Economic Theory (2021) 72:1297–1337
https://doi.org/10.1007/s00199-020-01313-w

RESEARCH ARTICLE

Prices versus auctions in large markets

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Received: 25 April 2019 / Accepted: 17 September 2020 / Published online: 22 September 2020

Abstract
This paper studies the use of posted prices and auctions in a large dynamic market with many short-lived sellers and long-lived buyers. Although a reserve-price auction maximizes the expected revenue, the optimal revenue decreases when the market becomes more buyer-friendly; namely, when buyers survive longer, face fewer competitors, and become more patient. As the market becomes more buyer-friendly, the revenue advantage of a reserve-price auction over posting a price also decreases, but using posted prices would lead to sale and allocative inefficiencies.

Keywords Optimal mechanism · Reserve price · Auction · Posted price

JEL Classification D44 · C73 · C78

1 Introduction

In theory and practice, a standard auction with a well-chosen reserve price is a desirable revenue-enhancing choice for unit-supply sellers who face buyers with independent private valuations. This paper confirms the revenue optimality of the reserve-price auction in dynamic markets, but the optimal revenue decreases when the market becomes more competitive for sellers.

I study the steady-state equilibrium of a dynamic market in which infinitely many short-lived sellers and long-lived buyers who potentially have many chances to obtain a good are matched randomly. When sellers have no fixed cost of running any mechanism, a reserve-price auction is expected-revenue maximizing. The reserve price is

This paper is a revised version of Chapter 4 of my Ph.D. dissertation at the University of Chicago. Financial support by Yahoo! Key Scientific Challenges Fellowship and the National Science Foundation is gratefully acknowledged.

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shown to indicate market competitiveness: The equilibrium reserve price and auction revenue are lower in a more buyer-friendly market; a market is said to be more buyer-friendly if buyers survive longer, face fewer competitors, and become more patient. I generalize the monopolist's problem of Myerson (1981) by introducing a market in which buyers may have subsequent opportunities to buy a perfect substitute of the good. Along the way, I generalize virtual utility (Myerson 1981), which is interpreted as the auction marginal revenue curve (Bulow and Roberts 1989).

Although in theory an auction with an optimally chosen reserve price generates the highest expected revenue, in practice there is increasing use of alternative mechanisms in many markets of goods with close substitutes. This is because these alternative mechanisms have other operational advantages while, at the same time, achieving revenues close to optimal. For example, used car dealers use secret reserve-price auctions followed by bargaining (Larsen 2020); central banks offer liquidity loans via price menus and only use auctions during special times (Hu and Zhang 2020); and bargaining is prevalent in housing and labor markets (Atakan and Ekmekci 2014; Ekmekci and Zhang 2020; He et al. 2020). The effects of subsequent competition offer a plausible explanation, which I will elaborate on, for the rise of alternative mechanisms; in particular, posted price mechanisms, which rapidly took over eBay after they were introduced in the 2000s (Reynolds and Wooders 2009; Einav et al. 2018; Backus et al. 2020). In addition, many desirable properties of dynamic and double auctions have been suggested (Campbell and Levin 2006; Kagel et al. 2007; Peters and Severinov 2008).

A posted price mechanism has the desirable properties of immediacy and simplicity compared with an auction, which involves more active organization by sellers and more active participation by buyers over a longer period of time. The gain in revenue from an auction over a posted price is caused by buyer competition, but is reduced when the market becomes more buyer-friendly. More precisely, the difference between the expected revenue of the reserve-price auction and that of simply posting the reserve price as the transaction price decreases when the distribution of buyers' potential price offers improves first-order stochastically in favor of buyers.[1] When a buyer has subsequent opportunities to purchase a good, her willingness to pay each seller is driven below her intrinsic value for the good. This, in turn, affects the expected revenues of different sales mechanisms and possibly alters a seller's profit-maximizing choice if there is a higher fixed cost associated with running an auction.

When sellers have heterogeneous costs of running auctions, some of them may switch to simpler mechanisms. For a sufficiently large cost of running an auction, sellers who live for one period post prices.[2] However, posted prices result in sale and allocative inefficiencies: Compared with the optimal reserve-price auction, (i) a good is less likely to be sold by the seller using the optimal price mechanism, and (ii) there

[1] This result is related to the key insight of Wang (1993) that the choice between an auction and a posted price depends on the steepness of the marginal revenue curve.

[2] Sellers who can live longer may first try to auction repeatedly, sell the item at a fixed price before they use an auction, or dynamically adjust prices. An auction followed by a posted price if it is not sold via the auction is never profit-maximizing (Crémer et al. 2009; Zhang 2015a, b, 2020). When buyers incur costs and/or buyers are risk-averse, alternative mechanisms also arise (Lee and Park 2016; Cao et al. 2018; Delnoij and De Jaegher 2020).

is no guarantee that the buyer with the highest value will receive the good. These inefficiencies result in market-wide externalities for future sellers and buyers.

In summary, the paper makes three contributions. First, it characterizes the revenue maximizing mechanism when buyers have subsequent purchasing opportunities. Second, the equilibrium of the setting in which buyers have value-dependent outside options is uniquely characterized, with definitive comparative statics results that offer guidance for auction platform design. And third, it draws an analogue between auction and posted price marginal revenue curves, and investigates the welfare effects in a dynamic setting when auctions and posted prices exist simultaneously.

The frictional matching process distinguishes the study from the directed search literature that studies competing sellers who simultaneously announce mechanisms to attract buyers who can choose which mechanism to participate in. In equilibrium, with mild technical conditions, each seller runs an auction with a reserve price lower than the Myerson (1981) reserve price (Burguet and Sákovics 1999; Pai 2010), and as the number of sellers approaches infinity, the equilibrium mechanism is the efficient auction (McAfee 1993; Peters and Severinov 1997). Although the literature yields beautiful technical results, it is hardly applicable to the real world: Not all agents can meet one another because of geographic or time constraints, and a lower reserve price may be a result of buyers' subsequent purchasing opportunities rather than a tool to attract buyer participation.

Sale inefficiency—buyers who value the goods more than the sellers do not receive the goods—is a result of the frictional matching process. Similar frictional matching models explain natural unemployment (Diamond and Maskin 1979) as well as non-trivial prices in bargaining (Rubinstein and Wolinsky 1985) and auctions (Wolinsky 1988).[3]

The idea that subsequent purchasing opportunities reduce buyers' willingnesses to pay is explored theoretically by Wolinsky (1988), Kittsteiner et al. (2004), Zeithammer (2006), and Said (2011). Said (2011) considers a dynamic market setting in which bidders arrive randomly and incorporate beliefs about current and future dynamics to shade their bids in efficient second-price auctions. He assumes that sellers are restricted to running efficient auctions and buyers—receiving new value draws for each imperfect substitute—have value-independent outside options. In comparison, this paper studies a general equilibrium in which buyers have value-dependent outside options, so that in equilibrium buyers with higher valuations have better opportunities and exit the market faster.

More recently, markets with buyers with subsequent outside options are explored empirically by Hendricks and Sorensen (2018), Backus and Lewis (2019), Coey et al. (2019), and Bodoh-Creed et al. (2020). The general equilibrium approach renders the problem technically more difficult, because equilibrium existence and uniqueness are not easily guaranteed. For example, Hendricks and Sorensen (2018) resort to a partial equilibrium. Coey et al. (2019) prove the existence of an equilibrium in a dynamic model with endogenous entry and mechanism choices by sellers.

[3] Relatedly, Satterthwaite and Shneyerov (2007, 2008) show that the transaction price converges to the competitive Walrasian price as the length of trading period shortens to zero—in other words, as the search frictions disappear.

The rest of the paper is organized as follows. Section 2 introduces the basic dynamic setup and defines the equilibrium. Section 3 solves a monopolist's problem when buyers have subsequent outside options, generalizing the symmetric independent private values case of Myerson (1981), and examines the effects of changes in subsequent outside options. Section 4 characterizes the unique equilibrium, and Sect. 5 presents comparative statics results. Section 6 introduces the posted price and discusses the welfare effects when auctions and posted prices coexist in the market. Section 7 concludes, and the appendix collects omitted proofs and details.

2 Model

In this section, I describe the setup, comment on modeling choices, and define the equilibrium.

2.1 Setup

There are countably infinite periods: $t = 0, 1, \ldots$. At the beginning of each period, there is a measure 1 of unit-supply (male) sellers and an integer measure n of unit-demand (female) buyers in a homogeneous good market. I refer to n as the buyer–seller ratio. All agents are risk-neutral, expected utility maximizers with quasilinear preferences, and have a common discount factor δ for the next period. Each seller lives for one period and has value zero for the good. Each buyer possibly lives forever and has a persistent private value v, independently and identically drawn from value distribution F with a positive density f on support $[0, 1]$. Let H_t denote period t's active buyer value distribution. Active buyers in each period consist of old and newborn buyers, except in period 0, when all buyers are newborn (i.e., $H_0 = F$).

The market proceeds as follows. At the beginning of each period t, buyers and sellers are randomly matched. The number of buyers each seller is matched with is deterministically n. Each seller chooses a sales mechanism.

Seller j chooses a direct anonymous mechanism (DAM) $M_{j,t}$ in which buyers cannot be distinguished by their ages or identities but only by reported values. It consists of probability assignment and cost functions $\{P_i(\cdot), C_i(\cdot)\}_i$, which, for any vector of value reports $\mathbf{z} = (z_1, \ldots, z_n)$, specifies each buyer's probability of winning and payment. Therefore, buyer i's expected probability of winning and expected payment by reporting z_i are

$$\overline{P}_i(z_i) = \int_{\mathbf{z}_{-i}} P_i(z_i, \mathbf{z}_{-i})\, dH_{-i}(\mathbf{z}_{-i}),$$

and

$$\overline{C}_i(z_i) = \int_{\mathbf{z}_{-i}} C_i(z_i, \mathbf{z}_{-i})\, dH_{-i}(\mathbf{z}_{-i}),$$

respectively, where $H_{-i}(\mathbf{z}_{-i}) = \prod_{j \neq i} H_j\left(z_j\right)$.

