Pre-matching gambles

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A B S T R A C T

This paper investigates pre-matching gambles and provides a new reason to gamble: matching concerns. Examples of pre-matching gambles include occupational choices before the marriage market, college major choices before the labor market, and portfolio management before attracting future clients in the financial market. I show that people make risky investments they would not have made if not for their subsequent participation in a competitive matching market. A fundamental and unique feature of the competitive matching market, which I call the *competitive matching effect*, induces gambling. The paper also illustrates the relationship between social efficiency and inequality in this setting, and shows how progressive taxation eliminates social inefficiency, reduces inequality, and generates government revenue.

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"The lottery of the law [profession] ... is very far from being a perfectly fair lottery; and that as well as many other liberal and honorable professions, is, in point of pecuniary gain, evidently under-recompensed. Those professions keep their level, however, with other occupations; and, notwithstanding these discouragements, all the most generous and liberal spirits are eager to crowd into them."

Adam Smith, *Wealth of Nations* (1776)

1. Introduction

People gamble. College graduates pick career paths with similar expected lifetime earnings but different earnings distributions. Young professionals quit steady jobs to become entrepreneurs. Fund managers build risky portfolios. Novices accept meager wages in professions such as acting, and survive on the hope of future stardom.

People gamble because their utility functions exhibit convexity (Friedman and Savage, 1948). Many reasons for convexity and gambling have been proposed: social status motives (Smith, 1776; Robson, 1992; Becker et al., 2005); reaching a

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subsistence level (Rubin and Paul, 1979); risk pooling (Bergstrom, 1986); marrying an additional partner in a polygamous society (Robson, 1996); and moving to a better location (Rosen, 1997).

I introduce a new reason to gamble: matching concerns. People gamble *before* they participate in a matching market such as the labor, marriage, or financial market. For example, (i) college students choose majors that provide different bundles of human capital to meet future employers' needs; (ii) the unmarried obtain wealth in risky ways to potentially attract better mates in the future; and (iii) investment managers build risky portfolios because financial investment returns not only augment cash flow but also attract future investors.

To the best of my knowledge, this paper is the first to study (the active choice of) gambling in an equilibrium one-to-one matching market. In the first stage, agents pick lotteries to change their matching characteristics, and in the second stage they match and bargain over their surplus so that a stable outcome—in which no pair of agents has an incentive to deviate from their current partners—is reached. Investment gambles in the first stage, the distribution of matching characteristics in the second stage, and the stable outcome—matching and surplus division—in the second stage are all endogenously determined in equilibrium. Studying these often overlooked risky investments in an equilibrium setting can yield surprising implications about private and social risk-taking.

The paper's three main results are as follows. First, it rationalizes risky investment choices before people enter the matching market. Second, the paper discovers a unique gamble-inducing effect inherent to competitive matching markets. Third, it derives implications about the trade-off between efficiency and inequality in this setting.

I demonstrate the existence and prevalence of profitable pre-matching gambles using examples. I start by presenting examples in which an agent may strictly prefer to take an actuarially unfair gamble. I follow with more examples in which (i) the supermodularity of the surplus function is not a crucial driver of gambling, and (ii) moderate risk aversion or moderate surplus concavity does not preclude agents from taking unfair risky pre-matching gambles.

Then, I show that the competitive nature of the matching market encourages and induces pre-matching risk-taking. The stability condition in the matching market implies that no pair of individuals can benefit from deviating from the current stable match. Reinterpreting the stability condition for each individual agent implies that *each agent is matched with the partner who gives him or her the highest attainable payoff.* Hence, for each realization of a gamble, an agent is matched with the partner who gives him or her the highest attainable payoff. Intuitively speaking, for an agent who gambles, *matching based on a realized characteristic is an additional benefit if the realized characteristic increases after gambling and is insurance if the realized characteristic decreases after gambling.* This gamble-inducing *competitive matching effect*, as I call it, is exclusive to the competitive matching market. I decompose the payoff gain of a gamble into the always gamble-inducing competitive matching effect and a term that increases in the convexity of the surplus function, which I call the *surplus contribution effect.* I provide conditions under which the competitive matching effect dominates the surplus contribution effect.

Finally, risk-taking, despite its usual negative connotations, is not necessarily undesirable in the current matching setting. Gambles can be beneficial to private as well as social welfare. I will present an example in which there are two equilibria: an efficient equilibrium in which everyone gambles and an inefficient equilibrium in which no one gambles. When multiple equilibria arise, carefully designed tax schemes can eliminate the inefficient equilibrium. I will show that a progressive tax scheme can effectively enhance social efficiency, reduce inequality, and generate positive tax revenue.

After a brief literature review, the rest of the paper proceeds as follows. Section 2 provides examples in which agents could find profitable pre-matching gambles. Section 3 describes the model. Section 4 highlights how the competitive nature of the matching market can drive pre-matching gambles. Section 5 discusses the implications for efficiency, inequality, and progressive taxation. Section 6 concludes.

1.1. Related literature

This paper investigates pre-matching investments with uncertain returns, extending previous models in which investments yield deterministic returns (Peters and Siow, 2002; Cole et al., 2001; Iyigun and Walsh, 2007; Chiappori et al., 2009; Mailath et al., 2013; Nöldeke and Samuelson, 2015; Mailath et al., 2016; Dizdar, 2018). Recent papers (Bhaskar and Hopkins, 2016; Chade and Lindenlaub, 2018; Chiappori et al., 2018; Zhang, 2019) also consider investments with uncertain returns, but agents in their models *cannot* choose the level of uncertainty they are exposed to. To be more specific, Bhaskar and Hopkins (2016) use heterogeneous ex post returns from the same investments to guarantee a unique equilibrium in a nontransferable utility matching model. Chade and Lindenlaub (2018) focus on comparative statics when one side of the market can make risky investments, and gambling on both sides is discussed only to a limited extent. College education in Chiappori et al. (2018) and Zhang (2019) results in uncertain income returns, but there is no choice in the models to mitigate or amplify the uncertainty.

This paper provides a new reason to gamble. Smith (1776) postulates that the primary reason for gambling is the social status associated with wealth. Modern treatment begins with Friedman and Savage (1948), who argued that utility over money is convex within a certain range, and people take risks if their wealth lies within that range. Their explanation for the convexity is that people gamble with wealth because of social status considerations, as wealth gain raises not only consumption but also one's social status. The social status concern is investigated in a market setting by Becker et al. (2005) and in a dynamic market setting by Ray and Robson (2012). This paper improves on Becker et al. (2005), who investigate the gambling incentives when social status can be bought and traded in an explicit or implicit market. Their paper emphasizes that the complementarity between money and status for utility is key to prompt risk-taking behavior that results in unequal

wealth distributions, and the fixed supply of "status goods" is also key to prompt people to compete and gamble. I show that surplus complementarity is no longer key in a two-sided hedonic market, and the fixed supply of status goods is also not important, because both sides can gamble and the wealth distributions are endogenously determined as a result.

This paper contributes to the idea that the marriage market can generate gambling. Rubin and Paul (1979) and Robson (1996) discuss incentives to gamble in order to obtain additional wives. Because of the discreteness of the number of wives a man can have, he would rather risk unfair lotteries to obtain enough wealth and resources for an additional wife than let some resources to remain idle for inefficient use. Those papers rely on both assumptions of nontransferable utility and polygamy to justify gambling. In contrast, this paper is in a transferable utility, one-to-one matching setting. In other words, previous papers consider how *quantity* can affect a man's gambling incentive in a polygamous society, whereas I consider how *quality* can induce similar gambling incentives in a monogamous society. In addition, the model herein can be extended to offer a set of testable implications on differences in risk-taking behavior by gender and marital status, which are elaborated on in a separate paper (Zhang, 2020).

