G Auction took place between March 6, 2000 and April 27, 2000. During the one and half months intermittent with recessions in weekends and holidays, there were 150 rounds during which thirteen bidders bid for five licenses.

Spectra that compose the licenses are either paired or unpaired. Paired spectra provide separate frequencies for communication from the base station to the mobile phone and from the mobile phone to the base station, while the unpaired spectrum uses the same frequency for both directions. The paired spectrum composes of a bit of spectrum in the lower frequency and a bit of spectrum in a higher frequency band, so that is enables duplex, a situation that mobile phones allow simultaneous two-way transfer of data. It is often denoted as 2×10 MHz, meaning 10MHz from lower and 10 MHz from upper frequency bands.

Five licenses are categorized into three types in terms of its spectrum composition. License A consists of 2×15 MHz of paired spectrum and 5 MHz of unpaired; B consists of 2×15 MHz of paired spectrum. Licenses C to E are identical with 2×10 MHz of paired spectrum plus 5 MHz of unpaired spectrum. License A was reserved for a new entrant into the market, which in turn excluded 2G incumbents Vodafone (37.3%), Cellnet (BT3G) (30.1%), Orange (17.2%), and One2One (15.4%) from bidding[2]. Because of the restrictions imposed, final price of item B was higher than that of item A, which has five additional unpaired spectrum.

[2]percentages in parentheses are their market shares in the second-generation market according to Rothschild and Sons (1999)

Each license was set an opening price on March 6. Each round, every non-leader bidder who has not dropped out of the auction is eligible to bid for at most one item, with a minimum based on current leading price. 5% initially, the minimum increment percentage was dropped to 1.5% by the end of auction. A bidder is allowed to pass at most three rounds by requesting waivers. If the bidder's bid for a license is the highest, the bidder becomes the price leader for the license and automatically ineligible to bid for next round but stays the price leader if no new bid was submitted. At the same time, its bid amount becomes the current price for the license.

Descriptive Statistics of Bids

Company	A	B	C	D	E	C-E	Jump Bids	Increment*	Total	Leader	Waiver	Round Withdrawn
TIW	12	5	17	20	15	52	23	4.6%	69	81	0	A
Vodafone	-	43	0	0	0	0	2	1.2%	43	107	0	B
BT3G	-	33	24	33	17	74	0	-	107	43	0	C
One2One	-	0	37	44	24	105	3	0.9%	105	45	0	D
Orange	-	18	0	1	26	27	40	2.1%	45	105	0	E
NTL	25	0	14	15	4	33	19	1.9%	58	91	0	150
Telefonica	6	1	14	23	6	43	36	1.2%	50	80	2**	133
Worldcom	5	0	23	24	10	57	26	1.2%	62	58	0	121
One.Tel	2	0	29	34	16	79	4	2.3%	81	16	3	100
Epsilon	0	0	25	21	12	58	29	0.7%	58	36	3	98
Spectrumco	21	5	8	10	4	22	1	4.5%	48	47	1	97
3GUK	4	0	22	24	14	60	21	0.7%	64	27	3	95
Crescent	0	0	31	37	14	82	2	2.8%	82	8	3	94
TOTAL	75	105	244	286	162	692	206	1.7%	872	744		

Table 1: Number of bids placed on each of the licenses by each bidder, and the number of round that the bidders have withdrawn are shown above. *It is the average extra increment of jump bids made by each bidder; **One recess day requested before Round 107.

Auction Results

License	Paired (MHz)	Unpaired (MHz)	Company	Opening Prices (millions)	Winning Prices (million)
A	2×15	5	TIW	£ 1.25	£ 4,384.70
B	2×15	–	Vodafone	£ 1.07	£ 5,964.00
C	2×10	5	BT3G	£ 0.89	£ 4,030.10
D	2×10	5	One2One	£ 0.89	£ 4,003.60
E	2×10	5	Orange	£ 0.89	£ 4,095.00

Table 2: Amount of unpaired and paired spectrum contained in licenses, opening prices, eventual winners for licenses, and winning bids.

After 150 rounds, 872 bids (75 on A, 105 on B, 692 on C-E), and more than 300 lead changes, eventual winners were TIW for license A, Vodafone for B, BT3G for C, One2One