Buyer i of value v, after knowing what mechanism $M_{j,t}$ she participates in, plays a feasible strategy $\sigma_{i,t}(v, M_{j,t}) \in [0,1]$. If buyer i obtains the good from the seller she is matched with, she is a winner ($i \in W_t$) and exits the market; otherwise, she is a loser ($i \in L_t$) and survives with probability s to period $t+1$; I refer to s as the survival rate. Newborn buyers enter the market at the beginning of period $t+1$ to replace the winners and exiting losers to keep a constant measure n of buyers.

Let $l_{i,t}\left(v|\sigma_{i,t}\left(v, M_{j,t}\right), \sigma_{-i,t}\left(\cdot, M_{j,t}\right), M_{j,t}\right)$ denote value v buyer i's expected probability of losing when she plays $\sigma_{i,t}\left(v, M_{j,t}\right)$ and her opponents play $\sigma_{-i,t}\left(\cdot, M_{j,t}\right)$ in the mechanism $M_{j,t}$ she participates in, and $l_t(\{\sigma_i(\cdot,\cdot)\}_i, \{M_{j,t}\}_j)$ denote the proportion of losers in period t given the agents' behaviors. For expositional convenience, I subsequently denote them by $l_{i,t}(v)$ and l_t, but keep in mind that they depend on the behaviors of all active agents. Period $t+1$ active buyers are composed of newborn buyers and surviving losers from period t, so the expected value distribution of active buyers in period $t+1$ is

$$\mathbb{E}\left[H_{t+1}(v)\,|H_t\right] = (1 - s \cdot l_t) \cdot F(v) + s \cdot H_t(v|L_t), \tag{1}$$

where $H_t(v|L_t)$ is the value distribution of losing buyers.

2.2 Remarks

Before defining the equilibrium, I remark on the following modeling choices: (i) frictional matching technology with a deterministic number of buyers per seller, (ii) potentially multiple goods held by a seller, (iii) a continuum of anonymous agents, (iv) the restriction to direct anonymous mechanisms, and (v) the constant size of the market. A reader comfortable with these assumptions can skip the justifications in this subsection without loss of understanding of the setup.

Remark (i). Wolinsky (1988) employs a similar frictional matching technology to reflect information and search frictions. All buyers are randomly matched to sellers in his model, so the number of competitors each buyer faces is stochastic. If the matching process is stochastic, the number of buyers a seller is matched with follows a Poisson distribution. That is, the probability that a seller is matched with $k \in \{0, 1, 2, \ldots\}$ buyers is $n^k \cdot e^{-n}/k!$. Adopting the stochastic matching technology would only add computational and expositional complexities but no additional insight to the paper, so I restrict my attention to uniform matching with a deterministic buyer–seller ratio.

Remark (ii). A seller can hold multiple goods in the same period, and as long as he does not own a positive measure of the goods in the market, he has no market power and his actions cannot alter subsequent market compositions. Without loss of generality, each good can be considered to be sold separately.

Remark (iii). Two channels of learning arise in finite markets but not in infinite markets. In a market with a finite number of buyers, there is a significant probability that a buyer's opponents may become her opponents again in subsequent periods. Thus, a bidder can infer from her bid information about the values of her opponents, regardless of whether the transaction price is announced in an auction (Kittsteiner et al. 2004). In a market with a finite number of sellers (hence, a finite number of items for sale), a buyer who knows whether the item from the mechanism she participated in has been sold helps her to learn about a nontrivial portion of the market. Information asymmetry arises in sellers facing markets with asymmetric bidders who know different amounts of information from the history of actions and outcomes despite symmetric newborn distributions. Milgrom and Weber (2000) study the setting with n buyers and the setting with one seller holding $k \leq n$ goods, and show that these learning effects make the expected price path a martingale. Said (2012) shows that a simultaneous ascending auction can alleviate these effects resulting from stochastic arrival of the agents.

Remark (iv). The assumption of a continuum of agents and restriction to the direct anonymous mechanisms guarantee information symmetry among all participating agents. Bodoh-Creed et al. (2020) show that the equilibrium in a model with a continuum of agents is approximated by that in a model with a finite number of agents, as the number of agents approaches infinity. In the current setting, the sellers have symmetric beliefs about buyers' values. Anonymous mechanisms further restrict sellers from discriminating by buyers' ages, which potentially reveal buyers' losing histories and thus values.

Remark (v). The assumption that the masses of buyers and sellers remain constant is without loss of generality. Suppose instead that the market size grows each period so that the masses of buyers and sellers both increase by $g > 0$. This market growth effect is actually equivalent to decreasing the buyer's survival rate by $1/(1+g)$. Instead of $s \cdot l_t$, there is only $s \cdot l_t/(1+g)$ of surviving losers in period $t+1$. The value distribution of active buyers is then

$$\mathbb{E}\left[H_{t+1}(v)\,|H_t\right] = \left(1 - \frac{s}{1+g} \cdot l_t\right) F(v) + \frac{s}{1+g} H_t \cdot (v|L_t)\,. \tag{2}$$

When $s' = s/(1+g)$, the buyer value evolution coincides with Eq. 1. Although each buyer survives with probability s, the addition of a market growth rate of $g > 0$ essentially changes the survival rate to $s' = s/(1+g)$.

2.3 Equilibrium

For the remainder of the section, I define the solution concept I intend to solve, stationary symmetric sequential equilibrium (SSSE). In equilibrium, buyers and sellers of the same value play the same payoff-maximizing strategies in each period when the expected active buyer value distribution remains stationary, and buyers' and sellers' beliefs are updated according to equilibrium behavior.

Agents' strategies and beliefs are defined as follows. Since all sellers are identical and all buyers are symmetric, I restrict my attention to symmetric behavior in which every seller chooses the same mechanism M_t and every value v buyer behaves according to the same strategy, $\sigma_t(v, M_t)$; that is, agents are only distinguished by their types but not by indices. In period t, each agent has belief $\mu_t\left(\{H_{t'}, M_{t'}, \sigma_{t'}\}_{t'=t}^{\infty}\right)$ about the active buyer value distribution, the seller mechanism choice, as well as the buyer strategies for the current period and for every subsequent period.

Agents' payoffs are specified as follows. Each seller gets expected revenue $r\left(M_t|\mu_t\right)$ by choosing mechanism M_t under belief μ_t. Each value v buyer receives total discounted expected utility $u\left(v, \sigma_t\left(v, M_t\right), \sigma_{-i,t}\left(\cdot, M_t\right)|M_t, \mu_t\right)$ when her opponents play $\sigma_{-i,t}\left(\cdot, M_t\right)$.

Subsequent active buyer value distributions, $\mathbb{E}\left[H_{t+1}\left(v\right)|H_t, M_t, \sigma_t\right]$, can be inferred from the buyers' and sellers' behaviors and prespecified market and matching dynamics. If the active buyer value distribution is stationary, the buyer composition will be expected to be the same across periods, so the stationary value distribution is $H_*\left(v\right) \equiv \mathbb{E}\left[H_t\left(v\right)|\mu_*\right] = \mathbb{E}\left[H_{t+1}\left(v\right)|\mu_*\right]$.

In particular, given the equilibrium mechanism M_* and strategy $\sigma_*\left(\cdot, \cdot\right)$, the stationary value PDF is a weighted average of newborn PDF f and the previous period PDF h_* in stationary equilibrium. The stationary value PDF and CDF satisfy

$$h_*\left(v\right) = \left(1 - s \cdot l_*\right) \cdot f\left(v\right) + s \cdot l_*\left(v\right) \cdot h_*\left(v\right) \tag{3}$$

and

$$H_*\left(v\right) = \left(1 - s \cdot l_*\right) \cdot F\left(v\right) + s \cdot \int_0^v l_*\left(z\right) dH_*\left(z\right), \tag{4}$$

respectively, where the losing probabilities $l_*\left(\cdot\right)$ and l_* depend on the equilibrium seller and buyer behaviors and matching dynamics.

Now I can define the equilibrium in which (i) all agents play symmetric stationary strategies, (ii) the value distribution of active buyers is stationary, (iii) agents' beliefs about the equilibrium behaviors of other agents and the value distribution of active buyers are correctly updated, and (iv) every agent maximizes the expected payoff given beliefs:

Definition 1 $\left(M_*, \sigma_*, H_*, \mu_*\right)$ constitutes a stationary symmetric sequential equilibrium (SSSE) if

1. (Symmetry) Every seller runs the same mechanism M_* and every value v buyer plays according to strategy $\sigma_*\left(v, M\right)$ in mechanism $M \in \mathcal{M}$.
2. (Stationarity) Stationary value distribution is a martingale with respect to the equilibrium mechanism M_* and strategy σ_*: $H_*\left(v\right) = \mathbb{E}\left[H_{t+1}\left(v\right)|H_t = H_*, M_*, \sigma_*\right]$, where the stationary value PDF and CDF are determined by Eqs. 3 and 4, respectively.
3. (Consistency) Every agent has the correct belief μ_* about the stationary value distribution, equilibrium mechanism, and equilibrium strategy: $\mu_*\left(H_*, M_*, \sigma_*\right) = 1$.
4. (Sequential Rationality)
   (a) (Sequential Rationality of Sellers) Mechanism M_* maximizes each seller's expected profit with respect to belief μ_*: $\forall M \in \mathcal{M}, \pi\left(M_*|\mu_*\right) \geq \pi\left(M|\mu_*\right)$.
   (b) (Sequential Rationality of Buyers) Strategy $\sigma_*\left(\cdot, M\right)$ is a symmetric Bayes-Nash equilibrium in every mechanism M: $\forall v, \forall M \in \mathcal{M}, \forall \sigma\left(\cdot, M\right) \in \Sigma_M$,

$$u\left(v|M, \mu_*\right) \geq u\left(v, \tilde{\sigma}\left(v, M\right), \sigma_{-i,*}\left(\cdot, M\right)|M, \mu_*\right),$$

where $u(v|M,\mu_*) \equiv u\left(v, \sigma_*(v,M), \sigma_{-i,*}(\cdot,M)|M,\mu_*\right)$ denotes value v buyer's equilibrium expected payoff in M.