Finally, the paper sheds light on the trade-off between efficiency and inequality, and offers insights on how a progressive taxation scheme can fix the problems. Rosen (1997) shows agents' willingness to gamble when higher wages enable one to move to a larger city with more abundant resources. In relation to the literature that investigates wealth redistribution, the paper establishes an inevitable relationship between efficiency and inequality in a matching market.

2. Motivating examples

Let me begin with an example in which agents always want to make a risky investment before they enter a matching market. Consider a simple continuum version of the standard Becker-Shapley-Shubik perfectly transferable utility matching market model (Koopmans and Beckmann, 1957; Shapley and Shubik, 1972; Becker, 1973; Gretsky et al., 1992). Namely, let there be masses 1 of men and women who have characteristics x_m and x_w uniformly distributed on $[0, 1]$.[2] A man with characteristic x_m and a woman with characteristic x_w produce a surplus of $s(x_m, x_w) = x_m x_w$ when they match. Men and women in the matching market match frictionlessly and bargain to divide their surplus. Let $v_m(x_m)$ denote the payoff an x_m man gets, and $v_w(x_w)$ the payoff an x_w woman gets. In a stable outcome, when an x_m man and an x_w woman match, they divide their surplus between them: $v_m(x_m) + v_w(x_w) = x_m x_w$. In addition, for any pair of an x_m man and an x_w woman, stability requires that no pair of agents can generate more surplus than in their current match: $v_m(x_m) + v_w(x_w) \geq x_m x_w$ (Roth and Sotomayor, 1990; Galichon, 2016; Chiappori, 2017).

The stable outcome of this matching market is simple to characterize. The result is well known: Supermodularity of the surplus function results in positive assortative matching, and every x_m man matches with an $x_w = x_m$ woman. Every x_m man and his wife of the same characteristic $x_w = x_m$ produce a surplus of x_m^2. Because an $x_m = 0$ man and an $x_w = 0$ woman receive zero, the division of the surplus is pinned down for every pair: Each x_m man gets $x_m^2/2$ and each x_w woman gets $x_w^2/2$. That is, $v_m(x_m) = x_m^2/2$ and $v_w(x_m) = x_w^2/2$.

Clearly, the payoff functions are convex in the agents' characteristics. A man of characteristic 0.5 has a payoff of $0.5^2/2 = 0.125$. Suppose that he can make a risky investment before he enters the matching market to change his matching characteristic. Namely, he can take an investment that makes him either a man of characteristic 0.4 with probability 1/2 or a man of characteristic 0.6 with probability 1/2. This investment keeps his expected characteristic of 0.5. When he becomes of characteristic 0.4, his partner will be a woman of characteristic 0.4. When he becomes of characteristic 0.6, he will match with a woman of characteristic 0.6. With probability one-half, his payoff is $0.4^2/2 = 0.08$, and with the other one-half probability, his payoff is $0.6^2/2 = 0.18$. The expected payoff is 0.13, which is higher than 0.125 (the payoff without gambling). Therefore, he prefers such a risky investment to the safe investment. In fact, of all the risky investments that have the same expected characteristic of 0.5, he prefers to take the investment that has the highest risk! When he takes the gamble that makes him of characteristic 1 with probability one-half and characteristic 0 with probability one-half, his expected payoff is $\frac{1}{2}(1^2/2) + \frac{1}{2}(0^2/2) = 0.25$—double the payoff if he does not gamble at all!

Moreover, the gain from gambling is so high that he may prefer an investment that is not only riskier but also yields a lower expected characteristic. For example, a man of characteristic 0.5 prefers to take a gamble that yields characteristic 1 with probability p and characteristic 0 with probability $1 - p$ as long as $p > 1/4$. The gamble only has an expected characteristic of p; the gamble is unfair if $p < 1/2$. Moreover, even if the man is risk-averse in payoffs, he can still prefer to take the second-order stochastically dominated gamble as long as he is not too risk-averse.

Recognize that if such a matching market does not exist and the man's partner is fixed to have characteristic x_w, then all of the different mean-preserving spread investments $(0.5, \frac{1}{2} \circ 0.4 + \frac{1}{2} \circ 0.6, \frac{1}{2} \circ 0 + \frac{1}{2} \circ 1$, etc.) yield him exactly the same expected payoff ($\mathbb{E} v_m(x_m) = \mathbb{E}[x_m x_w - v_w(x_w)] = 0.5x_w - v_w(x_w)$). Without the matching market, it is never optimal for a man to take a second-order stochastically dominated gamble. The stability of the matching market plays a quintessential role in inducing gambling. Intuitively speaking, the stable matching market provides the gambling incentive, because for different realizations of the gambles, the man matches with a different partner. A gamble changes the man's matching characteristic. More importantly, it changes the partner the man is matched with in a systematic way that encourages gambling. Stability of the matching market induces such a gambling incentive.

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[2] I name the two sides in the model men and women for expositional ease. Obviously, agents on the two sides of the market can also be managers and workers, doctors and hospitals, schools and students, or buyers and sellers.

The possible conjecture that supermodularity of the surplus function and the resulting positive-assortative matching, as in the example above, is necessary for gambling is not entirely correct. Even if the surplus function is completely submodular, we would find the same pre-matching gambling incentives, as the example below demonstrates.

Example 1 *(Gambling when surplus function is submodular)*. Suppose that the surplus function is $s(x_m, x_w) = x_m + x_w - x_m x_w$, which is submodular in x_m and x_w. The stable matching is negative assortative; that is, a man of x_m matches with a woman of $x_w = 1 - x_m$. The payoffs satisfy for all $x_m \in [0, 1]$, $v_m(x_m) + v_w(1 - x_m) = 1 - x_m + x_m^2$. Because $v_m(x_m) = \max_{x_w}[s(x_m, x_w) - v_w(x_w)]$, by the Envelope Theorem, which will be derived in Section 4.2, $v'_m(x_m) = \frac{\partial s(x_m, x_w(x_m))}{\partial x_m} = 1 - x_w(x_m) = x_m$, where $x_w(x_m)$ represents the partner of an x_m man. Coupled with the boundary conditions $v_m(0) + v_w(1) = v_m(1) + v_w(0) = 1$, we have $v_m(x_m) = x_m^2/2 + 1/4$ and $v_w(x_w) = x_w^2/2 + 1/4$. ∎

Another possible conjecture—that the surplus is homogeneous of degree $r > 1$ is necessary for gambling—is also not correct. Gambling incentives exist even when the surplus function is homogeneous of degree $r < 1$, as the following example demonstrates. The distribution of characteristics on both sides of the market plays a role.

Example 2 *(Gambling when the surplus function is homogeneous of degree $r < 1$)*. Suppose the surplus function is $s(x_m, x_w) = x_m^{1/2} x_w^{1/2}$ and the distributions of men's and women's characteristics are different: $\widehat{F}_m(x_m) = x_m^2$ and $\widehat{F}_w(x_w) = x_w$. In the stable outcome, each x_m man matches with an $x_w = x_m^2$ woman, and the payoffs are $v_m(x_m) = \frac{1}{3} x_m^{\frac{3}{2}}$ and $v_w(x_w) = \frac{2}{3} x_w^{\frac{3}{4}}$.[3] Since the payoff function for men is convex, men want to gamble–even to take extreme gambles–although the surplus function is homogenous of degree 1. Even if the surplus function is homogeneous of degree $r < 1$, $s(x_m, x_w) = x_m^{r/2} x_w^{r/2}$, and the distributions of men's and women's characteristics are $\widehat{F}_m(x_m) = x_m^2$ and $\widehat{F}_w(x_w) = x_w$, men's payoff function is $v_m(x_m) = \frac{2}{2+r} x^{1 + \frac{r}{2}}$, which is always convex when $r > 0$. For certain distributions, both men and women have incentives to gamble. ∎

In fact, the gambling incentive exists not only independent of the surplus function, but also absent the particular distributions of the matching characteristics in the market, the masses of men and women in the market, and the dimensionality of the characteristics.