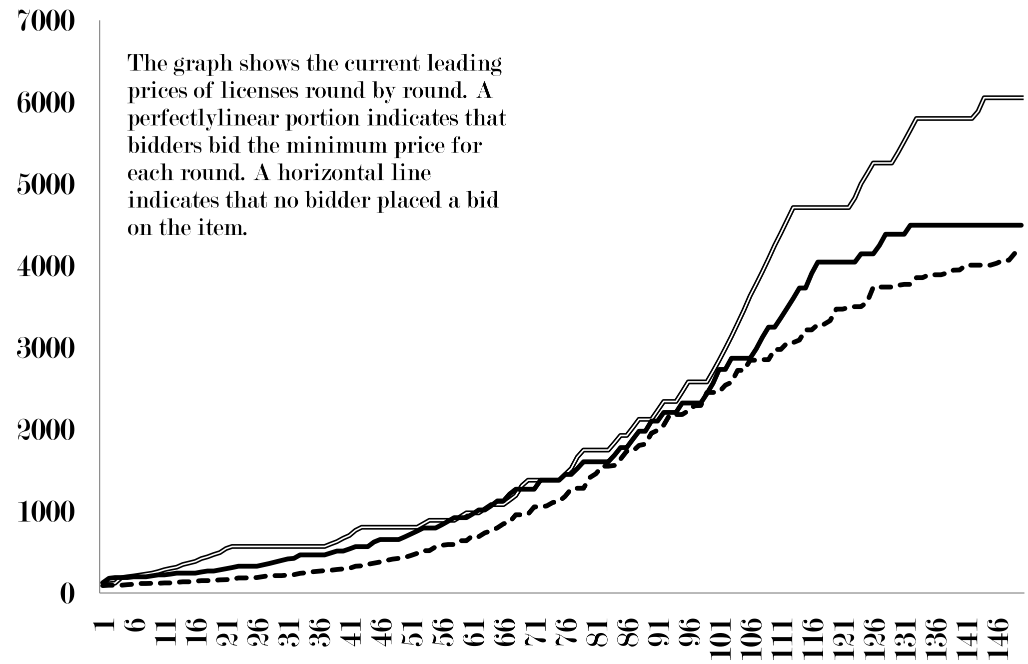

Figure 1: Evolution of leading prices

for D, and Orange for E. Table 1 lists the number of bids placed on each license by each bidder, jump bids submitted, average extra increment of the jump bids, total number of bids, number of rounds in which the bidder is a leader thus ineligible to bid, number of waivers requested, and round number in which the bidder withdraws completely from the auction. The firm who were incumbents in Second Generation market are bold. Table 2 lists amount of unpaired and paired spectrum contained in licenses, opening prices, eventual winners for licenses, and winning bids.

Evolution of current prices is shown in Figure 1. The graph shows the current leading prices of licenses A, B, and CDE which represents the largest of prices of C, D, and E. A perfectly linear portion indicates that bidders bid the minimum price in those rounds, and a horizontal line indicates that no bidder has placed a bid on the item.

All information on the data set is obtained from the official website created and maintained by the Radiocommunications Agency, the branch of UK government that oversaw the entire auction in 2000, `http://www.ofcom.org.uk/static/archive/spectrumauctions/auction/auction_index.htm`. The dataset includes rules of the auction and round by round summaries that are subdivided into pre-round and end-round reports. They contain information such as current price, minimum bid amount, current price leader, number of waivers requested by each bidder, duration of the bid, eligible bidders, bids posted by each bidder in a round, and tie bids. Detailed analyses of the jump bids are done by the author.

4 Theory

This section describes some of the fundamental rules that a risk-neutral, profit-maximizing bidder should abide to in a simultaneous ascending auction. These rules are obvious and self-explanatory in some sense, but they are nevertheless essential to the econometric set-up that follows.

4.1 Bidders' Behaviors

Every firm i is assumed to have a privately held, independent value $v_i(k)$ for each of the K objects being auctioned. We may incorporate a time value t, with t representing round of bids or simply time in continuous bidding models, but here for simplicity and assumption that bidder's valuation on each object does not vary when bidding is taking place, variable t is not considered, thus $v_i(k) = v_i(k,t)$ for all t.

Definition 4.1. *In the beginning of each round,* **current leading prices** *are denoted by* $p(k,t)$*.* **Minimum percentage increment** $\underline{m}(k,t)$ *for each round is announced* [3]*. Then the* **minimum bidding** *on object k in round t is*

$$\underline{b}(k,t) = p(k,t)(1 + \underline{m}(k,t)). \tag{4.1}$$

Definition 4.2. *Suppose the bidder bids $b_i(k,t)$ in round t, then its* **surplus** *or* **value** *if he becomes the leader and eventually wins the auction with the bid, is*

$$\pi_i(k,t) = v_i(k) - b_i(k,t). \tag{4.2}$$

From the observation of thirteen firms' behaviors, Cramton (2001) reports to the National Audit Office that "Most of the bidders pursued a strategy of bidding on the license that represented the best value." Therefore, the first principle that a risk-neutral, profit-maximizing company should adhere to is

Proposition 1 (Principle of Surplus Maximization)**.** *Given for each object k, current leading prices $p_i(k,t)$ and minimum bids $\underline{b}_i(k,t)$, a firm should bid on the object $k^*(t)$ that yields the highest surplus. That is,*

$$k^*(t) = \arg\max_k \pi_i(k,t). \tag{4.3}$$

[3] The minimum percentage increments can differ by items, but in the UK auction, the minimum increment remained identical for different items.

A utility-maximizing firm should choose to bid on the item with the highest value. However, there is argument that since the licenses' sizes are proportional and firms are investors or need to attract investments, they choose to bid on the item with the highest rate of return. Though the argument is not completely invalid, since the decision is treated as a utility-maximizing problem instead of financial one, Proposition 4.3 is a good starting point.

Thereafter, in order to maximize $\pi_i(k)$ with $v_i(k)$ fixed, $b_i(k)$ needs to be minimized. There is no upper bound for a bid $b_i(k)$, but a rational, risk-neutral bidder tries to maximize his surplus by Proposition 1. In other words, for a firm that follows Proposition 1, it should make a decision on which item to bid by assuming its bidding a minimum amount required and bid that amount thereafter. This brings to our second fundamental principle.

Proposition 2 (Principle of Minimum Bid)**.** *When a bidder makes a bid on object k in round t , it bids the minimum $\underline{b}_i(k,t)$. In discrete price auctions, in order to avoid ties, a bidder may bid an amount "ϵ" greater than $\underline{b}_t(k)$, where ϵ could be thousands of dollars depending on $p_i(k,t)$. The optimal bid $b_i^*(k,t)$ is then defined as*

$$b_i^*(k^*(t),t) \equiv \underline{b}(k^*(t),t) + \epsilon, \tag{4.4}$$

where $b_t(k^*)$ is as defined in equation 4.1, k^* in equation 4.3 and $\epsilon \to 0^+$.