Note that type-symmetric strategies do not need to be assumed, but rather arise as an equilibrium outcome. Since all of the optimal incentive-compatible mechanisms yield the same revenue and the same expected buyer payoffs, all sellers will choose the same mechanism in equilibrium and all buyers will choose the same equilibrium strategy.

Characterizing the equilibrium requires (i) characterizing the stationary value distribution, (ii) solving the monopoly profit-maximizing problem with value-dependent outside options, and (iii) solving buyers' equilibrium strategies when they have subsequent purchasing opportunities. In essence, in the dynamic setting, the existence of subsequent markets affects both sellers' mechanism choices and buyers' behaviors, and subsequent market conditions depend on buyer composition H_*.

In Sect. 3, I solve a generalized version of the monopolist's problem and specify the buyer's equilibrium strategy in the optimal mechanism and the agents' expected equilibrium payoffs. The SSSE is characterized and proven to exist uniquely in Sect. 4, and comparative statics results on the survival rate, buyer–seller ratio, discount factor, and buyer value distribution are presented in Sect. 5.

3 Seller's problem

In this section, I study the revenue maximization of a monopolist who faces symmetric buyers in the presence of a subsequent market in which the identical good is expected to be sold by other sellers. I show that the optimal mechanism is implementable by a standard reserve-price auction, but the reserve price changes with respect to the condition of the subsequent market. In particular, the reserve price is always lower than the monopoly reserve price in Myerson (1981), whose symmetric independent private values case is subsumed in the current setup. Moreover, the current setup also incorporates the seller's problem in the SSSE as a special case. I generalize the marginal revenue curve defined by Bulow and Roberts (1989) to the current setting, and use it to determine the optimal reserve price.[4]

Suppose there are n buyers and one seller, whom I refer to as a monopolist. In addition to the assumptions above, the value distribution is assumed to have a decreasing inverse hazard rate:

Assumption 1 $d\eta(F(v))/dv \leq 0$, where $\eta(F(v)) \equiv (1-F(v))/f(v)$.

The setting so far is the same as the symmetric independent private values setting in Myerson (1981), which I refer to as the pure monopoly setting. I refer to the (unique) ρ_{mon} that satisfies $\eta(F(v)) = 0$ as the pure monopoly reserve price or pure monopoly reserve type; the distinction will become apparent when I add a subsequent market.

[4] The general revenue-maximizing mechanism with ex ante *a*symmetric buyers and related results regarding the incentive compatibility, buyer indifference, and revenue equivalence of different mechanisms are derived and presented in Appendix B.

An auction with the pure monopoly reserve price maximizes the expected revenue (Myerson 1981).

Now I add a subsequent market in which each buyer is expected to receive one price offer drawn from distribution Λ with associated density λ on support $\left[\underline{x}, \overline{x}\right]$. Since Λ incorporates all of the information about subsequent periods, I simply refer to the subsequent market by the distribution Λ. To avoid interruptions by technical details, I assume that buyers are symmetric and the subsequent market distribution is continuous and differentiable.

The setting incorporating a subsequent market incorporates a wide range of settings. Myerson (1981) essentially solves the monopolist's problem when there is no price offer lower than 1 in the subsequent market ($\Lambda(x) = 0$ for all $x \leq 1$) and/or buyers discount any subsequent market infinitely ($\delta = 0$). In the SSSE, a buyer's continuation payoff is her total discounted expected utility from all mechanisms she participates in future periods. All subsequent opportunities she faces can be characterized by her expected payment, and there is a probability assigned to each subsequent opportunity. Hence, the set of all subsequent opportunities can be summarized by a cumulative distribution function, namely, Λ. In particular, the set of all subsequent opportunities each buyer faces in equilibrium is captured by the equilibrium subsequent market Λ_*.

Although all buyers face the same subsequent market, they may receive different realized price offers and may make different decisions based on their own valuations when facing the same realized price offers. In the subsequent market—when there is no further purchasing opportunity—a buyer purchases the good if and only if her value exceeds the realized price offer she receives; note that a buyer's value and willingness to pay coincide in the subsequent market because there is no future purchasing opportunity for the buyer. Therefore, a value v buyer's expected payoff in the subsequent market Λ is

$$\underline{u}_{\Lambda}(v) = \int_{\underline{x}}^{v} (v - x)\, d\Lambda(x) = \int_{\underline{x}}^{v} \Lambda(x)\, dx, \tag{5}$$

where the second equality follows from integration by parts. The subsequent expected payoff increases whenever the buyers receive lower price offers with higher probabilities, so I say that Λ is more buyer-friendly than $\tilde{\Lambda}$ if Λ is first-order stochastically dominated by $\tilde{\Lambda}$.

Definition 2 A subsequent market is more buyer-friendly than another if and only if it is first-order stochastically dominated by the other: $\Lambda \succeq_{\mathrm{B}} \tilde{\Lambda}$ if and only if $\Lambda(x) \geq \tilde{\Lambda}(x)\, \forall x$.

A value v buyer will not pay more than the discounted expected utility if she waits, so her maximum willingness to pay (WTP) a seller is

$$w_{\Lambda}(v) = v - \delta \cdot \underline{u}_{\Lambda}(v).$$

The w_Λ function is differentiable, weakly increasing, and weakly concave.[5] The monopolist's problem with a subsequent market corresponds to a monopolist's problem with buyers' values transformed according to an increasing and concave function. Conversely, for any concave value transformation $\tilde{v}(\cdot)$, there is a subsequent market Λ that induces a WTP function $w_\Lambda(\cdot) = \tilde{v}(\cdot)$.

In essence, the monopolist maximizes expected revenue with respect to n buyers who have WTP $w(v)$ drawn from the distribution $\tilde{F}_\Lambda(\tilde{v}) = F\left(w_\Lambda^{-1}(\tilde{v})\right)$, which has a density of $\tilde{f}_\Lambda(\tilde{v}) = f\left(w_\Lambda^{-1}(\tilde{v})\right)/w'\left(w_\Lambda^{-1}(\tilde{v})\right)$. Therefore, by Myerson (1981), the optimal mechanism is to run a standard auction with the reserve price determined by a transformed virtual utility curve. The virtual utility in this setting is defined as the competitive auction marginal revenue (MR) in the presence of subsequent market Λ,

$$\mathrm{MR}_\Lambda^{\mathrm{A}}(v) = w_\Lambda(v) - \eta\left(\tilde{F}_\Lambda(w_\Lambda(v))\right) = w_\Lambda(v) - \eta(F(v))\,w'_\Lambda(v). \tag{6}$$

Proposition 1 (The optimal mechanism) *Let $A_\Lambda(\rho)$ represent a standard auction that implements reserve price $w_\Lambda(\rho)$ in the presence of subsequent market Λ. The revenue-maximizing mechanism in the presence of subsequent market Λ is $A_\Lambda^* \equiv A_\Lambda\left(\rho_\Lambda^*\right)$, where $MR_\Lambda^A\left(\rho_\Lambda^*\right) = 0$.*

Not all buyers who have values above the reserve price $w_\Lambda(\rho)$ will participate in the auction, but only buyers who have willingness to pay above the reserve price will participate. They have values above ρ, which I call the reserve type. Absent a subsequent market, a buyer's value is her WTP, so the reserve price $w_\Lambda(\rho)$ and reserve type ρ coincide. The optimal reserve price when there is a subsequent market is lower than the pure monopoly reserve price. The relation of the reserve type to the probability of sale, $1 - F^n(\rho)$, is apparent: The higher the optimal reserve type, the lower the probability of sale, and the lower the expected sale efficiency to transfer the good from sellers to buyers.

Proposition 2 *The optimal reserve price in the presence of a subsequent market is lower than the optimal pure monopoly reserve price.*

Remark 1 The optimal competitive reserve price is equal to the pure monopoly reserve price when there is no price offer (weakly) lower than the pure monopoly reserve price in the subsequent market, because the competitive auction marginal revenue curve is unchanged for values below the pure monopoly reserve price.

The following numerical example demonstrates that I cannot compare the optimal reserve price and optimal reserve type across different markets, as a more buyer-friendly subsequent market does *not* guarantee a lower optimal reserve price or reserve type.

[5] To see the weak monotonicity and weak concavity, take the first and second derivative of w_Λ: $w'_\Lambda(v) = 1 - \delta\underline{u}'_\Lambda(v) = 1 - \delta\cdot\Lambda(v) \geq 0$ and $w''_\Lambda(v) = -\delta\cdot\underline{u}''_\Lambda(v) = -\delta\cdot\lambda(v) \leq 0$.