The stability condition is the central driving force of the gambling incentive. This is best seen from a reinterpretation of the stability condition. Stability guarantees that no pair of agents wants to deviate from their current match and form a new match. For any individual agent, *stability guarantees that each agent in the market is matched with the partner that gives him or her the highest personal payoff*, although the partner may not be ranked the best by sheer personal characteristic. Although each agent is free to give up more personal gain to match with a partner who has a higher characteristic, it is not in his or her best interest to do so.

The mathematical formulation of the economic argument in the previous paragraph is as follows. Stability dictates that a particular x_m^* man who matches with an x_w^* woman has a payoff of $v_m(x_m^*) = s(x_m^*, x_w^*) - v_w(x_w^*)$. If he matches with any other $x_w \neq x_w^*$ woman and pays her the competitive payoff $v_w(x_w)$, his payoff would be $s(x_m^*, x_w) - v_w(x_w)$. By the stability condition, for any x_m and x_w, $v_m(x_m) + v_w(x_w) \geq s(x_m, x_w)$. Hence, $v_m(x_m^*) \geq s(x_m^*, x_w) - v_w(x_w)$ for any $x_w \neq x_w^*$. The argument holds for any x_m^* and for any distribution of men and women.

Let's see how this reinterpretation of the stability condition helps us understand the gambling incentive when the surplus is $x_m x_w$, independent of the distribution of men and women. An x_m^* man matches with an x_w^* woman (note that I am not relying on the positive assortative matching resulting from surplus supermodularity), and they divide the surplus, $v_m(x_m^*) = x_m^* x_w^* - v_w(x_w^*)$. Suppose that the man takes a gamble that makes him $x_m^* - \epsilon$ with probability $1/2$ and $x_m^* + \epsilon$ with probability $1/2$. After he realizes the outcome from the gamble, he will get a new partner according to the new outcome. No matter who the new match is, we know by stability that he is guaranteed a payoff $x_m x_w^* - v_w(x_w^*)$ by matching with x_w^*. That is, for any realization of x_m, $v_m(x_m) \geq x_m x_w^* - v_w(x_w^*)$. The expected payoff of the gamble is thus bigger than $\frac{1}{2}[(x_m - \epsilon)x_w^* - v_w(x_w^*)] + \frac{1}{2}[(x_m + \epsilon)x_w^* - v_w(x_w^*)] = x_m x_w^* - v_w(x_w^*)$, which is exactly the payoff $v_m(x_m^*)$ without gambling. As long as either $x_m^* + \epsilon$ or $x_m^* - \epsilon$ does not match with x_w^*, gambling yields a strictly bigger expected utility.

3. Model

Let me set up the general model and define the equilibrium of the model in this section. There is a continuum of men and women with innate characteristics $\widehat{x}_m \in \widehat{X}_m \subset \mathbb{R}^{N_m}$ and $\widehat{x}_w \in \widehat{X}_w \subset \mathbb{R}^{N_w}$, which are possibly multidimensional. Measures $\widehat{\mu}_m$ and $\widehat{\mu}_w$ describe the measures of men's and women's innate characteristics, respectively. The mass of men,

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[3] The derivation of the stable payoffs is as follows. The total surplus an x_m man and his partner $x_w = x_m^2$ generate is $x_m^{\frac{3}{2}}$. The stable payoff satisfies a condition that will be fully derived in Section 4.2: $v'_m(x_m) = s_1(x_m, x_w(x_m))$. According to the condition, $v'_m(x_m) = \frac{1}{2} x_m^{-\frac{1}{2}} (x_m^2)^{\frac{1}{2}} = \frac{1}{2} x_m^{\frac{1}{2}}$. Using the boundary condition $v_m(0) = 0$, we derive men's stable payoff function to be $v_m(x_m) = \frac{1}{3} x_m^{\frac{3}{2}}$.

$\widehat{\mu}_m(\widehat{X}_m)$, and that of women, $\widehat{\mu}_w(\widehat{X}_w)$, are not necessarily equal.[4] All agents are risk-neutral and derive utilities from the payoffs in the matching market.

3.1. Gambling phase

Each agent of innate characteristic $\widehat{x}$ can pick a gamble $\gamma(\cdot|\widehat{x}) \in \Gamma(\widehat{x})$ that alters his or her innate characteristic $\widehat{x}$, where γ represents the probability measure of realized matching characteristics. Assume the degenerate investment gamble that keeps the innate characteristic (i.e., $\gamma_0(\widehat{x}|\widehat{x}) = 1$) is always feasible. In addition, for each $\widehat{x}$, the set of feasible investments $\Gamma(\widehat{x})$ is compact. All gambles have the same expected characteristic: $\int x d\gamma(x|\widehat{x}) = \widehat{x}$. Let $\sigma_m : \widehat{x}_m \mapsto \gamma \in \Gamma(\widehat{x}_m)$ denote men's investment strategy and $\sigma_w : \widehat{x}_w \mapsto \gamma \in \Gamma(\widehat{x}_w)$ denote women's investment strategy. Let σ_m and σ_w be measurable. Let X_m and X_w denote the full support of possible realized characteristics. Since keeping the innate characteristic is always feasible, any innate characteristic is also a possible realized characteristic: $X_m \supseteq \widehat{X}_m$ and $X_w \supseteq \widehat{X}_w$. After the agents choose the investments and before they enter the matching market, their matching characteristics x_m and x_w are realized.

3.2. Matching phase

Let μ_m and μ_w represent the measures of men's and women's realized characteristics after they make their investment decisions. Let $s(x_m, x_w)$ denote the surplus produced by a match between an x_m man and an x_w woman. Suppose that remaining single yields zero surplus.[5] Assume that s is continuous in the two arguments. In the matching market (μ_m, μ_w), men and women frictionlessly match and bargain. They divide their surplus in a perfectly transferable way.

A stable outcome of the matching market consists of a matching measure denoted by μ on $X_m \times X_w$ and payoff functions $v_m : X_m \to \mathbb{R}_+$ and $v_w : X_w \to \mathbb{R}_+$. Measure $\mu(\widetilde{X}_m, \widetilde{X}_w)$ describes the measure of matches between $x_m \in \widetilde{X}_m$ and $x_w \in \widetilde{X}_w$, and $v_m(x_m)$ and $v_w(x_w)$ describe the payoff an x_m man and an x_w woman get, respectively. The following conditions are satisfied: (i) μ has marginals μ_m and μ_w; (ii) $v_m(x_m) + v_w(x_w) = s(x_m, x_w)$ when $(x_m, x_w) \in \text{supp}(\mu)$ where supp represents the support of the measure; (iii) $v_m(x_m) + v_w(x_w) \geq s(x_m, x_w)$ for any $(x_m, x_w) \in \text{supp}(\mu_m) \times \text{supp}(\mu_w)$[6]; and (iv) for any $x_m \in X_m$ and $x_w \in X_w$ outside of the supports $\text{supp}(X_m)$ and $\text{supp}(X_w)$, the payoff functions are defined as

$$v_m(x_m) = \max_{x_w \in \text{supp}(X_w)} [s(x_m, x_w) - v_w(x_w)], \text{ and } v_w(x_w) = \max_{x_m \in \text{supp}(X_m)} [s(x_m, x_w) - v_m(x_m)].$$

Condition (i) is the feasibility condition for the matching. Conditions (ii) and (iii) are the conditions under which any matched pair splits their surplus, and that two agents not matched with each other cannot deviate and form a pair to have both of their payoffs strictly improve. Condition (iv) deals with defining the payoff of the characteristics outside of the post-gambling characteristics support. The payoff functions need to be well-defined on the entire support of possible realized characteristics in order for the agents to make gambling decisions.