The principle is justified in the British 3G auction process. In the earlier rounds, many firms bid \$10,000 higher than minimum required in order to avoid three- or four-way ties. However, after several bidders withdrew from the auction, even such small deviation from minimum bid is no longer evident. Most absurdly, One2One made a jump bid of £121.21 million to signify the company's name. In addition, as shown in Table 1, number of jump bids made are small compared to the total number of bids made. The average percentage increment is minimal, comparably zero for many firms. The biggest percentage increment is 4.6% made by TIW's 23 jump bids. However, if we examine the data more closely, we find that TIW made some huge bids in the early rounds to speed up the auction and stay as the leader of one of the items C, D, and E, possibly for media attention. Avery (1998) among others argues that firms do use jump bids for signaling in auction, but such claims are not evident in this auction data. Therefore, in this paper, we make the following assumption:

Remark 4.3. *Jump bids in the British 3G auction were made by the bidders only to avoid ties, speed up the auction, or catch media's attention, but do not serve any means of signaling, colluding or threatening.*

Therefore, it is clear that a rational bidder would bid the minimum price required on the item that yields the highest value to him. However, when should it stop bidding, that is, waive, or withdraw entirely from the auction? This brings another principle for bidder.

Proposition 3 (Principle of Withdrawing)**.** *A bidder stops bidding when its surplus from bidding on all available objects is lower than that of its alternative opportunity, or outside option,* Π_i*. Mathematically, a firm waives or withdraws,*

$$k^*(t) = 0 \quad \textit{when } \pi_i(k,t) < \underline{\pi}_i \quad \forall\, k. \tag{4.5}$$

Generally, $\underline{\pi}_i \geq 0$. For a general enough theoretical models, we can allow $\underline{\pi}_i \in [-\infty, +\infty)$, but for empirical and practical purposes, a firm should stop bidding when it expects negative profit from winning any of the items in the bids though it is possible that a firm withdraws when there is negligible profit or certain profit lower than engaging in alternative project. In the UK3G auction, it can be assumed that $\underline{\pi}_i = 0$ for all firms i as there is no evidence that a firm has alternative option. In addition, waivers and requests for recess days are distinguished from withdrawals.

Remark 4.4. *Waivers and recess days are used by firms to aggregate fundings for deposit. Requesting waiver does not necessarily imply that a firm has* $\pi_i(k,t) < \underline{\pi}_i, \quad \forall k \in \{1, ..., K\}$.

Notice that there are possibly different reasons of waiver from withdrawals. Though all bidders except one (Telefonica) withdrew in the immediate round after exhausting three waivers, waivers could be used differently from withdrawal. First, since the payment involved is huge, deposit was requested during the auction. Firms need time to aggregate the fundings required, so waivers serve as a time "buffer". Budget constraint is a major problem in the auction. Börgers and Dustmann (2005) attribute the cluster of withdrawals occurred between rounds 94 and 100 to budget constraints of the firms. In addition, bidding behavior by BT3G suggests that it is trying to attract investors by maximizing its rate of return.

4.2 Bidders' Values

Each object consists of certain distinctive properties that differentiate the products. Each characteristic has its relative importance. Therefore, an object with characteristics $j \in \{1, ..., M\}$, with m_j amount of each character with different values c_j per unit, has a total value of

$$v_i(k) = \sum_{j=1}^{M} c_j m_j(x). \tag{4.6}$$

In the British auction, compositions of the objects only differ by their spectrum length and distinctions whether they are paired or unpaired. Because of the strong diminished marginal values present in the three types of items, a three-parameter model of γ_i, α_i and β_i can be introduced to estimate values of firms on objects. Values of the three different items (licenses C, D, and E are identical) are

$$\begin{aligned} v_i(A) &= 20\gamma_i + 10\alpha_i + 5\beta_i, \\ v_i(B) &= 20\gamma_i + 10\alpha_i, \\ v_i(C) &= 20\gamma_i + 5\beta_i, \end{aligned} \tag{4.7}$$

where γ_i denotes value of the common paired spectrum per MHz, α_i value of extra paired spectrum per MHz, present in items A and B, and β_i value of unpaired spectrum per MHz. There are reports that claim at least $15 - 20$ MHz is required, but 25 MHz is more than sufficient to set up the necessary network, so we observe a sharp diminishing marginal utility in the extra spectrum length. Therefore, the common spectrum has different values from the differentiated parts.

In contrast, Plott and Salmon (2004) specified the parameters as

$$\begin{aligned} v_i(A) &= 25\gamma_i + 10\alpha_i, \\ v_i(B) &= 25\gamma_i + 5\alpha_i, \\ v_i(C) &= 25\gamma_i, \end{aligned} \tag{4.8}$$

The advantage of my set-up is the ability to distinguish all three types of licenses. With just two parameters, it is not easy to see infer from the result that the spectrum lengths are the only determining factors, as licenses A and B only differ by the amount of "α_i-type" spectra.

4.3 Maximum Number of Bids

The three fundamental principles lead naturally to prediction of auction duration based on opening prices and minimum bid increment.

$$\begin{aligned} p(k,T) &= (1 + \underline{m}_t)^{b_k} p(k,0), \\ b_k &= \Big(\ln p_k(T) - \ln p_k(0)\Big) \Big/ \Big(1 + \underline{m}_t\Big), \\ \bar{b} &= \max_k b_k \end{aligned} \tag{4.9}$$

where $p(k,T)$ is the predicted equilibrium price, or the estimation of the maximum valuation from all bidders. Since $\underline{m}_t(k)$, the minimum percentage increment, changes during most auctions, the estimation does not correspond to the actual number of bids perfectly.

5 Econometric Application

According to the three Propositions, whenever there is positive surplus, the bidder should bid on the item k^* with the highest surplus in the current round, with the minimum price $\underline{b}_t(k^*)$. The mechanism through which the model can identify the values a bidder has is through the observed "switching" behaviors . However, when surpluses of items are similar, bidders tend to make errors and do not optimize their winnings. Therefore, a probabilistic function depending on surpluses of items by a bidder, is constructed. Since many rounds are observed, estimations of the parameters are based on as much as information as possible. Therefore, aggregate loss functions derived from the probabilistic functions are implemented to obtain the parameters γ_i, α_i, and β_i.

5.1 Bounds on γ_i, α_i, and β_i

When different items are chosen, we can infer the bounds on valuations of items by bidders, and in turn bounds on the parameter estimates. We illustrate how the bounds can be achieved through a hypothetical bid profile of bidder i. Observe the bid history of few rounds of this theoretical auction setting in Table 3.