Example 1 Suppose there are $n = 2$ buyers with values drawn uniformly from [0, 1], that is, $F(x) = x$. In the first market, they may receive a uniform price offer between 0 and 1:

$$\Lambda_1(x) = x \quad x \in [0, 1],$$

and in the second market, they may receive a lower price offer with a lower probability:

$$\Lambda_2(x) = \begin{cases} 2x^2 & \forall x \in [0, 0.5] \\ x & \forall x \in (0.5, 1] \end{cases}.$$

Therefore, the first market is more buyer-friendly. However, the optimal reserve type and price are higher in the first market: $\rho^*_{\Lambda_1} \approx 0.4227 > \rho^*_{\Lambda_2} \approx 0.4221$ and $w_{\Lambda_1}(\rho^*_{\Lambda_1}) \approx 0.333 > w_{\Lambda_2}(\rho^*_{\Lambda_2}) \approx 0.303$.

Finally, I can calculate buyers' equilibrium strategies and payoffs in the standard auctions in the presence of a subsequent market. First-price and second-price auctions (as well as Dutch, English, and all-pay auctions) with the same reserve prices generate the same revenues, from the generalized revenue equivalence and buyer indifference result (Corollary 1 in Appendix B), which is a corollary of the characterization of incentive-compatible mechanisms (Proposition 12 in Appendix B). In the second-price auction, a buyer bids her WTP rather than her value. In the first-price auction, each buyer continues to shade her bid, but also according to her WTP.

Proposition 3 (Equilibrium in standard auctions) *Consider a standard reserve-price auction $A(\rho)$ with n buyers in the presence of subsequent market Λ. A value $v \geq \rho$ buyer in the second-price auction bids her willingness to pay $w_\Lambda(v)$. In the first-price auction, the equilibrium bidding strategy of buyer of value $v \geq \rho$ is*

$$\sigma(v, A(\rho)) = w_\Lambda(v) - \int_\rho^v \frac{F^{n-1}(z)}{F^{n-1}(v)} dw_\Lambda(z). \tag{7}$$

The total discounted payoff of a buyer of value $v \geq \rho$ is

$$u(v|A(\rho)) = \int_\rho^v F^{n-1}(z)\, dw_\Lambda(z) + \delta \cdot \underline{u}_\Lambda(v), \tag{8}$$

and the seller's expected revenue is

$$r(A(\rho)) = \int_\rho^1 MR^A_\Lambda(v)\, dF^n(v). \tag{9}$$

These equilibrium characterizations of bidder behaviors and the characterization of the optimal auction directly apply to the dynamic setting.

4 Equilibrium

In this section, I characterize the equilibrium and prove its existence and uniqueness under mild technical conditions. Based on results from the previous section, every seller runs the same reserve-price auction. Higher-value buyers are more likely to win the auctions and consequently exit the market at faster rates, resulting in lower-value buyers staying longer and a stationary value distribution that is first-order stochastically dominated by the newborn value distribution.[6] Such an equilibrium exists and is unique under mild conditions. The convergence to the equilibrium from the initial period is also discussed.

The equilibrium is characterized by the optimal reserve type and the stationary value distribution. Since buyers' subsequent purchasing opportunities in the SSSE can be summarized by a subsequent market Λ_*, which I will characterize, Proposition 1 shows that the optimal mechanism is a standard auction with reserve type ρ_* determined by equating the equilibrium auction marginal revenue to zero:

$$\mathrm{MR}_*^{\mathrm{A}}(\rho_*) \equiv w_*(\rho_*) - \frac{1 - H_*(\rho_*)}{h_*(\rho_*)} \cdot w_*'(\rho_*) = 0,$$

where $w_*(v)$ is value v buyer's equilibrium WTP. However, since buyers' realized payments are all higher than ρ_*, by Remark 1, the equilibrium reserve type is the same as the equilibrium reserve price, and is determined by

$$\rho_* - \frac{1 - H_*(\rho_*)}{h_*(\rho_*)} = 0. \tag{10}$$

The equilibrium reserve price is always positive, in contrast to the result obtained by McAfee (1993) whereby it converges to zero. As the results in the next section will demonstrate, the equilibrium reserve price indicates the level of market competitiveness and varies with different parameters of the model. Competitive forces alleviate matching frictions by reducing the equilibrium reserve price to be close to the efficient level of zero. Sellers are able to set positive reserve prices even if there is no excess demand. When $n = 1$, this setting is a dynamic bargaining setting with an equal measure of buyers and sellers, so the model is similar to that of Rubinstein and Wolinsky (1985).

As a result, a value v buyer wins the auction if and only if her value is above the equilibrium reserve type ρ_* and no opponent has a value above v, so her probability of winning is $\overline{P}_*(v) = 1 - \mathbf{1}_{v>\rho_*} \cdot H_*^{n-1}(v)$. Conversely, a value v buyer's losing probability $l_*(v)$ is 1 if $v \le \rho_*$ and is $1 - H_*^{n-1}(v)$ if $v > \rho_*$. The equilibrium mass of losing buyers is then

$$l_* = \int_0^1 l_*(v)\, dH_*(v) = 1 - \int_{\rho_*}^1 H_*^{n-1}(v)\, dH_*(v) = 1 - \frac{1}{n} \cdot \left[1 - H_*^n(\rho_*)\right].$$

[6] Similarly, in Hendricks and Sorensen (2018), the equilibrium value distribution is first-order stochastically dominated by the newborn value distribution.

In other words, measure $1 - H_*^n(\rho_*)$ of sellers sell their goods every period. Because sellers live for one period, $1 - H_*^n(\rho_*)$ is also the probability of sale of all goods in the market.

The stationary value distributions can be determined by Eqs. 3 and 4 by substituting for $l(v) = 1 - \overline{P}(v)$, a value v buyer's expected probability of losing in a direct anonymous mechanism. A value v buyer's expected utility of reporting z is $\overline{P}_*(z) \cdot v - \overline{C}_*(z)$ in the mechanism and is $\overline{P}_*(v) \cdot v - \overline{C}_*(v)$ if she does not win. The stationary value distributions are determined as follows.

$$h_*(v) = \left[1 - s \cdot \int_0^1 \left(1 - \overline{P}_*(z)\right) dH_*(z)\right] \cdot f(v) + s \cdot \left(1 - \overline{P}_*(v)\right) \cdot h_*(v), \tag{11}$$

$$H_*(v) = \left[1 - s \cdot \int_0^1 \left(1 - \overline{P}_*(z)\right) dH_*(z)\right] \cdot F(v) + s \cdot \int_0^v \left(1 - \overline{P}_*(z)\right) dH_*(z). \tag{12}$$

Given the equilibrium losing probability and the proportion of losers, the stationary value distributions characterized by Eqs. 3 and 4 are

$$h_*(v) = \frac{\left[1 - s \cdot \left(1 - \frac{1}{n} \cdot \left(1 - H_*^n(\rho_*)\right)\right)\right] \cdot f(v)}{1 - s \cdot \left(1 - \mathbf{1}_{v>\rho_*} \cdot H_*^{n-1} \cdot (v)\right)}, \tag{13}$$

$$H_*(v) = \left[1 + \frac{s}{1-s} \cdot \frac{1}{n} \cdot \left(1 - H_*^n(\rho_*)\right)\right] \cdot F(v) - \mathbf{1}_{v>\rho_*} \cdot \frac{s}{1-s} \cdot \frac{1}{n} \cdot \left(H_*^n(v) - H_*^n(\rho_*)\right). \tag{14}$$

In particular, the stationary distribution at the equilibrium reserve type is determined by

$$\frac{h_*(\rho_*)}{f(\rho_*)} = \frac{H_*(\rho_*)}{F(\rho_*)} = 1 + \frac{s}{1-s} \cdot \frac{1}{n} \cdot \left(1 - H_*^n(\rho_*)\right). \tag{15}$$

As a consequence of the auction's allocative efficiency that the buyer with the highest value above the reserve type is allocated the good, higher-value buyers exit the market faster, resulting in a stationary value distribution that is first-order stochastically dominated by the newborn value distribution.[7]

[7] To see the result of first-order stochastic dominance, rearrange Eq. 14:

$$\frac{H_*(v)}{F(v)} = \left[1 + \frac{s}{1-s} \cdot \frac{1}{n} \cdot \left(1 - H_*^n(\rho_*)\right)\right] \Big/ \left[1 + \mathbf{1}_{v>\rho_*} \cdot \frac{s}{1-s} \cdot \frac{1}{n} \cdot \frac{H_*^n(v) - H_*^n(\rho_*)}{H_*(v)}\right].$$

Since $\left[H_*^n(v) - H_*^n(\rho_*)\right] / H_*(v)$ achieves its maximum $1 - H_*^n(\rho_*)$ at $v = 1$, the RHS is bigger than 1, so $H_*(v) > F(v)$ for any v.

In summary, $(\mathrm{A}(\rho_*), \sigma_*, H_*, \mu_*)$ constitutes an SSSE. Each seller runs a reserve-price auction $\mathrm{A}(\rho_*)$ with the reserve price ρ_* determined by Eqs. 10 and 15, which also pin down $H_*(\rho_*)$. The stationary value distribution H_* is characterized by Eqs. 10 and 14, given $H_*(\rho_*)$. Each buyer reports truthfully in each incentive-compatible direct anonymous mechanism (i.e., bids $w_*(v)$ in a second-price auction by Proposition 3), and the equilibrium belief is the same across periods: $\mu_*(\mathrm{A}(\rho_*), \sigma_*, H_*) = 1$.

Such an equilibrium exists when losing buyers exit the market sufficiently fast. When enough higher-value newborns enter the market to ensure the active value distribution with decreasing inverse hazard rate, a reserve-price auction remains optimal. Sufficient conditions are as follows.

Assumption 2 The newborn value distribution is weakly convex: $F''(v) \geq 0$.