3.3. Equilibrium

The primitives of the model consist of the measures of innate characteristics $\widehat{\mu}_m$ and $\widehat{\mu}_w$, the feasible set of gambles $\Gamma(\cdot)$, and the surplus function s. An equilibrium

$$(\sigma_m^*, \sigma_w^*, \mu_m^*, \mu_w^*, \mu^*, v_m^*, v_w^*)$$

consists of equilibrium investment strategies σ_m^* and σ_w^*, equilibrium measures μ_m^* and μ_w^* of characteristics, and equilibrium matching outcome (μ^*, v_m^*, v_w^*) such that (i) the equilibrium strategies σ_m^* and σ_w^* maximize the agents' expected payoffs; (ii) the equilibrium measures of characteristics, μ_m^* and μ_w^*, are induced by the equilibrium strategies σ_m^* and σ_w^*; and (iii) the equilibrium matching market outcome (μ^*, v_m^*, v_w^*) is a stable outcome of the equilibrium matching market (μ_m^*, μ_w^*). An equilibrium exists by Zhang (2015).

Theorem 1. *An equilibrium exists.*

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[4] However, we can always add a few degenerate agents who produce zero surplus in marriages on the short side to balance the two sides.

[5] To relax this assumption, suppose the payoff of being single is not zero, and it depends on each individual's ex post matching characteristic. Define the payoff of a type x_m man being single as $s_m(x_m)$, with $s_w(x_w)$ similarly defined for any woman. Because s_m and s_w are the reservation payoffs, they get at least these payoffs regardless of who they marry. An x_m man's total payoff becomes $s_m(x_m) + v_m(x_m)$. The convexity of s_m would contribute to the incentive to gamble in straightforward ways. Therefore, to focus on the gamble-inducing market effects, I mute this effect in the main text of the paper.

[6] To include the possibility that agents choose to remain single and they satisfy the participation constraint, I augment X_m and X_w to include type $\emptyset$ and define $v_m(\emptyset) = v_w(\emptyset) = s(x_m, \emptyset) = s(\emptyset, x_m) = 0$, so that $v_m(x_m) \geq 0$ and $v_w(x_w) \geq 0$.

4. Competitive matching effect

4.1. Dominant extreme gambles under linear surplus

When the surplus function is linear in matching characteristics, it is a weakly dominant strategy to pick the extreme lotteries that realize either the highest possible characteristic or the lowest possible characteristic. It follows that there exists an equilibrium in which every man and every woman chooses the extreme lottery when the surplus function is bilinear.

Definition 1. An **extreme lottery** $\gamma(\cdot|\widehat{x})$ for an agent of innate characteristic $\widehat{x}$ is to realize only the highest or the lowest possible characteristic. The gamble yields $\overline{x}$ with probability p and $\underline{x}$ with probability $1-p$ so that $p \cdot \overline{x} + (1-p) \cdot \underline{x} = \widehat{x}$.

Note that when x is multidimensional, p is a vector.

Proposition 1. If the surplus function $s(x_m, x_w)$ is linear in x_m, every man's unique weakly dominant strategy is the extreme lottery.

First, when the surplus is linear in men's characteristic, every man prefers any fair gamble to the degenerate gamble.

Lemma 1. If $s(x_m, x_w)$ is linear in x_m, every man prefers a gamble to any second-order stochastically dominant gamble that generates the same expected characteristics.

Note that the result is independent of μ_m and μ_w, the distributions of characteristics.

Proof of Lemma 1. For an $\widehat{x}_m$ man, the expected payoff of taking the degenerate gamble $\gamma_0(\cdot|\widehat{x})$ is $u_m(\gamma_0|\widehat{x}) = v_m(\widehat{x})$, and the expected payoff of taking any other fair gamble $\gamma(\cdot|\widehat{x})$ is $u_m(\gamma|\widehat{x}_m) = \int v_m(x_m) d\gamma(x_m|\widehat{x}_m)$. Suppose $\widehat{x}_m$ is matched with an x_w^* woman, and they divide the surplus:

$$v_m(\widehat{x}_m) = s(\widehat{x}_m, x_w^*) - v_w(x_w^*).$$

By stability, for any $x_m \neq \widehat{x}_m$,

$$v_m(x_m) \geq s(x_m, x_w^*) - v_w(x_w^*),$$

that is, any other man's payoff cannot be worse than matching with the x_w^* woman. Plugging in the inequality, the expected utility of taking gamble γ is

$$u_m(\gamma|\widehat{x}_m) = \int v_m(x_m) d\gamma(x_m|\widehat{x}_m)$$
$$\geq \int [s(x_m, x_w^*) - v_w(x_w^*)] d\gamma(x_m|\widehat{x}_m) = \int s(x_m, x_w^*) d\gamma(x_m|\widehat{x}_m) - v_w(x_w^*).$$

Since s is linear in x_m,

$$\int s(x_m, x_w) d\gamma(x_m|\widehat{x}_m) = s\left(\int x_m d\gamma(x_m|\widehat{x}_m), x_w\right).$$

Because $\gamma(x_m|\widehat{x}_m)$ is fair, $\int x_m d\gamma(x_m|\widehat{x}_m) = \widehat{x}_m$. Hence,

$$u_m(\gamma|\widehat{x}_m) \geq s(\widehat{x}_m, x_w^*) - v_w(x_w^*) = v_m(\widehat{x}_m) = u_m(\gamma_0|\widehat{x}_m). \quad \square$$

To prove Proposition 1—that a second-order stochastically dominated fair gamble γ is always preferred to a second-order stochastically dominant fair gamble γ'—recognize that the second-order stochastically dominated gamble γ can be decomposed into the second-order stochastically dominant gamble γ' and another non-degenerate gamble. As the proof of Lemma 1 demonstrates, a gamble is always preferred to no gamble. Regardless of the innate characteristic, a man always prefers to have taken an even riskier gamble! The detailed proof is shown below.

Proof of Proposition 1. Consider a gamble γ and a gamble γ' that second-order stochastically dominates gamble γ. Let x_γ denote the random variable of the dominated gamble and $x_{\gamma'}$ the random variable of the dominant gamble. There exists a random variable z such that $x_\gamma = x_{\gamma'} + z$, $\mathbb{E}[z|x_{\gamma'}] = 0$. That is, x_γ can be obtained by adding mean-zero noise to $x_{\gamma'}$. Let $x_w(x_m)$ denote the partner an x_m man is matched with (if the match is not pure, so that a man can be matched to women of different characteristics with positive probability, randomly pick any one of the women to match). For any $\widehat{x}_m$,

$$v_m(\widehat{x}_m) = s(\widehat{x}_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m)).$$

And for any $x_m \neq \widehat{x}_m$,

$$v_m(x_m) \geq s(x_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m)).$$

The inequality, linearity of the surplus function, and decomposition of x_γ into $x_{\gamma'}$ and z are used in the following derivation:

$$
\begin{aligned}
u_m(\gamma|\widehat{x}_m) &= \int v_m(x_m) d\gamma'(x_m|\widehat{x}_m) \\
&= \int [s(x_m, x_w(x_m)) - v_m(x_w(x_m))] d\gamma'(x_m|\widehat{x}_m) \\
&= \int \left[\left[\int s(x_m + z, x_w(x_m)) d\gamma_z(z) - v_m(x_w(x_m)) \right] d\gamma'(x_m|\widehat{x}_m) \right. \\
&= \int \int [s(x_m + z, x_w(x_m)) - v_m(x_w(x_m))] d\gamma_z(z) d\gamma'(x_m|\widehat{x}_m) \\
&\leq \int \int v_m(x_m + z) d\gamma_z(z) d\gamma'(x_m|\widehat{x}_m) \\
&\leq \int v_m(x_m) d\gamma(x_m|\widehat{x}_m) \\
&= u_m(\gamma'|\widehat{x}_m).
\end{aligned}
$$

Therefore, I show that $u_m(\gamma|\widehat{x}_m) \leq u_m(\gamma'|\widehat{x}_m)$. □

Proposition 2. *Suppose the surplus function $s(x_m, x_w)$ is bilinear. There exists an equilibrium in which every agent takes the extreme gamble.*

Proof of Proposition 2. Because $s(x_m, x_w)$ is linear in x_m, by Proposition 1, every man's weakly dominant strategy is to choose the extreme lottery. Similarly, because $s(x_m, x_w)$ is linear in x_w, every woman's weakly dominant strategy is to choose the extreme lottery. Therefore, everyone choosing the extreme lottery is an equilibrium. □

Two comments follow. First, note that bilinearity does not imply supermodularity of the surplus function. Whereas $s(x_m, x_w) = x_m x_w$ is bilinear and strictly supermodular, $s(x_m, x_w) = x_m + x_w - x_m x_w$ is bilinear and strictly submodular. The weak dominance of extreme gamble holds when the surplus function is bilinear, even if it is not supermodular. Second, the equilibrium in which everyone takes the extreme gamble is not necessarily the unique equilibrium. Section 5 is based on an example with multiple equilibria.