Round	Item Bid on	$p_t(A)$	$p_t(B)$	$p_t(A) - p_t(B)$
1	A	1000	850	150
3	A	1200	1050	150
6	B	1300	1100	200
10	A	1500	1400	100

Table 3: Hypothetical bid profiles.

When the bidder bids on A in Round t, it indicates that

$$\pi_i(A, t) = v_i(A) - p_i(A, t) \geq \pi_i(B, t) = v_i(B) - p_i(B, t). \tag{5.1}$$

Rearranging, we have

$$v_i(A) - v_i(B) \geq p_i(A, t) - p_i(B, t). \tag{5.2}$$

This is true for every round that the bidder has bid on A. So $v_A - v_B \geq 150$. On the other hand, when the bidder chooses B, the inequality is reversed. Therefore, $v_A - v_B \leq 200$. Together, item A is worth 150 to 200 more than item B. Then substitute in the parameter representation of the items' values, we obtain the parameter bounds. For a firm in the British

3G auction,

$$
\begin{aligned}
&\text{A is chosen:} && 5\beta \geq p_A - p_B, \\
& && 10\alpha \geq p_A - p_B, && (5.3)\\
&\text{B is chosen:} && 5\beta \leq p_A - p_B, \\
& && 10\alpha - 5\beta \geq p_B - p_C, && (5.4)\\
&\text{C is chosen:} && 10\alpha \leq p_A - p_C, \\
& && 10\alpha - 5\beta \leq p_B - p_C. && (5.5)
\end{aligned}
$$

5.2 Estimations of γ_i, α_i, β_i, and the Bidders' Values

In the previous subsection, procedures in restricting bounds for parameters are described. Though the argument and bounds that can be obtained look attractive to zone in the actual values of the parameters, it cannot be implemented deterministically because bidders tend to err or to make arbitrary decision when faced with small surplus difference. In addition, bidders may choose to bid on the item with second highest value surplus in earlier rounds to avoid ties or even other items for strategic reason. Consequently, a probabilistic choice rule is defined in Equation 5.6 below, with $f^+ = \max\{f, 0\}$ and λ_i represents the bidder's propensity to respond. When there is some profit in some items in round t, the probability of bidder i to bid on item k in that round is defined as

$$\rho_i(k,t) \equiv \frac{\left(\pi_i(k,t)^+\right)^{\lambda_i}}{\sum_1^K \left(\pi_i(k,t)^+\right)^{\lambda_i}}, \quad k \in \{1, \cdots, K\} \tag{5.6}$$

Notice that as the surplus of an item k grows relative to other items, the probability of choosing k rises proportionally. By this specification, item with a negative surplus is automatically assigned zero probability of being placed a bid on.

Common logit specification $\rho_i(k,t) = e^{(\pi_i(k,t))} \Big/ \sum\limits_{1 \leq k \leq K} e^{(\pi_i(k,t))}$ which could solve this problem, however, does not apply because the probabilities would be invariant to the addition of constants. That is, suppose there are only two items, such set up predicts that the probability of bidding on the item with surplus \$100 as compared to \$200 equals the probability of bidding on the item with surplus \$8,100 as opposed to the item with surplus of \$8,200.

Instead of trying to incorporate the possibility of bidding on items with negative surpluses, focus is placed on distinguishing more sharply the items with positive profits. λ_i, the bidder's propensity to respond, is introduced. The bigger is λ_i, less likely the bidder will make a mistake. The probabilities assigned to bidding \$100 and \$200 surpluses equal those when facing the choice between \$10,000 and \$20,000, respectively.

Extend the probabilistic choice rule as defined in Equation 5.6 by incorporating the choice of withdrawal as if bidding on item $k = 0$ with surplus $\underline{\pi}_i$, the alternative profit as defined in equation 4.5. For $k = 0, \cdots 5$, with $f^+ \equiv \max\{f, 0\}$, if $\max \pi_i(k,t) > \underline{\pi}_i$,

$$\rho_i(k,t) = \begin{cases} \left(\pi_i^+(k,t)\right)^{\lambda_i} \Big/ \left[\underline{\pi}_i^{\lambda_i} + \sum_{j=1}^{5} \left(\pi_i^+(j,t)\right)^{\lambda_i}\right], & 1 \le k \le 5 \\ 1 - \sum_{j=1}^{K} \rho_i(j,t) & k = 0 \end{cases} \tag{5.7}$$

Otherwise when $\max \pi_i(k,t) < \underline{\pi}_i$, by Proposition 3, the bidder should withdraw; so, $\rho_k = 0$ for each of $k = 1, \cdots, 5$, and $\rho_0 = 1$. Since in UK3G auction, it is assumed that $\underline{\pi}_i = 0$, so that $\pi_i^+(j,t) = \max\{\max_{k=\{1,\ldots,5\}} \pi_i(k,t), \pi_i\}$, and there are five items, the equation can be simply represented as

$$\rho_i(k,t) = \begin{cases} \left(\pi_i^+(k,t)\right)^{\lambda_i} \Big/ \sum_{j=1}^{5} \mathbf{1}_i(j,t) \cdot \left(\pi_i^+(j,t)\right)^{\lambda_i}, & 1 \le k \le 5 \\ 1 - \sum_{l=1}^{5} \rho_i(k,t) & \text{else} \end{cases} \tag{5.8}$$

The indicator function $\mathbf{1}_i(k,t)$ is introduced to indicate a bidder i's restraint from placing bid on object k. It can be also used when there is belief that a firm would not bid on any item it is eligible to bid on. For example, in British 3G auction, four incumbents of 2G network were forbidden from bidding on the largest item in terms of spectrum, object A. Therefore, $\mathbf{1}_i(A,t) = 0$ for the four 2G incumbent firm in all periods t; after several rounds, we can infer certain small firms will not bid on the licenses with more spectrum length.