Assumption 3 The survival rate is sufficiently small: s and δ satisfy $\delta s^2 - 2s + 1 \geq 0$. Equivalently, $s \leq \left(1 - \sqrt{1-\delta}\right)/\delta$ for all δ.

These are reasonable assumptions. Although a convex distribution assumption is more restrictive than that of a monotone hazard rate, it includes some standard distributions such as uniform and exponential distributions. Although in this paper I do not have a buyer entry stage, buyers with higher values should be more likely to enter the market in any model that incorporates an entry. The assumption on the survival rate and discount factor is not restrictive. First, as long as $s \leq 1/2$, the equilibrium exists regardless of δ. And for a higher discount factor, the range of survival rates that support a unique equilibrium becomes larger. For example, for $\delta = 0.95$, the condition is satisfied as long as $s \leq 0.876$. Note that these assumptions are by no means necessary conditions for existence or uniqueness, but rather are sufficient conditions. For example, how values above ρ_{mon} are distributed does not affect equilibrium existence or uniqueness.

Proposition 4 *When Assumptions 2 and 3 hold, there exists a unique SSSE.*

Consider the convergence to this stationary equilibrium. This equilibrium is reached from periods of plays in which the reserve prices monotonically decrease over periods. Starting with the initial period in which all buyers are newborns, sellers run auctions with a reserve price higher than the equilibrium price, because buyer value distribution first-order stochastically dominates the stationary distribution and buyers face a worse subsequent market than buyers in the steady state. However, as time progresses, lower-value buyers congest the market and expect a more buyer-friendly environment in future periods, and sellers post lower reserve prices in response. The sequence of reserve prices monotonically converges to the equilibrium reserve price. The monotonicity of inverse hazard rate of the active buyer value distribution in each period is guaranteed because the subsequent market improves and the active buyer distribution decreases as higher-value buyers win more often and exit faster.

Proposition 5 *A value v buyer's equilibrium WTP is*

$$w_*(v) = v - \mathbf{1}_{v \geq \rho_*} \cdot \int_{\rho_*}^{v} \frac{s \cdot \delta \cdot H_*^{n-1}(z)}{1 - s \cdot \delta + s \cdot \delta \cdot H_*^{n-1}(z)} dz. \tag{16}$$

Hence, by Eq. 5, when the discount factor is interpreted as $s \cdot \delta$, in equilibrium, it is as if each buyer faces one subsequent market

$$\Lambda_* (x) = \mathbf{1}_{v > \rho_*} \cdot \frac{H_*^{n-1}(x)}{1 - s \cdot \delta + s \cdot \delta \cdot H_*^{n-1}(x)} \quad \forall x > \rho_*. \tag{17}$$

As I have explained, the equilibrium subsequent market summarizes the buyer-friendliness of the market. In the next section, I use the change in the equilibrium subsequent market to explore effects of changes in different parameters of the model—namely, the survival rate, buyer–seller ratio, and discount factor—on the equilibrium reserve price, stationary value distribution, buyer payoffs, seller revenue, and sale efficiency.

5 Comparative statics

In this section, I investigate how the equilibrium is affected by the respective changes in (i) buyer survival rate, (ii) buyer–seller ratio, (iii) discount factor, and (iv) newborn value distribution. The three parameters, (i)–(iii), and one distribution, (iv), parsimoniously summarize the model, and each represents different characteristics of the market.

I consider the changes in the sellers', buyers', and social planner's equilibrium welfare, respectively. It should not be a surprise that sellers and buyers always exhibit conflicts of interests, but it is not obvious that the social planner's welfare aligns with the sellers'. In general, changes that benefit sellers and harm buyers include decreasing buyer survival rate, increasing competition, and making buyers less patient. However, from a social planner's point of view, harming the buyers improves the allocative efficiency, because the probability of sale increases.

A seller's expected revenue from his optimally chosen auction A (ρ_*) is

$$r_* = \int_{\rho_*}^{1} \left[w_* (v) - \eta \left(H_* (v) \right) \cdot w_*' (v) \right] dH_*^n (v) , \tag{18}$$

and a value v buyer's total discounted expected utility is

$$\underline{u}_* (v) = \mathbf{1}_{v \geq \rho_*} \cdot \int_{\rho_*}^{v} \frac{s \cdot \delta \cdot H_*^{n-1}(z)}{1 - s \cdot \delta + s \cdot \delta \cdot H_*^{n-1}(z)} dz. \tag{19}$$

For the comparative statics results, I need to characterize the changes in the reserve price of the optimal auction, in the stationary buyer value distribution, and in the buyer's WTP.

Different types of efficiencies are associated with a sale mechanism. If the good is transferred from the seller who values the good at zero to any of the buyers, I say the transaction is sale efficient. If the sale occurs, and it is transferred to the buyer with the highest value, I say the sale is allocatively efficient. For example, a zero-reserve second-price auction is both sale efficient and allocatively efficient, because the good always ends up in the hands of the buyer with the highest valuation. A positive reserve-price auction is not always sale efficient but is allocatively efficient, because the buyers' competition results in the allocative efficiency. However, a positive posted price—including the optimal posted price—is neither sale nor allocatively efficient. A posted price results in more ex post sale inefficient allocation than the optimal auction because the optimal reserve price is lower than the optimal posted price, and it may also result in allocative inefficiency because the good does not necessarily end in the hands of the buyer with the highest value.

The total social welfare is the discounted sum of buyers and sellers across all periods, but a little more careful consideration reveals that the equilibrium probability of sale is the key indicator of social efficiency. Since allocatively efficient auctions (mechanisms that assign the good to the highest valued agent if not withheld by the seller) are used, and buyers and sellers divide the surplus, the good being transferred is more preferred than otherwise, because the good being held by sellers is the most allocatively inefficient allocation. Hence, social efficiency monotonically increases in sale probability.

The platform designer's profit is closely tied to sale and allocative efficiency. If his profit is proportional to the total social surplus he generates, given that a mechanism is allocatively efficient—as is the case in an auction—higher sale efficiency results in higher total welfare.[8] If instead the designer charges a participation fee for new agents who arrive in the market, he hopes that more new agents will arrive, which is realized when the equilibrium probability of sale increases.

Therefore, in order to consider the changes in welfare, I solve for the comparative statics of these outcome variables: the reserve price, the probability of sale, stationary buyer value distribution, buyers' payoffs, and sellers' revenue. I summarize these results in the next three propositions. All of the changes have definite signs and monotonic effects over the range, as long as an additional convexity assumption on the newborn value distribution is imposed. It further restricts Assumption 2, which guarantees equilibrium existence and uniqueness, but not by much: Any distribution with CDF $F(v) = v^k$ for any $k \geq 1$, including the uniform distribution, still satisfies the assumption.

Assumption 4 $v \cdot f(v)/F(v)$ is increasing for any $v < \rho_{\mathrm{mon}}$.

First, let us consider the change in the buyer survival rate. Variations in the buyer survival rate may reflect those in the elasticities of demand across different types of goods: People are more likely to keep searching for a cell phone until they get one, but they may quit searching for a book if they cannot find it at a satisfactory price. The survival rate can also represent search friction in the market. A higher survival rate means that current buyers can easily find a perfect substitute in the existing market. Lastly, recall that the market size growth rate is inversely related to the survival rate, so an increase in the survival rate corresponds to a slower market expansion or faster market contraction.

Suppose that the survival rate increases so that the good becomes more inelastically demanded, the consumer's search friction decreases, or the consumer base

[8] The idea that the platform designer's maximum profit is proportional to the total welfare appears in Oi (1971) and Armstrong (1999), for example.

remains relatively more stable. When sellers survive to the next period with a higher probability, the equilibrium stationary value distribution will be depressed because lower-value buyers crowd the market, preventing newcomers from entering. The equilibrium reserve price sellers choose goes down as a result. The probability of sale, which is positively related to the equilibrium reserve price, decreases. Buyers' expected utilities increase, as they can search longer, and the seller's revenue decreases.

Proposition 6 (Survival rate) *When the survival rate s increases, the reserve price decreases, the probability of sale decreases, the stationary value distribution shifts down first-order stochastically, buyers' payoffs increase, and sellers' revenue decreases.*

The proof relies on Eqs. 10 and 15, which pin down the equilibrium reserve price and the stationary value distribution at the equilibrium reserve price, respectively. Algebraic rearrangements of the differentiation by the implicit function theorem with the continuity of the solution guaranteed specifies the change in the reserve price, and with Assumption 4, the sign is definite. The ensuing changes follow from the change in reserve price. The proofs of the following two propositions follow the same method, with the last one particularly easy, since the discount factor does not affect the equilibrium conditions.

A decrease in the buyer–seller ratio—in other words, a reduction in the relative number of buyers to sellers—indicates that the relative demand decreases or the relative supply increases. Similar to the increase in survival rate, if the buyer–seller ratio decreases, buyers are better off and sellers are worse off. What is not obvious is that although the reserve price decreases, the probability of sale decreases as well: The demand plummets more than what the supply side can optimally adjust to.

Proposition 7 (Buyer–seller ratio) *When the buyer–seller ratio n decreases, the reserve price decreases, the probability of sale decreases, the stationary value distribution shifts down first-order stochastically, buyers' payoffs increase, and sellers' revenue decreases.*

Finally, let us examine the effects of change in the discount factor. If the discount factor increases, buyers become more patient, or the interval between time periods decreases and buyers have more subsequent purchasing opportunities in the imminent future. The stationary value distribution and the reserve price are not affected by the change in the discount factor: Sellers do not change their optimal mechanism, as the equilibrium reserve price only depends on the survival rate and the buyer–seller ratio.