4.2. Isolating the competitive matching effect

The motivating example in Section 2 and the results, which show that extreme gambles are the unique dominant strategies under linear surplus, suggest that the stable payoff functions are inherently convex and that the stable and competitive organization of the matching market itself contributes to the payoff convexity. I will show below that an inherent competitive matching effect always generates convexity in the stable payoff functions, and thus induces pre-matching gambling. Notably, this gamble-inducing effect arises solely as a byproduct of the stability conditions, and does not depend on any other structural assumption.

In essence, the stable matching based on realized characteristics provides an implicit but persistent benefit for gambling. In any stable outcome, a man cannot do strictly better by matching with a partner different from the one he is matched with and paying the new partner at least her competitive payoff. Every man is matched with the partner that maximizes his marital payoff, and every woman is matched with the partner that maximizes her marital payoff.

When a man gambles, no matter the realization, he is matched with a partner who guarantees him a (weakly) higher payoff than he would get otherwise with the partner he's matched with if he does not gamble. This competitive matching effect always brings an extra benefit for the agent who gambles. Competitive matching based on realized characteristics is treated as a benefit for the agent who realizes a higher characteristic and as insurance if the realized characteristic is lower than before gambling.

Let me formally demonstrate the gamble-inducing competitive matching effect. Consider a matching market described by measures μ_m and μ_w of realized characteristics. Take any man x_m^*. Recall the stability conditions that stable matching and payoffs satisfy. For any woman $x_w \in \text{supp}(\mu_w)$,

$$v_m(x_m^*) \geq s(x_m^*, x_w) - v_w(x_w).$$

If an x_m^* man and an $x_w(x_m^*)$ woman are matched, i.e., $(x_m^*, x_w(x_m^*)) \in \text{supp}(\mu)$, then

$$v_m(x_m^*) = s(x_m, x_w(x_m^*)) - v_w(x_w(x_m^*)).$$

The right-hand side $s(x_m, x_w) - v(x_w)$ represents an x_m man's payoff if he marries an x_w woman and pays her the competitive market value $v_w(x_w)$. In the stable outcome, x_m^* is matched with $x_w(x_m^*)$ only if for all x_w,

$$s(x_m^*, x_w(x_m^*)) - v_w(x_w(x_m^*)) \geq s(x_m^*, x_w) - v_w(x_w).$$

That is, in any stable outcome, *every man marries the woman who gives him the highest possible private payoff*, i.e.,

$$v_m(x_m^*) = \sup_{x_w \in \text{supp}(\mu_w)} [s(x_m^*, x_w) - v_w(x_w)].$$

The same statement can be made about women: In any stable outcome, *every woman marries the man who gives her the highest possible private payoff*, i.e.,

$$v_w(x_w) = \sup_{x_m \in \text{supp}(\mu_m)} [s(x_m, x_w) - v_m(x_m)].$$

After a man of innate characteristic $\widehat{x}_m$ takes a gamble and realizes a characteristic x_m, his partner in the market changes, and the payoff he gets by matching with the new partner exceeds the payoff he gets by matching with the original pre-gamble partner $x_w(\widehat{x}_m)$, i.e.,

$$v_m(x_m) = s(x_m, x_w(x_m)) - v_w(x_w(x_m)) \geq s(x_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m)). \tag{1}$$

Compare a feasible fair lottery $\gamma(\cdot|\widehat{x}_m)$ for an $\widehat{x}_m$ man with his degenerate gamble γ_0. The difference between the expected payoffs from the two decisions is

$$u_m(\gamma|\widehat{x}_m) - u_m(\gamma_0|\widehat{x}_m) = \mathbb{E}_\gamma[s(x_m, x_w(x_m)) - v_w(x_w(x_m))] - [s(\widehat{x}_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))].$$

Subtract and add the same term $\mathbb{E}_\gamma[s(x_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))]$, the hypothetical expected payoff $\widehat{x}_m$ would receive by taking gamble γ, matching with woman $x_w(\widehat{x}_m)$, and transferring $v_w(x_w(\widehat{x}_m))$ to the partner. The expected payoff difference $u_m(\gamma|\widehat{x}_m) - u_m(\gamma_0|\widehat{x}_m)$ is rewritten as

$$\mathbb{E}_\gamma[s(x_m, x_w(x_m)) - v_w(x_w(x_m))] - \mathbb{E}_\gamma[s(x_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))]$$
$$+$$
$$\mathbb{E}_\gamma[s(x_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))] - [s(\widehat{x}_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))].$$

Consider the first two terms together and the last two terms together. The expected payoff difference is expressed as the sum of two effects,

$$\underbrace{\mathbb{E}_\gamma\{[s(x_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))] - [s(\widehat{x}_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))]\}}_{\text{surplus contribution effect}}$$
$$+$$
$$\underbrace{\mathbb{E}_\gamma\{[s(x_m, x_w(x_m)) - v_w(x_w(x_m))] - [s(x_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))]\}}_{\text{competitive matching effect}}.$$

The first combined term represents the difference between two expected payoffs: (1) the expected payoff $\widehat{x}$ gets by taking the gamble γ but matching with the partner $x_w(\widehat{x}_m)$ he would have matched with without gambling, and (2) the (expected) payoff of $\widehat{x}_m$ for not gambling and always matching with $x_w(\widehat{x}_m)$. Since the wife always gets her payoff $v_w(x_w(\widehat{x}_m))$ regardless of the realization of the characteristic, the first combined term can be simplified to

$$\underbrace{\mathbb{E}_\gamma[s(x_m, x_w(\widehat{x}_m))] - s(\widehat{x}_m, x_w(\widehat{x}_m))}_{\text{surplus contribution effect}}.$$

If the surplus function is convex in x_m—i.e., the marginal surplus increases as a man's characteristic increases—then the effect is positive. If the surplus function is concave—i.e., the marginal surplus decreases as the man's characteristic increases—then the effect is negative. I call this term the **surplus contribution effect**, since its sign depends on the slope of the marginal surplus function (equivalently, the convexity of the surplus function). If the surplus function is linear in x, then this term is always zero and this effect does not affect gambling incentives at all. Therefore, the previous results—namely, Lemma 1 and Proposition 1—on beneficial gambling under a bilinear surplus function must be driven by the second effect.

The second combined term represents the expected payoff difference from optimal partner matching based on different gambling realizations. For any realized x_m,

$$[s(x_m, x_w(x_m)) - v_w(x_w(x_m))] - [s(x_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))]$$

represents the difference between (i) the maximal payoff man x_m can get by matching with $x_w(x_m)$ and (ii) the possibly non-optimal payoff x_m gets by matching with $x_w(\widehat{x}_m)$. By Equation (1) and that $s(x_m, x_w(x_m)) - v_w(x_w(x_m))$ is the maximal payoff for x_m, the payoff difference is nonnegative for any realized x_m. Since the payoff difference is nonnegative for any realization, the expected payoff difference over all possible realizations is always nonnegative. I call the second term the **competitive matching effect**, because the payoff gain comes from matching the agent to an optimally chosen partner in the competitive market. This competitive matching benefit gives a persistent reason for agents to gamble.