A quadratic loss function is used in order to avoid hypersensitivity (Friedman 1983), which is problematic in the case when many of the predicted probabilities will be close 0 or 1. Log-likelihood function is not used here because it will induce strange results because of its separate treatment of high and low prediction events, as well as its treatment of events with zero probability. The loss function of bidder i in round t from bidding on item k is defined as

$$LF_i(k,t) = 2\rho_i(k,t) - \sum_{j=0}^{5} \rho_j^2(t), \quad t \in T_i(t^*) \tag{5.9}$$

where $T_i(t^*)$ is a subset of $\{1, \cdots T\}$, where an element in the subset represents the round in which bidder i is eligible to bid up to round t^*. Full information set is $T_i(T)$. For a particular t in $T_i(t^*)$, the loss function takes range $[-1, 1]$, with -1 represents the situation when the bidder bids on an item he should not have bid at all when there is only one item he should have bid on, and $+1$ represents the situation when the bidder bids on the only item he should have bid on. Values that range in between represent all bidding situations.

The equation specifies the algorithm from which the parameters of γ, α, β are obtained.

$$\begin{aligned}\gamma_i^*, \alpha_i^*, \beta_i^* &= \underset{\gamma_i,\alpha_i,\beta_i}{\operatorname{argmax}} \sum_{t \in T_i(x)} f_i(t) LF_i(\tilde{k}, t) \\ &= \underset{\gamma_i,\alpha_i,\beta_i}{\operatorname{argmax}} \sum_{t \in T_i(x)} f_i(t) \Big[2\rho_i(\tilde{k}, t) - \sum_{j=0}^{5} \rho_j^2(t) \Big], \end{aligned} \tag{5.10}$$

where $\tilde{k}$ indicates the item that the bidder is observed to bid on, so $\rho_i(\widehat{k}, t)$ is the predicted probability of the item on which the bidder actually bids in period t. For full data estimate, $x = T$, the number of rounds in the auction so that all behaviors observed are included in the estimation. When partial data is needed x is set to be a smaller number than T.

$f_i(t)$ indicates the different weights that can be put on each round that the bidder to eligible for, in order to most closely reflect the seriousness of the bidders. In the runs presented below, $f_i(t) = 1$; that is, equal weight is assigned to each round of decision.

Therefore, an equal weighted sum of each round's loss function is adopted. The function aggregates all the decisions made by a particular firm. A firm should behave according to the above-mentioned principles as much as possible. Therefore, $\gamma_i, \alpha_i, \beta_i$ are adjusted to maximize the aggregated loss functions. Ideally, λ_i should be subject to change because it is indeed a variable, but since with three variables the global surface is hard to find an absolute maximum, including a power term would result in extreme difficulty of accurate estimation.

Another strong assumption about the behaviors of the bidders is independence by rounds. Since the bidders try to obtain the item with highest present-value profit, its action is only dependent on his evaluations in each current round, but not based on history of past surpluses. Then by Equations 4.7, bidders' private values on each objects are derived after $\gamma_i, \alpha_i, \beta_i$ are estimated. Thereafter, an estimated efficient outcome can be derived with equilibrium price. Then the estimated number of rounds can be estimated by equation 4.9.

5.3 Estimation of the Number of Rounds

The parameter estimation procedure identified above is the same as the one used by Plott and Salmon (2004) in essence. However, because of modifications and extensions of their model including the introduction of the indicator functions, the addition of weighting factor according to rounds, consideration of alternative profit as part of the probabilistic choice function, the estimation model is more full-fledged and generalized to be more widely applied to simultaneous ascending bids.

Plott and Salmon (2004) indicates that their model is a reasonable parameter specification given that the only characteristics distinguishing different objects, but since the modifications only improve upon theirs withou distorting basic properties, the model described above is appropriate as well. Paired and unpaired spectrum types are treated separately. Furthermore, using three parameters differentiating three types of the five items can help to indicate any preference besides the difference of spectrum.

6 Results

In this section, estimation results, $\widehat{\alpha}_i, \widehat{\beta}_i, \widehat{\gamma}_i$, bidders' values, predictions on final outcomes, and predicted auction duration are presented and discussed. First, I use the full data set to derive bidders' value estimation to determine how reasonable they are compared to the actual results. Then, partial data sets are used to test the predictive power of the model in terms of evaluation and discrete predictions of winners.

The Solver function in Microsoft Office Excel 2007 is used to derive the following estimates of $\widehat{\alpha}_i, \widehat{\beta}_i, \widehat{\gamma}_i$. Standard deviations were obtained by "jackknife" procedure in Plott and Salmon (2004). I omit the procedure in this paper as the standard deviations do not differ much from bidder to bidder and they do not serve any indication for the model. $\lambda_i = 20$ is chosen simply for the reason that it implies a very strong degree of responding. α_i, β_i are both assumed to have upper bound of £2000.0 million per MHz, and lower bound of £0 per MHz. γ_i, in addition, is assumed to have a lower bound of £6 million per MHz because of the starting price. Four complete sets of runs were performed with different combinations of round numbers and λ, the propensity of bidders to respond to optimal strategy. Tables of results and discussions follow.

Predictions and Estimations from different combinations of rounds and λ

Licenses	Actual	$\lambda = 1$, Rd 150	$\lambda = 20$ Rd 150	$\lambda = 20$, Rd 75	$\lambda = 20$, Rd 100	Actual Values	Full Estimates
A	TIW	TIW	**NTL**	**NTL**	TIW	£4,384.70	£4495.80
B	Vodafone	Vodafone	Vodafone	Vodafone	Vodafone	£5,964.00	£6102.59
C	BT3G	BT3G	BT3G	**Epsilon**	BT3G	£4,030.10	£4030.10
C	One2One	One2One	**TIW**	One2One	One2One	£4,003.60	£4003.60
C	Orange	Orange	Orange	Orange	Orange	£4,095.00	£4393.15

Table 4: Predicted and Actual winner of four runs, and estimated values from full data set are presented. **Bold** items are wrong predictions.