Proposition 8 (Discount factor) *When the discount factor δ increases, the reserve price, the probability of sale, and stationary value distribution are not affected, buyers' payoffs increase, and sellers' revenue decreases.*

Overall, when the survival rate increases and/or the buyer–seller ratio decreases, buyers' expected utilities increase since their WTPs are depressed. Sellers' revenue decreases, as they face a buyer composition that is first-order stochastically worse (as a result of the precedent sellers' responses to a more buyer-friendly market environment). However, social sale efficiency decreases in response, resulting in a lower probability of sale. When buyers become more patient, the market composition does not change, but sellers' revenue will be hit hard, as buyers are more willing to wait for future chances.

Finally, I investigate the effects of a change in newborn buyer value distribution. I restrict my attention to a class of distributions for tractability. When buyers value the good more, sellers can increase reserve prices, but the probability of sale increases nonetheless.

Proposition 9 (Buyer value distribution) *Suppose the buyer value distribution is $F = v^k$ for $k \geq 1$. As k increases, the reserve price increases, the probability of sale increases, the stationary value distribution increases, buyers' utility decreases, and sellers' revenue increases.*

6 Posted prices and their market effects

Though reserve-price auctions are proven to be revenue maximizing in the model, posted prices may offer the following advantages, not explicitly modeled above. Posted prices may be (i) *less costly*: A fee is associated with setting a reserve price, but there is no such fee for simply setting a price; (ii) *more immediate*: Buyers can buy an item at a fixed price immediately (labeled "Buy It Now" on eBay), but must wait for the conclusion of an auction to obtain the good; (iii) *less ambiguous and uncertain in transaction price*: Buyers know exactly how much they need to pay for a posted price; (iv) *simpler*: Buyers do not need to think about their exact willingness to pay for a good, because they only need to think about whether they are willing to pay above the posted price to obtain the good, and sellers do not need to think about the reserve price to set or pay attention to the auctioning process; and (v) *more preferred*: Sellers and buyers may simply prefer one mechanism over another.

Each of the reasons stated above can be modeled in a more elaborate manner in a separate paper; Einav et al. (2018) provide a parsimonious model that captures these differences between prices and auctions. To be agnostic about the exact reasons in reality, I take a reduced-form approach by simply assuming that there is an additional cost associated with running an auction. The auction cost can be positive, zero, or negative, and can vary by seller. I conclude the section by providing a welfare result about the market with coexisting prices and auctions. Most importantly, posted prices may result in sale and allocative inefficiencies.

Einav et al. (2018) document that auctions were widely used in the early 2000s because buyers were excited about auctions, but were used less often when the goods being sold changed. I offer an alternative explanation: When there are more and better purchasing opportunities for buyers in subsequent periods, the revenue advantage from running an auction relative to posting a price decreases. Even without a change in sellers' underlying preferences for auctions versus prices, sellers are more likely to use posted prices in a more buyer-friendly market.

Proposition 10 *When the subsequent market becomes more buyer-friendly or buyers become more patient, the auction premium decreases for any critical type.*

Furthermore, I investigate the welfare effects when sellers have heterogeneous auction costs and can choose posted prices. Each seller chooses either a price or an auction, but has a heterogeneous fixed cost associated with running an auction. The cost c is independently and identically drawn from the cost distribution G on support $[-1, 1]$ with positive PDF g. The expected profit is the expected revenue minus the auction cost. The matching and market processes remain the same as in the previous setting. The modified definition and characterization of the equilibrium are detailed in Appendix C.

When sellers have heterogeneous auction costs, as posted prices are introduced, allocative inefficiency increases but the change in sale efficiency is ambiguous. High-cost sellers are the ones using posted prices, and they get a strictly lower profit than those who use auctions. High-value buyers' expected utilities are also depressed. If sellers and buyers decide whether to enter the market based on ex ante expected utilities, the expected prevalence of posted price mechanisms may deter their entrance, and possibly causes the market to collapse if there is a sufficiently competitive rival platform. The usage rate of auctions has rapidly decreased from over 95% at the beginning of 2003 to less than 25% at the end of 2010 to 21% in 2016 (Einav et al. 2018; Backus et al. 2020).[9] In the same period, the growth of eBay has stalled. The increasing prevalence of inefficient posted prices offers a possible reason for the relatively slow growth. Facilitating buyer bids by allowing bidding bots, reducing the fixed costs of auctions for sellers and buyers, and deterring posted prices may be several options worth trying.[10]

7 Conclusion

The paper shows that a reserve-price auction remains a desirable choice for a seller facing uncertain demand and uncertain future competition. The reserve price, determined by a generalized marginal revenue curve, not only screens for buyers' willingnesses to pay but also indicates market competitiveness. The optimal posted price is shown to be determined in a similar way as the optimal reserve price.

Although the optimal mechanism remains a reserve-price auction, subsequent outside options reduce the desirability of an auction and curtail some of the most important features of an auction. The advantage of auction comes from the surplus extracted from the possible high valuations of the buyers, but subsequent purchasing opportunities

[9] The share of auctions in active listings declined from 95.98% at the beginning of 2003 to 21.85% at the beginning of 2011, and the share in revenue declined from 90.16 to 51.58% (Einav et al. 2018). A nontrivial proportion of sellers—from 1% in 2005 to 12% in 2016—switched to bargaining (i.e., eBay's "Best Offer" platform) (Backus et al. 2020).

[10] I do not characterize any theoretical convergence to SSSE, but empirically, one episode suggests that the speed of convergence to SSSE with prices and auctions is rather fast: 3 weeks in Ockenfels and Roth (2004). A deck of cards with each showing a most wanted terrorist was issued by the US military to its solders on April 11, 2003, but was sold on eBay and retail immediately (on day 2, April 12, and day 3, April 13, respectively). The deck, for its novelty, origin, and rarity, sold for nearly $70 in the first week, and mostly by auction. However, the US Playing Cards Company released identical copies of the cards for $5.95 and received publicity. Prices dropped to competitive prices in 3 weeks, and gradually higher proportion of decks were sold through posted prices.

make the current item dispensable, especially for buyers with high values for the item. A mechanism particularly immune to such turns in market conditions is a posted price mechanism in which the transaction price is not dependent on buyers with high willingness to pay. However, posted prices are allocatively inefficient and impose market externalities that affect sale efficiency and active buyer composition, which lowers profits for future sellers.

I will offer some concluding remarks by focusing on the study's limitations and possible avenues for future work. One issue is the seller's commitment in case he does not sell, which is embodied by the assumption I maintained throughout the paper: Sellers live for only one period, and thus choose a static selling mechanism. When the transaction costs of posting to intermediaries are high or when the items for sale a have high depreciation rate or face intense competition from substitutable goods, a seller is better off using one-shot mechanisms. Reputation and rating systems also punish sellers for repeatedly selling low-quality goods. Furthermore, many goods that have substitutable competition and upgrades have steep price drops in a short period, which essentially prevents a seller from choosing a dynamically optimal pricing rule or repeated auctions. A dynamic model that considers possible resale is also of potential interest.

Although I suggest several possibilities for the rise in auction cost including risk aversion, a discount factor, fixed cost, and idiosyncratic taste, the simple, one-dimensional cost imposed is seemingly ad hoc and does not tackle the more fundamental question of *why* an auction is less desirable for a rational agent who should supposedly only care about revenue. Work on dynamic mechanism design and revenue management may provide more micro-founded justifications. Competing sellers may result in alternative mechanism choice and information revelation incentives (Rezende 2018; Troncoso-Valverde 2018).

An SSSE is proven to exist uniquely in various settings, but I have not considered the speed of convergence to such stationary equilibrium behavior from an initial value distribution. If the speed of convergence to the SSSE is low, then it will be important to investigate the adjustment on the equilibrium path to see, for example, how sellers switch from auctions to posted prices. Furthermore, I do not investigate whether it is guaranteed that any change in the market environment results in global convergence to the new SSSE.

Finally, a line of empirical research warrants investigation, especially in dynamic markets with prices and auctions. Equilibrium characterizations are already complicated, such that definitive comparative statics results were not obtainable because of the complexity and nonmonotonicity that resulted from sellers' possible switch to alternative mechanisms. Numerical simulations and empirical work can be conducted to investigate different inefficiencies and externalities; quantify the effects such as changes in revenues and probabilities of sale; and test the validity of the assumptions made in this paper. Recent papers make significant progress toward that end (Hendricks and Sorensen 2018; Backus and Lewis 2019; Coey et al. 2019; Backus et al. 2020; Bodoh-Creed et al. 2020).

Acknowledgements I thank the editor (Nicholas Yannelis), two referees, Jon Eguia, Brent Hickman, Eiichiro Kazumori, Maciej Kotowski, Markus Möbius, Arijit Mukherjee, Roger Myerson, Hugo Sonnenschein, and Glen Weyl for valuable suggestions, and especially Scott Kominers and Phil Reny for continuous advice. Discussions at the Stony Brook Game Theory Festival, University of Chicago Economic Theory and Industrial Organization Working Groups, Midwest Theory Meeting, and Econometric Society Winter Meeting, as well as the hospitality of Microsoft Research New England are gratefully acknowledged.

Compliance with ethical standards

Conflict of interest The authors declare that there is no conflict of interest.