Note that the competitive matching effect does not depend on any assumption about the shape of the surplus function or the distributions of the matching characteristics, but rather comes solely from the stability condition. The stability condition is reinterpreted to be a competitive condition, such that the stability guarantees the agents a competitive maximal payoff from their partners.

To emphasize that stability induces gambles regardless of the shape of the surplus function, let's take a price-theoretic approach to see why, in particular, the surplus supermodularity assumption is dispensable. Consider a matching market in which the matching characteristics are one-dimensional. Suppose that the mass distributions F_m and F_w that represent the distributions of the one-dimensional characteristics are strictly increasing and twice differentiable on full supports $X_m \in \mathbb{R}_+$ and $X_w \in \mathbb{R}_+$ and that s is strictly supermodular and twice differentiable. The stable matching is positive assortative: $x_w(x_m) = F_w^{-1}(F_m(x_m))$ is strictly increasing and bijective. Moreover, v_m and v_w are differentiable. An x_m man's payoff is the surplus x_m and $x_w(x_m)$ produce together net the payoff of $x_w(x_m)$ woman,

$$v_m(x_m) = s(x_m, x_w(x_m)) - v_w(x_w(x_m)).$$

Each man is paired with the woman who maximizes his payoff, so we have the first-order condition

$$s_2(x_m, x_w(x_m)) - v'_w(x_w(x_m)) = 0. \tag{2}$$

Let's examine the convexity of the continuous and twice differentiable payoff function v_m. Differentiate $v_m(x_m)$ with respect to x_m, and utilize the envelope theorem:

$$v'_m(x_m) = s_1(x_m, x_w(x_m)) + [s_2(x_m, x_w(x_m)) - v'_w(x_w(x_m))]x'_w(x_m).$$

By the first-order condition in Equation (2), the second term is zero, and $v'_m(x)$ is simply the marginal surplus of x_m given the optimal partner $x_w(x_m)$,

$$v'_m(x_m) = s_1(x_m, x_w(x_m)),$$

a standard and widely known result in matching literature (Galichon, 2016; Chiappori, 2017). Differentiate $v'_m(x_m)$ with respect to x_m,

$$v''_m(x_m) = \underbrace{s_{11}(x_m, x_w(x_m))}_{\text{surplus contribution effect}} + \underbrace{s_{12}(x_m, x_w(x_m))x'_w(x_m)}_{\text{stable rematching effect}}.$$

The two terms correspond to the two effects described above: The surplus contribution effect and the competitive matching effect. The second term—the competitive matching effect—is unambiguously weakly positive. When the surplus function is strictly supermodular, $s_{12} > 0$, the stable matching is positive assortative, so $x'_w(x_m) > 0$ and the effect is positive.

When the surplus is strictly supermodular, it is straightforward to understand gambling incentives to improve expected equilibrium marginal surplus. With a supermodular surplus function, an agent's own marginal surplus increases in the partner's characteristic, so the agent has an incentive to take a fair gamble to be matched with a better partner and enjoys a higher marginal surplus. The gambling incentives in the one-sided hedonic market in Rosen (1997) and Becker et al. (2005), for example, crucially depend on the assumption of complementarity.

Although the gambling incentives in the matching market can be justified in the same way when the surplus is supermodular, that does not imply that gambling incentives crucially depend on the supermodularity assumption. Take the extreme opposite case in which the surplus function is submodular, $s_{12} < 0$ (e.g., $s(x_m, x_w) = x_m + x_w - x_m x_w$ for $x_m, x_w \in [0, 1]$). An agent's marginal surplus *decreases* as the partner's characteristic increases. However, in the matching market, the agent is matched with a partner based on the realized characteristic. A higher realized characteristic results in a partner with a *lower* characteristic when the surplus is submodular. Consequently, when an agent realizes a higher characteristic, his competitively matched partner has a lower characteristic than if he does not take the gamble; when an agent realizes a lower characteristic, his stably assigned partner has a higher characteristic. Isolating the competitive matching effect, $s_{12} < 0$ and $x'_w(x_m) < 0$ imply $s_{12}(x_m, x_w(x_m))x'_w(x_m) > 0$.

We see from the elaborations above that the competitive matching effect always contributes to the convexity of the payoff function regardless of the underlying surplus function. Suppose the surplus function is strictly supermodular for certain pairs (x_m, x_w) and strictly submodular for other pairs (x_m, x_w). When $s_{12} > 0$, the stable matching is locally positive assortative. On the other hand, when $s_{12} < 0$, the stable matching is locally negative assortative. Therefore, $s_{12}(x_m, x_w(x_m))x'_w(x_m) \geq 0$ for all pairs of (x_m, x_w). As long as the surplus contribution effect is not significantly negative, the competitive matching effect always encourages gambling behavior.

An important condition that guarantees a strict gambling incentive for agents and how much gambling the agents do is the degree of diversity on the opposite side of the market. Take the extreme case in which all women are born identical and do not gamble. Then $x_w(x_m) = x_w(x'_m)$ $\forall x_m \neq x'_m$. The competitive matching effect disappears completely, because it crucially depends on the positive probability that men match with different partners after gambling. Conversely, if the other side of the two-sided market is diverse, then gambling becomes more attractive. Mathematically,

$$v_m(x_m) = [s(x_m, x_w(x_m)) - v_w(x_w(x_m))] = \sup_{x_w \in \mathrm{supp}(\mu_w)} [s(x_m, x_w) - v_w(x_w)].$$

As the diversity of women's matching characteristics increases, $\mathrm{supp}(\mu_w)$ expands, and any man's optimal payoff weakly increases. The payoff gain due to competitive matching increases without affecting the magnitude of the surplus contribution effect. The degree of diversity is important in guaranteeing the uniqueness of the equilibrium with socially efficient investments in Cole et al. (2001). It also will play a crucial role in this model. In Section 5, I present an example with homogeneous agents on both sides of the market and multiple equilibria.

The dissection into the surplus contribution effect and the competitive matching effect helps us understand our results in the previous subsection with a bilinear surplus function. When the surplus function is linear in men's characteristics, the surplus contribution effect disappears for men. When the surplus function is bilinear (linear in both men's and women's characteristics), the surplus contribution effect disappears for both men and women. The convexity of the stable payoff function hinges on the competitive matching effect. In the special case of a bilinear surplus function, $s_{11} = s_{22} = 0$, the surplus contribution effect is mute and the stable payoff functions exhibit weak convexity generally and strict convexity when the agents are sufficiently heterogeneous.

To see how much surplus concavity and homogeneity in matching characteristics compete with the competitive matching effect, consider the following tractable example with a simple condition under which gambles are profitable.

Example 3 *(Surplus contribution effect versus competitive matching effect).* Suppose the innate characteristics are distributed according to cumulative distribution functions $F_m(\widehat{x}_m) = \widehat{x}_m$ and $F_w(\widehat{x}_w) = \widehat{x}_w^{1/\alpha}$ on $[0, 1]$. The marital surplus function is $s(x_m, x_w) = x_m^{r_m} x_w^{r_w}$. The stable matching is represented by $x_w(x_m) = x_m^{\alpha}$ and, equivalently, $x_m(x_w) = x_w^{1/\alpha}$. Stable payoffs are

$$v_m(x_m) = \frac{r_m}{r_m + \alpha r_w} x_m^{r_m + \alpha r_w} \quad \text{and} \quad v_w(x_w) = \frac{\alpha r_w}{r_m + \alpha r_w} x_w^{r_m/\alpha + r_w}.$$

Men's stable payoff functions are derived from $v'_m(x_m) = s_1(x_m, x_w(x_m)) = r_m x_m^{r_m + \alpha r_w - 1}$ for any $x_m \in [0, 1]$ and $v_m(0) = 0$. Women's stable payoff functions are derived from $v_w(0) = 0$ and $v'_w(x_w) = s_2(x_m(x_w), x_w) = r_w x_w^{r_m/\alpha + r_w - 1}$ for any $x_w \in [0, 1]$.