6.1 Full Data Set Estimates

The first set of results is on the full data estimate with $\lambda_i = 1$ and $\lambda_i = 20$ for all firms (I should denote all propensity of bidder to respond as λ, because all bidders are assumed to have the same value). The first one results in a linear response probabilistic function and the second a high propensity of bidder's response with $\lambda = 20$. The results are shown in Tables 5 and 6, respectively.

In Table 5, we have estimates of the three parameters and values of each bidder, their highest actual bids on items. Since items C, D, and E are identical, there is no reason to believe that they differ in values because no firm should have preference over any one particular license, so v_c in the table represents the values of bidders on items C to E. "Percent" shows the percentage of correct predictions of actual behaviors by the model, and P&S is the correct prediction made by the full data set estimate in Plott and Salmon (2004) with $\lambda = 20$. Result shows that

Full data set estimates with $\lambda = 1$

Bidders	$\widehat{\gamma}_i$	$\widehat{\alpha}_i$	$\widehat{\beta}_i$	$\widehat{v}_A$	$\widehat{v}_B$	$\widehat{v}_C$	$\tilde{v}_A$	$\tilde{v}_B$	$\tilde{v}_{C-E}$	Percent	P&S*
TIW	105	73	334	4496	2827	3765	4385	330	3590	63.8%	16.0%
Vodafone	175	261	38	6292	6103	3685	NA	5964	-	100.0%	100.0%
BT3G	178	237	96	6405	5927	4030	NA	5799	4030	36.4%	77.0%
OneTwoOne	133	132	268	5323	3981	4004	NA	-	4004	99.0%	99.0%
Orange	168	251	208	6902	5860	4393	NA	1583	4095	40.0%	42.0%
NTL	178	52	80	4494	4092	3971	4278	-	3971	48.3%	46.0%
Telefonica	166	19	37	3684	3498	3498	3003	113	3668	65.4%	53.0%
Worldcom	158	0	0	3170	3168	3168	3774	-	3173	87.7%	87.0%
One.Tel	59	0	203	2188	1175	2188	1788	-	2181	94.0%	96.0%
Epsilon	59	0	178	2080	1188	2080	-	-	2072	75.4%	73.0%
Spectrum Co.	65	151	300	4312	2812	2800	2101	1378	2100	55.1%	59.0%
3GUK	60	0	183	2116	1201	2116	2001	-	1650	82.1%	80.0%
Crescent	91	0	0	1819	1819	1819	-	-	1819	100.0%	99.0%

Table 5: Estimates of $\gamma_i, \alpha_i, \beta_i$, estimated values of licenses $\widehat{v}_A, \widehat{v}_B, \widehat{v}_C$ are presented. Observed values of highest actual bids v^*_A, v^*_B, v^*_{C-E} are also included. Percentages of correct predictions of decisions are compared to those in Plott and Salmon (2004) (indicated as P&S) in the table.

Conclusion 6.1. *The model with* $\lambda = 1$*,*

1. *has no serious bound violation for the 13 bidders.*

2. *accurately predicts winners and final prices.*

3. *accurately predicts* 72.5% *of the actual choices made, not far from the* 78% *predicted by Plott and Salmon (2004) with* $\lambda = 20$.

Since true values of γ_i, α_i, and β_i are not known, we cannot measure how close these estimations come to true values. Therefore, a simple consistency check is to examine whether the bidders follow Proposition 3, Principle of Withdrawal. Due to the nature of the maximization of linear probability sum, the estimated values of the bidders are close to the highest value that the bidders have actually bid. The only fairly significant violation from the actual bidding behavior is Telefonica. Value of Telefonica on item C is £3498 million, whereas the latest bid is £3668. This difference in estimation is possibly caused by the fact that Telefonica rejoined bids until Round 129, after requesting waivers twice in Rounds 105 and 106.

Next check if the estimated values can determine whether or not they match with the final price and allocation results. The prediction results are combined in Table 4. The bound violations do not influence the final outcomes in terms of the predictions of final prices and winners. The full data set estimation model with $\lambda = 1$ predicts accurately the final winners of all five items. The estimated final prices are the actual final prices during the real auction. Plott and Salmon (2004) only accurately predicts three out of five licenses because of the anomalous value estimation of NTL which is the last to drop out the auction. This a result of the extra parameter used in this paper's model.

However, the percent of actual choices abiding to Proposition 1 on page 7 is lower than that predicted by Plott and Salmon (2004). This effect is due to the choice of parameter, propensity to respond to be $\lambda = 1$. When $\lambda = 1$, the loss functions do not necessarily produce big difference, so when the bidder does not choose the item with the highest surplus, loss function does not punish the mistake harshly, resulting in possible sacrifice in accuracy for bigger sum of overall probability.

Similarly, we examine the model's estimations and predictive power in $\lambda = 20$ case. The labels are the same as in Table 5.

Conclusion 6.2. *Full data estimate model with* $\lambda = 20$*,*

1. *has all bidders except Vodafone satisfying the bound conditions.*
2. *predicts three out of five winners and inaccurate prices.*
3. *predicts* 78.0% *of the actual choices made, the same as Plott and Salmon (2004).*

Vodafone's bound violation is caused by the nature of the estimation method and lack of switching behaviors. In order for the Solver function to maximize the likelihood function, parameters that make up item B, α and γ are boosted and the parameter not associated with item B, β is annealed, resulting in all three parameters to be pegged to upper and