A Omitted proofs

Proof of Proposition 1 It follows as a corollary of Proposition 12 in Appendix B when all the value distributions are identically F. When the WTP function $w(\cdot)$ is not continuously differentiable, the optimal reserve type ρ^* satisfies $\mathrm{MR}^{\mathrm{A}}\left(\rho^*_+\right) \geq 0$ and $\mathrm{MR}^{\mathrm{A}}\left(\rho^*_-\right) \leq 0$. The existence and uniqueness of ρ^* are guaranteed by the monotonicity of the auction marginal revenue curve. □

Proof of Proposition 2 The optimal reserve type is determined by

$$w_\Lambda\left(\rho^*_\Lambda\right)/w'_\Lambda\left(\rho^*_\Lambda\right) = \eta\left(F\left(\rho^*_\Lambda\right)\right).$$

The LHS is increasing and the RHS is decreasing. If I show that the LHS is greater than ρ^*_Λ, then $\rho^*_\Lambda < \rho_{\mathrm{mon}}$. For any v, $w_\Lambda(v)/w'_\Lambda(v) \geq v$ if $w_\Lambda(v) - v \cdot w'_\Lambda(v) \geq 0$, which is satisfied because

$$\begin{aligned} w_\Lambda(v) - v w'_\Lambda(v) &= v - \delta \cdot \int_{\underline{x}}^{v} \Lambda(x)\, dx - v \cdot (1 - \delta \cdot \Lambda(v)) \\ &= v \cdot \delta \cdot \Lambda(v) - \delta \cdot \int_{\underline{x}}^{v} \Lambda(x)\, dx \geq 0, \end{aligned}$$

where the inequality follows from $\Lambda(x) \leq \Lambda(v)$ for all $x \leq v$. □

Proof of Proposition 3 A value v buyer's utility by bidding according to the strictly increasing bidding function $\sigma(z)$ is

$$u(v, \sigma(z), \sigma_{-i}(\cdot)\,|\mathrm{A}(\rho)) = F^{n-1}(z) \cdot (v - \sigma(z)) + l(z) \cdot \delta \cdot \underline{u}_\Lambda(v),$$

where $l(z) = 1 - \mathbf{1}_{v \geq \rho} \cdot F^{n-1}(z)$ is the expected probability of losing by reporting z. Rearrange,

$$u(v, \sigma(z), \sigma_{-i}(\cdot)\,|\mathrm{A}(\rho)) = F^{n-1}(z) \cdot (w_\Lambda(v) - \sigma(z)) + \delta \cdot \underline{u}_\Lambda(v). \quad (20)$$

Differentiate Eq. 20 with respect to z. The buyer optimality condition requires that it is zero at $z = v$. Coupled with the boundary condition that $\sigma(\rho) = w_\Lambda(\rho)$, the equilibrium bidding function is obtained. The candidate bidding function is indeed the equilibrium, as the utility is maximized at $z = v$. The equilibrium payoffs are obtained by replugging in the equilibrium bidding function, and the revenue is obtained by integrating the auction MR curve. □

Proof of Proposition 4 I first show that there exists a unique solution of ρ_* to Eqs. 10 and 15, and it is the only candidate equilibrium reserve type. It is then shown to be the unique optimal reserve type that a seller chooses in the equilibrium. Coupled with the stationary value distributions described by Eqs. 13 and 14, it constitutes an equilibrium.

First, I show the existence and uniqueness of ρ_* and $H_*(\rho_*)$ for Eqs. 10 and 15. If every seller chooses the same reserve-type ρ auction (not necessarily optimal), the stationary value distribution resulted from it is characterized by Eq. 14, with ρ instead of ρ_*. In particular, $x \equiv H_*(\rho)$ is determined by Eq. 15,

$$x \Big/ \left[1 + \frac{s}{1-s} \cdot \frac{1}{n} \cdot \left(1 - x^n\right)\right] = F(\rho).$$

LHS is monotonically strictly increasing in x, ranging from 0 (when $x = 0$) to 1 (when $x = 1$). Since RHS is a fixed number between 0 and 1 for any ρ, there is a unique solution $x(\rho) = H_*(\rho)$ for each ρ, and is strictly increasing in ρ.

In order to show that Eq. 10 has a solution, it is sufficient to show that LHS is increasing in ρ, and it holds when $1/\eta(H_*(\rho))$ is increasing,

$$\begin{aligned}\frac{h_*(\rho)}{1 - H_*(\rho)} &= \frac{H_*(\rho)}{1 - H_*(\rho)} \cdot \frac{f(\rho)}{F(\rho)} \\ &= f(\rho) \cdot \left[\frac{1}{1 - H_*(\rho)} + \frac{s}{1-s} \cdot \frac{1}{n} \cdot \frac{1 - {H_*}^n(\rho)}{1 - H_*(\rho)}\right],\end{aligned}$$

where both equalities follow from Eq. 15. Density f is increasing by Assumption 2 and the second term is increasing in ρ because $H_*(\rho)$ is increasing. Overall, LHS of Eq. 10 continuously and monotonically increases from $-1/f(0)$ to 1 as ρ increases from 0 to 1. Therefore, the solution to the system of equations, ρ_*, is unique.

In fact, it is the only candidate equilibrium. Any ρ can be the symmetric equilibrium reserve type but the stationary distribution from such choice of ρ is pinned down by solution $H_*(\rho) = x(\rho)$. Only the pairs $(\rho, x(\rho))$ satisfy stationarity condition and only one pair is candidate equilibrium by the seller's optimality condition that determines the reserve price. Finally, it is sufficient to show that under $H_*(\rho_*)$, ρ_* is indeed the unique optimal reserve price.

The reserve type ρ_* is optimal if $\mathrm{MR}_*^{\mathrm{A}}(v)$ is increasing, so it is sufficient to show that $h_*(v)/[(1 - H_*(v)) \cdot w_*'(v)]$ is increasing.

$$\frac{d \log\left[h_*(v) / \left[(1 - H_*(v)) \cdot w_*'(v)\right]\right]}{dv} = \frac{h_*'(v)}{h_*(v)} + \frac{h_*(v)}{1 - H_*(v)} - \frac{w_*''(v)}{w_*'(v)}.$$

By Eq. 3,

$$(1 - s \cdot l_*(v)) \cdot h_*(v) = (1 - s \cdot l_*) \cdot f(v),$$

which implies

$$(1 - s \cdot l_* (v)) \cdot h'_* (v) - s \cdot l'_* (v) \cdot h_* (v) = (1 - s \cdot l_*) \cdot f' (v) .$$

Therefore,

$$\frac{h'_* (v)}{h_* (v)} = \frac{f' (v)}{f (v)} + \frac{s \cdot l'_* (v)}{1 - s \cdot l_* (v)}.$$

Differentiate Eq. 22:

$$w''_* (v) = w'_* (v) \cdot \frac{\delta \cdot s \cdot l'_* (v)}{1 - \delta \cdot s \cdot l_* (v)}.$$

Pulling together, I get

$$\begin{aligned}
&\frac{f' (v)}{f (v)} + \frac{s \cdot l'_* (v)}{1 - s \cdot l_* (v)} + \frac{h_* (v)}{1 - H_* (v)} - \frac{\delta \cdot s \cdot l'_* (v)}{1 - \delta \cdot s \cdot l_* (v)} \\
&= \frac{f' (v)}{f (v)} + \frac{h_* (v)}{1 - H_* (v)} + \frac{(1-\delta) \cdot s \cdot l'_* (v)}{(1 - s \cdot l_* (v)) \cdot (1 - \delta \cdot s \cdot l_* (v))} \\
&= \frac{f' (v)}{f (v)} + h_* (v) \cdot \Bigg[\frac{1}{1 - H_* (v)} - \frac{(1-\delta) \cdot s}{(1 - s \cdot l_* (v)) \cdot (1 - \delta \cdot s \cdot l_* (v))} \\
&\quad \cdot (n-1) \cdot H_*^{n-2} (v) \Bigg].
\end{aligned}$$

Note that $\frac{1}{1-H_*(v)} \geq \sum_{j=0}^{n-2} H_*^j (v) \geq (n-1) H_*^{n-2} (v)$, so the term in the square brackets is greater than 0 if

$$\frac{(1-\delta) \cdot s}{(1 - s \cdot l_* (v)) (1 - \delta \cdot s \cdot l_* (v))} \leq 1,$$

where the LHS is upper bounded by $\frac{(1-\delta)\cdot s}{(1-s)\cdot(1-\delta\cdot s)}$, which is achieved when $l_* (v) = 1$. The numerator is smaller than the (positive) denominator when Assumption 3 holds. Coupled with Assumption 2 that guarantees f' to be positive, the auction marginal revenue curve is strictly increasing, so ρ_* is the only solution to $\mathrm{MR}_*^{\mathrm{A}} (\rho) = 0$, thus the only equilibrium. □

Proof of Proposition 5 Because the equilibrium behavior and the value distribution of active buyers are stationary, the total discounted expected payoff of a value $v > \rho_*$ buyer is the same as her continuation payoff, which is the same as the equilibrium expected payoff in $\mathrm{A} (\rho_*)$, which by Eq. 8 is,

$$\underline{u}_* (v) = u (v|\mathrm{A} (\rho_*)) = \int_{\rho_*}^{v} \left[H_*^{n-1} (z) \cdot w'_* (z) \right] dz + s \cdot \delta \cdot \underline{u}_* (v) . \tag{21}$$

Her WTP is her value net her present value of expected continuation payoff depressed by her survival rate and discount factor,

$$w_*(v) = v - s \cdot \delta \cdot \underline{u}_*(v) = v - \frac{s \cdot \delta}{1 - s \cdot \delta} \int_{\rho_*}^{v} \left[H_*^{n-1}(z) \cdot w_*'(z) \right] dz, \tag{22}$$

where Eq. 22 is derived from Eq. 21. Differentiate both sides of Eq. 22 and rearrange:

$$w_*'(v) = \frac{1 - s \cdot \delta}{1 - s \cdot \delta \cdot \left(1 - H_*^{n-1}(v)\right)} = \frac{1 - s \cdot \delta}{1 - s \cdot \delta \cdot l_*(v)}.$$