Consider an $\widehat{x}_m$ man who chooses a fair gamble γ. The expected payoff gain is

$$\mathbb{E}_\gamma[v_m(x_m)] - v_m(\widehat{x}_m).$$

According to the decomposition, the surplus contribution effect is

$$\mathbb{E}_\gamma[s(x_m, x_w(\widehat{x}_m))] - s(\widehat{x}_m, x_w(\widehat{x}_m)),$$

which is

$$\mathbb{E}_\gamma\left[x_m^{r_m} \widehat{x}_m^{\alpha r_w}\right] - \widehat{x}_m^{r_m}\widehat{x}_m^{\alpha r_w} = \left[\mathbb{E}_\gamma\left[x_m^{r_m}\right] - \widehat{x}_m^{r_m}\right]\widehat{x}_m^{\alpha r_w} = \mathbb{E}_\gamma\left\{x_m^{r_m}\widehat{x}_m^{\alpha r_w} - \widehat{x}_m^{r_m + \alpha r_w}\right\}.$$

As mentioned above and as one can see directly from the expression, the surplus contribution effect is positive/zero/negative if r_m is greater than/equal to/smaller than 1.

The competitive matching effect is

$$\mathbb{E}_\gamma\left\{[s(x_m, x_w(x_m)) - v_w(x_w(x_m))] - [s(x_m, x_w(\widehat{x}_m)) - v_w(x_w(\widehat{x}_m))]\right\}$$

$$= \mathbb{E}_\gamma\left\{\left[s(x_m, x_m^{\alpha}) - v_w(x_m^{\alpha})\right] - \left[s(x_m, \widehat{x}_m^{\alpha}) - v_w(\widehat{x}_m^{\alpha})\right]\right\}$$

$$= \mathbb{E}_\gamma\left\{\left[\frac{r_m}{r_m + \alpha r_w} x_m^{r_m + \alpha r_w}\right] - \left[x_m^{r_m}\widehat{x}_m^{\alpha r_w} - \frac{\alpha r_w}{r_m + \alpha r_w}\widehat{x}_m^{r_m + \alpha r_w}\right]\right\}$$

$$= \mathbb{E}_\gamma\left\{\frac{r_m}{r_m + \alpha r_w} x_m^{r_m + \alpha r_w} + \frac{\alpha r_w}{r_m + \alpha r_w}\widehat{x}_m^{r_m + \alpha r_w} - x_m^{r_m}\widehat{x}_m^{\alpha r_w}\right\}.$$

The positive competitive matching effect dominates the negative surplus contribution effect if and only if

$$\frac{r_m}{r_m + \alpha r_w} \left\{ \mathbb{E}_\gamma \left[x_m^{r_m + \alpha r_w} \right] - \left[\widehat{x}_m^{r_m + \alpha r_w} \right] \right\} \geq 0,$$

which boils down to

$$r_m + \alpha r_w \geq 1.$$

Hence, in this setting, the surplus function can still be concave (i.e., $r_m < 1$) for men to have incentives to gamble, but the exact condition for the surplus contribution effect not to be too negative is $r_m \geq 1 - \alpha r_w$. For women, the necessary and sufficient condition for the competitive matching effect to dominate the surplus contribution effect is $r_w \geq 1 - r_m/\alpha$.   ∎

Finally, the gamble-inducing competitive matching effect is unique to the matching market. The competitive matching effect is similar to a substitution effect. The competitive matching rematches a man with a woman when his characteristics change, whereas when the total income changes, there is substitution between the goods consumed. But they also differ in a crucial way. The competitive matching effect drives gambling, but the substitution effect in general does not. The matching market is special in the following sense. Consider a competitively organized market. A vector of goods $\{1, \cdots, N\}$ is available, and a bundle is denoted by $\mathbf{x} = (x_1, \cdots, x_N) \in \mathbf{X}$. Suppose the supply of goods is fixed. Every person is endowed with (possibly different) wage earnings $w_i \in \mathbb{R}_+$ and (possibly different, "well-behaved") utility function $u_i : \mathbf{X} \to \mathbb{R}_+$. They are price takers and denote the vector of prices by $\mathbf{p}$.

The utility a person i derives when he has income w is the maximal utility he can derive by consuming the optimal bundle of goods,

$$u(w) \equiv \max_{\mathbf{x}} u(\mathbf{x}) \quad \text{s.t. } \mathbf{p} \cdot \mathbf{x} \leq w.$$

Let $\mathbf{x}(w)$ denote the optimal bundle of goods when a person has income w.

Consider gambling before making a purchasing decision in the competitive consumption market. Suppose a person starts with income $\widehat{w}$ and can take a fair gamble on the income. The person purchases the goods after gambling. Then the utility difference between gambling and not gambling is

$$\mathbb{E}[u(w)] - u(\widehat{w}),$$

and expressed in terms of the explicit utility function,

$$\mathbb{E}[u(w)] - u(\widehat{w}) = \mathbb{E}[u(\mathbf{x}(w))] - u(\mathbf{x}(\widehat{w})).$$

Add and subtract the term $\mathbb{E}\left[u\left(\frac{w}{\widehat{w}}\mathbf{x}(\widehat{w})\right)\right]$, i.e., the expected utility if the agent simply consumes the feasible bundle given any income w by shifting proportionally with respect to his income $\widehat{x}$. The difference becomes

$$\mathbb{E}[u(\mathbf{x}(w))] - \mathbb{E}\left[u\left(\frac{w}{\widehat{w}}\mathbf{x}(\widehat{w})\right)\right] + \mathbb{E}\left[u\left(\frac{w}{\widehat{w}}\mathbf{x}(\widehat{w})\right)\right] - u(\mathbf{x}(\widehat{w})).$$

It combines to have two terms,

$$\underbrace{\mathbb{E}\left[u(\mathbf{x}(w)) - u\left(\frac{w}{\widehat{w}}\mathbf{x}(\widehat{w})\right)\right]}_{\text{income effect}} + \underbrace{\mathbb{E}\left[u\left(\frac{w}{\widehat{w}}\mathbf{x}(\widehat{w})\right) - u(\mathbf{x}(\widehat{w}))\right]}_{\text{substitution effect}}.$$

If the consumption bundles are homogeneous of degree 1, then the gambling incentive does not exist in a competitive consumption market in contrast to the competitive matching market.

5. Progressive taxation for efficiency and equality

In this section, I use an example to illustrate an inherent efficiency-equity trade-off in the matching setting. I present an example in which there exist two equilibria: (1) an efficient equilibrium in which everyone gambles and the inequality in the matching characteristics is maximal, and (2) an inefficient equilibrium in which no one gambles and the inequality in the matching characteristics is minimal. This demonstrates that risk-taking can be socially desirable. It also demonstrates the link between efficiency and inequality. I then propose a remedy—a progressive taxation scheme—that eliminates the inefficient equilibrium, encourages the efficient equilibrium, reduces the inequality resulting from equilibrium gambling, and generates a positive tax revenue.

Suppose that mass 1 of men and mass 1 of women are all born homogeneous with characteristic 2. Each can take an investment gamble before participating in the matching market. Each can take either the degenerate investment or the

gamble that makes him or her of characteristic 1 with probability one-half or characteristic 3 with probability one-half. The surplus function is $s(x_m, x_w) = x_m x_w$. The sets of characteristics are thus $X_m = X_w = \{1, 2, 3\}$.

Two quite different equilibria arise. In one equilibrium, no one takes a gamble and the matching characteristics are homogeneous. In another equilibrium, everyone takes the gamble and the matching characteristics are heterogeneous.

In the no-gambling equilibrium, everyone enters the matching market with his or her innate characteristic without gambling, so mass 1 of characteristic 2 men and mass 1 of characteristic 2 women are in the matching market. All men and women marry and divide their surplus equally. The equilibrium payoff functions are $v_m^*(1) = v_w^*(1) = 0$, $v_m^*(2) = v_w^*(2) = 2$, and $v_m^*(3) = v_w^*(3) = 4$. Under these equilibrium payoffs, no one can strictly benefit from taking a gamble, as the payoff function $v_m^*(x) = v_w^*(x) = 2(x - 1)$ is linear in the characteristics.