Full data set estimates with $\lambda = 20$

Bidders	$\widehat{\gamma}_i$	$\widehat{\alpha}_i$	$\widehat{\beta}_i$	$\widehat{v}_A$	$\widehat{v}_B$	$\widehat{v}_C$	Percent	P&S*
TIW	206.26	6.86	0	4193.82	4193.82	4125.2	63.8%	16.0%
Vodafone	2000	2000	0	60000	60000	40000	100.0%	100.0%
BT3G	139.98	156.85	246.08	5598.59	4368.18	4030.1	75.7%	77.0%
One2One	133.076	131.93	268.42	5322.94	3980.86	4003.6	99.0%	99.0%
Orange	336.55	144.78	164.46	9001.14	8178.84	7553.31	42.2%	42.0%
NTL	527.10	71.73	143.45	11976.6	11259.3	11259.3	44.8%	46.0%
Telefonica	166.64	18.62	33.12	3684.65	3519.03	3498.4	50.0%	53.0%
Worldcom	158.4	0	2.2	3179	3168	3179	88.7%	87.0%
One.Tel	105	0	0	2100	2100	2100	94.0%	96.0%
Epsilon	103.61	0	0	2072.2	2072.2	2072.2	75.4%	73.0%
Spectrumco	85.97	14.29	20	1962.34	1862.34	1819.4	55.1%	59.0%
3GUK	37.62	2.87	179.1	1676.49	781.196	1647.79	82.1%	80.0%
Crescent	90.97	0	0.01	1819.4	1819.35	1819.4	100.0%	99.0%

Table 6: $\widehat{\gamma}_i, \widehat{\beta}_i, \widehat{\alpha}_i$ are estimated by setting $\lambda = 20$. Values of different items are derived from the estimated parameters. P&S indicates the percentage of correct predictions of bidders' behaviors by Plott and Salmon (2004).

lower bounds, respectively. Consistency check on Proposition 3 is satisfied for lower bounds of parameters.

The predictions of winners and final prices are not accurate, however. It is predicted that NTL wins item A, and TIW is bumped to win one of C, D, and E (Table 4, page 15). This is the result of setting λ to be 20, which gives estimate of higher values than final closing prices. In particular, Orange and NTL are estimated to have a much higher valuation than the closing price, partly because of their frequent switching behaviors. The model predicts accurately more than 50% of the actual choices made by all firms except Orange (42.2%) and NTL (44.8%). Orange's inaccurate prediction is caused by its mysterious fondness for item E. Out of the 27 times Orange bids on items C-E, it bids on item E 26 times, thus violating assumption of identical valuation for C-E and Principle of Surplus Maximization.

The high precisions in prediction with $\lambda = 1$ and higher estimations of values compared to the final prices lead us to conclude that

Conclusion 6.3. *Model with $\lambda = 1$ can be used to verify the full data set estimates and bidding behaviors whereas model with $\lambda >> 1$ can be used to have some predictive power.*

Therefore, in the subsequent subsections, partial data set estimation models with $\lambda = 20$ are used instead of with $\lambda = 1$ in order to give closer prediction of valuations by bidders.

Partial data estimates with $\lambda = 20$ in Rounds 75 and 100

	Round 75						Round 100					
Bidders	$\widehat{\gamma_i}$	$\widehat{\alpha}_i$	$\widehat{\beta}_i$	$\widehat{v}_A$	$\widehat{v}_B$	$\widehat{v}_C$	$\widehat{\gamma_i}$	$\widehat{\alpha}_i$	$\widehat{\beta}_i$	$\widehat{v}_A$	$\widehat{v}_B$	$\widehat{v}_C$
TIW	55	17	0	1273	1273	1105	59	0	232	2339	1181	2339
Vodafone	155	611	0	9217	9217	3108	155	611	0	9217	9217	3108
BT3G	40	56	49	1611	1364	1052	93	72	79	2976	2580	2253
One2One	40	49	62	1594	1286	1105	63	97	195	3202	2226	2228
Orange	78	98	115	3121	2545	2139	137	158	229	5462	4316	3879
NTL	64	41	397	3666	1682	3259	73	14	113	2157	1591	2017
Telefonica	51	0	0	1013	1013	1013	86	16	0	1877	1877	1716
Worldcom	55	0	0	1105	1105	1105	28	0	110	1105	553	1105
One.Tel	48	0	0	955	955	955	48	0	0	955	955	955
Epsilon	32	0	108	1179	638	1179	55	0	145	1819	1093	1819
Spectrumco	27	30	103	1348	832	1052	29	31	112	1448	889	1137
3GUK	24	8	125	1186	563	1105	42	3	162	1676	868	1648
Crescent	53	0	0	1052	1052	1052	74	0	0	1483	1483	1483

Table 7: $\widehat{\gamma}_i, \widehat{\alpha}_i$, $\widehat{\beta}_i$, all corresponding estimated using both 75- and 100- round data set.

6.2 Partial Data Set Estimates

The importance and effect of the model lie on its predictive power using only partial data set. For estimates of values, an accurate prediction of magnitude of closing prices is enough, and accurate prediction of final winners is preferred. In this section, results of partial data estimations from Rounds 75 and 100 are presented. These two round estimates are chosen because of the following reasons. First, Round 75 is chosen because it is exactly half way of the total number of rounds, with all bidders remaining in the auction. Second, Round 100 is chosen because four bidders (Crescent, 3GUK, Spectrum Co., and Epsilon) have dropped out of the auction. With nine bidders remaining, bidders become serious as possibility of other bidders ceasing to bid altogether as happened during rounds 95-98 when bidders start to waive or withdraw, becomes real. Therefore, behaviors can be regarded to closely reflect bidders' true strategies.

The results are presented in Table 7. We can conclude that

Conclusion 6.4. *Partial data set estimates*

1. *give inaccurate predictions of final winners in round 75, but very accurate predictions of final winners in round 100, identical to the full data set estimate model with* $\lambda = 1$.