In the gambling equilibrium, everyone gambles. The matching market is composed of mass 0.5 of characteristic 3 men, mass 0.5 of characteristic 1 men, mass 0.5 of characteristic 3 women, and mass 0.5 of characteristic 1 women. People of characteristic 3 are matched to each other, and people of characteristic 1 are matched to each other. The equilibrium payoff functions are $v_m^*(1) = v_w^*(1) = 0.5$, $v_m^*(2) = v_w^*(2) = 1.5$, and $v_m^*(3) = v_w^*(3) = 4.5$. Gambling yields an expected payoff of $(0.5 + 4.5)/2 = 2.5$, and no gambling yields an expected payoff of 1.5.

The gambling equilibrium is the more efficient equilibrium: The total surplus in the gambling equilibrium is $\frac{1}{2}(3)(3) + \frac{1}{2}(1)(1) = 5$, while the total surplus in the no-gambling equilibrium is only $1(2)(2) = 4$. In fact, the gambling equilibrium is the most efficient a social planner can achieve, and the no-gambling equilibrium is the least efficient a social planner can achieve, given the feasible gambling strategies.

An undesirable effect of the population-wide risky behavior is the ex post inequality in their characteristics and payoffs. A remedy for the problem is a progressive tax scheme on the payoffs. As we saw in Section 4, the payoff functions are strictly convex. A progressive tax scheme can flatten out the convexity of the stable payoff function. As long as the after-tax payoffs remain convex, incentives to gamble still exist. Moreover, a carefully designed progressive tax, besides reducing inequality, can also produce positive revenue and eliminate inefficient equilibria.

Consider the following progressive tax scheme. If a person's payoff is between 0 and 1, then he or she is provided a subsidy so that her after-subsidy payoff is 1. If a person's payoff is between 1 and 3, then he or she is not provided a subsidy, and neither is he or she taxed. If a person's payoff is above 3, then the portion of his or her payoff above 3 is taxed at rate 2/3. The after-tax payoffs in the gambling equilibrium are $v_m^\tau(1) = v_w^\tau(1) = 1$, $v_m^\tau(2) = v_w^\tau(2) = 2$, and $v_m^\tau(3) = v_w^\tau(3) = 3.5$ (the after-tax payoff of characteristic 2 changes because the most he or she can generate now is to match with 1 and get all the surplus produced). The payoff from gambling is $(1 + 3.5)/2 = 2.25$ and the payoff from no gambling is 2: thus, gambling is still desirable. Furthermore, the net tax revenue is $1 - 0.5 = 0.5$, as all of the 3s get taxed 1 and all the of 1s get a subsidy of 0.5: There is a positive tax revenue. Furthermore, the inefficient no-gambling equilibrium is eliminated. The after-tax payoffs in the no-gambling equilibrium are $v_m^\tau(1) = v_w^\tau(1) = 1$, $v_m^\tau(2) = v_w^\tau(2) = 2$, and $v_m^\tau(3) = v_w^\tau(3) = 3.5$. It is strictly better for people to gamble given these payoffs, as the risks of gambling are partially insured and the gains are large.

A regressive taxation scheme in this example can also eliminate the inefficient no-gambling equilibrium because it increases the convexity of the payoff function, but can only exacerbate the inequality in payoffs in the gambling equilibrium.

There are some discussions about the relationship between economic growth and income inequality. Rosen (1997) focuses on the complementarity between income and location. Becker et al. (2005) focus on the market for status and generate endogenous income distributions. Ray and Robson (2012) use an equilibrium growth model with endogenous risk-taking. However, none of these papers considers the gambling incentives in a matching market or the tradeoff between efficiency and inequality.

We can also use this example to think about the sunk investments made by workers and firms before they search for their partners in the labor market. The example demonstrates that making risky investments can be both privately and socially optimal. Furthermore, when the population is not taking the efficient level of risk and the economy, as a result, is not reaching the optimal level of growth, progressive taxation can induce workers and firms to take efficient risks. Furthermore, progressive taxation can also reduce the inequality that results from risk-taking and generate tax revenue.

6. Conclusion

The competitive structure of the matching market encourages voluntary risk-taking. These gambling behaviors can be profitable, even for risk-averse agents choosing actuarially unfair lotteries. The fundamental force that drives this type of behavior is competitive matching and bargaining between the two sides of the market; in the Becker-Shapley-Shubik setting, stability and transferability of utilities. This discovery also links efficiency and inequality in the matching market, and helps explain observed risk-taking behavior in the marriage market.

The following four extensions are worth exploring: restricting the transferability of utilities, adding search or informational frictions, solving for a model with a finite number of agents, and considering investments that may be unproductive.

I have shown the possibility of pre-matching gambles in a setting with perfectly transferable utility and stability. In general, pre-matching gambling arises with some degree of transferability (even imperfect transferability) of utilities, but it is unclear whether the result would hold without any transferability of utilities. If agents have ordinal preferences, and the matching among the agents is stable in the sense of Gale and Shapley (1962), we might also investigate agents' risk-taking

behaviors in pre-matching investment in these markets. However, utilities and fair lotteries are not straightforwardly defined with respect to ordinal preferences.

Furthermore, it would be interesting to show that the logic goes through in a model with search frictions. Adding search frictions should not change the basic logic whereby a man still may strictly benefit from lotteries and the sorting effect is still strong enough for people to take lotteries. The bilinearity of the surplus function may play a crucial role in the extensions. Burdett and Coles (1997) show that when the surplus function is bilinear, there is block matching in equilibrium. That is, agents on both sides are segregated into blocks of matching that can be viewed as classes. The matching technology and bargaining process may have different implications for the incentives on pre-matching investments.

I show below that players may gamble, even with only a finite number of participants in the market. It would also be interesting to fully characterize the equilibria when there is a finite number of agents on the two sides of the market. The continuum model I have used allows us to cleanly characterize the gamble-inducing effects. The key complication with a finite number of agents is that for different investment realizations, an agent will end up at different ranks in the market and would match with a different partner based on different realizations. When there is a finite number of agents, that necessarily changes the matching of other agents in the economy. Solving for the Bayesian Nash equilibria of a multiplayer game could be difficult.

Example 4 (*Gambling with a finite number of players*). Consider a simple four-player matching market with a type-2 man, a type-3 man, a type-2 woman, and a type-3 woman. Let the surplus function be $s(x_m, x_w) = x_m x_w$. Assume that they equally split their surplus whenever such a division is part of a stable outcome. Without gambling, type-2s match with each other and type-3s match with each other, and type 2s get 2 and type 3s get 4.5. If the type-2 man has a gamble that makes him type 0 with probability 1/2 and type 4 with probability 2, then he would take the gamble to get an expected payoff of 3, because he either becomes type 0 and matches with the type-2 woman to get a payoff of 0 or becomes type 4 and matches with the type-3 woman to get a payoff of $6 = 4 \times 3/2$. If the type-3 man has a gamble that makes him type 0 with probability 1/2 and type 6 with probability 1/2, taking the gamble would yield him an expected payoff of 5: When he becomes type 0, he gets zero payoff, and when he becomes type 6, he matches with the type-3 woman and gets at least a payoff 10, because he could match with the type-2 woman and gets $6 \times 2 - 2 = 10$, where 2 is the type-2 woman's payoff matching with the type-2 man. ■

Finally, this paper picks one way to model investments. The investments are productive and change agents' matching types. It is also possible and important to consider investments in matching markets in which agents' types are not observable. Agents would engage in totally or partially wasteful investments to signal their types or to expand their matching opportunities (Hoppe et al., 2009; Bidner, 2010; Dizdar et al., 2019).

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