2. *result in underestimations of final prices, but can be modified to have great result based on existing knowledge of auction.*

The inaccuracies mainly arise from the following areas. First, the inaccuracies arise mainly from bidders who show little switching behaviors. This results in unreliable estimates of many parameters associated with the items that bidders have not bid on. For example, Vodafone has not switched to bid any item other than B, revealing nothing about its valuation of γ or α, the paired spectrum. Second, because of the algorithm used, the value estimates of each bidder are close to the highest bids they have placed on an item. In terms of parameter estimates, the reason is an underestimation of γ_i for bidders who only bid on C, D, and E. Estimates in round 100 are better because several such bidders dropped out.

Results obtained by setting $\lambda = 20$ and using three parameters instead of two are better than those of Plott and Salmon (2004), though far from perfect. The problem of underestimation for majority of bidders still exists. The predicted closing prices range from £1105.1 to £3878 millions for items C-E, for example, lower than the actual final prices around £4100. Plott and Salmon (2004) approach the problem of underestimation by making Orange the price setter and adjusting other firms' valuations according to Orange's in an alternative estimate. I do not agree with this approach because this modification is partially based on observations in the final result.

6.3 Predictions versus Outcome

Predictions made by the four models presented are discussed in the two subsections above. Here I simply reiterate some of the observations made earlier. The full data set estimate model with $\lambda = 1$, as shown in the table below, shows very close estimation of values to the actual final prices. This observation is encapsulated in Conclusion 6.3. Predictions by full data set estimates with $\lambda = 1$ and partial data set estimate with $\lambda = 20$ are entirely correct while the other two models mispredicted two items each. The main source of error is NTL Mobile. Since it is the last to drop out of the race, there is reason to believe that the estimation of NTL's values on items is not greatly wrong, but the firm quit because of other factors, such as budget constraint and management conflict.

6.4 Maximum Number of Bids

Conclusion 6.5. *The predicted maximum number of bids is higher than the actual number of new high bids in full data set, but lower in partial data estimate.*

The actual number of new high bids observed in the auction was 355. If one takes the prices predicted from the full period estimates, assuming a constant minimum increment to the initial 5%, it is predicted to take 383 high bids to reach the closing prices. The prediction made at the round 100 would be 326 according to the straight price predictions, whereas the estimation from 75 rounds would predict a total of 258 new high bids. Existence of jump bids results in the inevitable overestimation of the maximum number of bids, and under-prediction of bidders' values in the partial data set results in the underestimation of the maximum number of bids.

7 Discussions

7.1 Significance

The paper contributes to the field of estimation of bidders' values in simultaneous ascending auction by implementing an econometric model built upon several straightforward economic principles. Principles of Surplus Maximization, of Bid Minimization, and of Withdrawal are all fundamental assumptions. The econometric model utilizing a probabilistic decision function and quadratic loss function is fairly easy to implement and understand in a basic manner as well.

The paper follows the theoretical principles and econometric approach of Plott and Salmon (2004) but differs from it the parameter specifications, and extends the model to be more applicable to other data sets of the simultaneous ascending auction with independent values of bidders.

The key of the paper lies in its importance in the ability to predict winners, final prices, and number of rounds during an auction with multiple rounds, from the parameter estimates based on the theoretical principles and econometric applications. The results obtained here are promising to a possible accurate in-process estimation of eventual outcome of a simultaneous ascending auction.

7.2 Possible Improvements

As observed in the Results section, there is problem of underestimation of the final closing price. Bidders may try to manipulate the estimation by making jump bids in earlier rounds, to confuse and intentionally violate the rules of auction by making jump and arbitrary bids when the bidders strongly believe the auction will not end soon so that he has low chance of ending up with an item he does not like. Meanwhile, the bidder who prefers the item may be so confused that he believes that his chance of winning is minimal, thus dropping out.

Though the model predicts fairly accurately results of the auction, the parameters do not necessarily reflect the actual value per MHz of paired or unpaired spectra as they intend to measure. For example, Vodafone has a high α estimate and places zero value to β. However, surely, it will take the extra 10 MHz if given for free. Therefore, here we must interpret his estimated valuations not because Vodafone wants the paired spectrum. Rather, the firm wants to sustain its market share and continue its position as the leading market player in 2G market with about 40% of market. Many estimates have shown zero value for either α or β. Therefore, an alternative model should be in place for the existing one of piecewise continuous function.

Shown from the estimates, there are no significant distinctions of preferences between paired and unpaired spectra, frequency band width thus becomes the only distinguishable factor of the three items. Since they are in numerical values with more bandwidths preferred over fewer. Therefore, a model of the value of the bidders as a continuous function of number of bandwidths can be implemented. For instance, the value of an item as a function of number of spectrum x could be

$$v_i(x) = C_1 \cdot \ln(C_2 \cdot x + 1). \tag{7.1}$$

Moreover, Börgers and Dustmann (2005) argued that BT3G adopted such strategy from Round 100 onward, because it sought to attract investors who would invest their funds in the project with the highest rate of return. It is possible that Principle of Maximization Surplus can be altered to or combined with **Principle of Rate of Return Maximization**, which claims the bidder bids on the item with the biggest rate of return, where

$$r_i(k) \equiv \frac{v_i(k)}{p_i(k)} - 1. \tag{7.2}$$

Finally, in the econometric model, an unequal weight of round can be used. Because there is strong evidence that bidders become more serious about their decisions towards the latter rounds, an equal weight put on Round 1 and Round 100 for example is not necessarily appropriate. A good candidate of $f_i(t)$ in Equation 5.10 is t with linear growth of weight, or $e^{\zeta t}$ where adjusting ζ would produce different weights. In the paper, we use a special case of $f_i(t) = e^{\zeta t}$ by setting $\zeta = 0$.

7.3 Conclusion

From the analyses and discussions, we see that advantages and disadvantages co-exist for the model of estimation. Overall, underlying principles and models are valid, but there is room for improvement in both general models and estimation of the British Third-Generation Auction data set. More complicated estimation models as well as theoretical assumptions can be developed to incorporate more complex auction formats, such as those in the United States with horizontal and vertical integration, and relaxation on number of items to bid on.

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