Hence, a value v buyer's equilibrium WTP is

$$w_*(v) = v - \mathbf{1}_{v \geq \rho_*} \cdot \int_{\rho_*}^{v} \frac{s \cdot \delta \cdot H_*^{n-1}(z)}{1 - s \cdot \delta + s \cdot \delta \cdot H_*^{n-1}(z)} dz. \tag{23}$$

□

Proof of Proposition 6 Equilibrium ρ_* and $H_*(\rho_*)$ satisfy

$$H_*(\rho_*) \equiv \left[1 + \frac{s}{1-s} \cdot \frac{1}{n} \cdot \left(1 - H_*^n(\rho_*)\right)\right] \cdot F(\rho_*) \tag{24}$$

$$\rho_* \cdot h_*(\rho_*) + H_*(\rho_*) \equiv 1 \tag{25}$$

Plugging the equality $H_*(\rho_*)/h_*(\rho_*) = F(\rho_*)/f(\rho_*)$ into Eq. 25,

$$H_*(\rho_*) = F(\rho_*) / \left[F(\rho_*) + \rho_* \cdot f(\rho_*)\right], \tag{26}$$

and into Eq. 24,

$$\frac{1}{F(\rho_*) + \rho_* \cdot f(\rho_*)} = 1 + \frac{s}{1-s} \cdot \frac{1}{n} \cdot \left[1 - \left(\frac{F(\rho_*)}{F(\rho_*) + \rho_* \cdot f(\rho_*)}\right)^n\right]. \tag{27}$$

This holds for all s and n at the equilibrium reserve price $\rho_*(s, n)$. Applying implicit function theorem to Eq. 26,

$$\frac{dH_*(\rho_*)}{ds} = \left(\frac{1}{1 + \frac{\rho_* \cdot f(\rho_*)}{F(\rho_*)}}\right)' \cdot \frac{d\rho_*}{ds} \tag{28}$$

has the opposite sign as $d\rho_*/ds$, because the sign of the first term is negative by Assumption 4. By implicit function theorem again, differentiate Eq. 27 with respect to s,

$$\left(\frac{1}{F(\rho_*)+\rho_*\cdot f(\rho_*)}\right)'\cdot\frac{d\rho_*}{ds}$$
$$=\left(\frac{s}{1-s}\right)'\cdot\frac{1}{n}\cdot\left[1-H_*^n(\rho_*)\right]-\frac{s}{1-s}\cdot H_*^{n-1}(\rho_*)\cdot\frac{dH_*(\rho_*)}{ds}.$$

Plug Eq. 28 in and rearrange:

$$\left[\left(\frac{1}{F(\rho_*)+\rho_*\cdot f(\rho_*)}\right)'+\frac{s}{1-s}\cdot H_*^{n-1}(\rho_*)\cdot\left(\frac{1}{1+\frac{\rho_*\cdot f(\rho_*)}{F(\rho_*)}}\right)'\right]\cdot\frac{d\rho_*}{ds}$$
$$=\left(\frac{s}{1-s}\right)'\cdot\frac{1}{n}\cdot\left(1-H_*^n(\rho_*)\right).$$

Since $\left(\frac{s}{1-s}\right)'>0$, the sign of $\frac{d\rho_*}{ds}$ is the same as

$$\left[\left(\frac{1}{F(\rho_*)+\rho_*\cdot f(\rho_*)}\right)'\right]+\left[\frac{s}{1-s}\cdot H_*^{n-1}(\rho_*)\cdot\left(\frac{1}{1+\frac{\rho_*\cdot f(\rho_*)}{F(\rho_*)}}\right)'\right].$$

The term in the first square brackets is negative because $f'(v)\geq 0$, and the term in the second square brackets is negative by Assumption 4. Therefore, $d\rho_*/ds<0$. The sign is strict because $\rho_*\cdot f(\rho_*)$ is strictly increasing.

The change in the probability of sale with respect to survival rate has opposite sign as that of $dH_*(\rho_*)/ds$, which has opposite sign as $d\rho_*/ds$, so it is negative.

By Eq. 14, for all $v<\rho_*$,

$$H_*(v)=F(v)\cdot\left[1+\frac{s}{1-s}\cdot\frac{1}{n}\cdot\left(1-H_*^n(\rho_*)\right)\right].$$

Since the term in the square brackets equals $1/\left(F(\rho_*)+\rho_*\cdot f(\rho_*)\right)$, it increases as s increases:

$$\frac{d\left[1/\left(F(\rho_*)+\rho_*\cdot f(\rho_*)\right)\right]}{ds}=\left(\frac{1}{F(\rho_*)+\rho_*\cdot f(\rho_*)}\right)'\cdot\frac{d\rho_*}{ds}>0,$$

as both multiplicands are negative. For all $v\geq\rho_*$,

$$H_*(v)=F(v)\cdot\left[1+\frac{s}{1-s}\cdot\frac{1}{n}\cdot\left(1-H_*^n(\rho_*)\right)\right]-\frac{s}{1-s}\cdot\frac{1}{n}\cdot\left(H_*^n(v)-H_*^n(\rho_*)\right).$$

Rearrange:

$$H_*(v) + \frac{s}{1-s} \cdot \frac{1}{n} \cdot H_*^n(v) = F(v) + F(v) \cdot \frac{s}{1-s} \cdot \frac{1}{n} \cdot \left(1 - H_*^n(\rho_*)\right) + \frac{s}{1-s} \cdot \frac{1}{n} \cdot H_*^n(\rho_*). \tag{29}$$

Differentiate with respect to s and let $x \equiv H_*(v)$, the LHS is

$$\frac{dx}{ds} + \frac{s}{1-s} \cdot x^{n-1} \cdot \frac{dx}{ds} + \left(\frac{s}{1-s}\right)' \cdot \frac{1}{n} \cdot H_*^n(v),$$

and the RHS is

$$F(v) \cdot \left(\frac{s}{1-s}\right)' \cdot \frac{1}{n} \cdot \left(1 - H_*^n(\rho_*)\right) + \left(\frac{s}{1-s}\right)' \cdot \frac{1}{n} \cdot H_*^n(\rho_*) + (1 - F(v)) \cdot \frac{s}{1-s} \cdot H_*^{n-1}(\rho_*) \cdot \frac{dH_*(\rho_*)}{ds}.$$

Rearrange, $\left(1 + \frac{s}{1-s} \cdot x^{n-1}\right) \cdot \frac{dx}{ds}$ equals

$$\left[F(v) \cdot \frac{1}{n} \cdot \left(1 - H_*^n(\rho_*)\right) - \frac{1}{n} \cdot \left(H_*^n(v) - H_*^n(\rho_*)\right)\right] \cdot \left(\frac{s}{1-s}\right)' + (1 - F(v)) \cdot \frac{s}{1-s} \cdot H_*^{n-1}(\rho_*) \cdot \frac{dH_*(\rho_*)}{ds}.$$

To show that $dx/ds \geq 0$, it suffices to show that the terms in the square brackets are positive, because $\frac{dH_*(\rho_*)}{ds} \geq 0$ is already shown. By Eq. 14, the term equals

$$[H_*(v) - F(v)] / \left(\frac{s}{1-s}\right) \geq 0,$$

where the inequality follows from the fact the newborn value distribution first-order stochastically dominates the stationary value distribution in equilibrium.

The buyer's utility then increases as the subsequent market becomes more buyer-friendly. For Eq. 19, the integrand increases as $H_*(v)$ increases for all $v > \rho_*$, s increases, and ρ_* decreases. On the other hand, the seller's revenue decreases because the willingnesses to pay of the buyers all decrease, and the buyer stationary value distribution first stochastically increases, resulting in less equilibrium probability of sale. □

Proof of Proposition 7 The comparative statics results still follow from the equilibrium conditions of Eqs. 24 and 25. Differentiate Eq. 27 with respect to n, the LHS is

$$\left(\frac{1}{F(\rho_*) + \rho_* \cdot f(\rho_*)}\right)' \cdot \frac{d\rho_*}{dn},$$

and the RHS becomes

$$\begin{aligned}
&\frac{s}{1-s}\cdot\left(-\frac{1}{n^2}\right)\cdot\left[1-H_*^n\left(\rho_*\right)\right] \\
&\quad+\frac{s}{1-s}\cdot\left[-n\cdot H_*^{n-1}\left(\rho_*\right)\cdot\frac{dH_*\left(\rho_*\right)}{dn}-H_*^n\left(\rho_*\right)\cdot\log\left(H_*\left(\rho_*\right)\right)\right] \\
&=-\frac{s}{1-s}\cdot n\cdot H_*^{n-1}\left(\rho_*\right)\cdot\frac{dH_*\left(\rho_*\right)}{dn} \\
&\quad-\frac{s}{1-s}\cdot\frac{1}{n^2}\cdot\left[1-H_*^n\left(\rho_*\right)+H_*^n\left(\rho_*\right)\cdot\log\left(H_*^n\left(\rho_*\right)\right)\right].
\end{aligned}$$

Equate the two sides and rearrange:

$$\begin{aligned}
&-\left[\left(\frac{1}{F\left(\rho_*\right)+\rho_*\cdot f\left(\rho_*\right)}\right)'+\frac{s}{1-s}\cdot n\cdot H_*^{n-1}\left(\rho_*\right)\cdot\left(\frac{1}{1+\frac{\rho_*\cdot f\left(\rho_*\right)}{F\left(\rho_*\right)}}\right)'\right]\cdot\frac{d\rho_*}{dn} \\
&=\frac{s}{1-s}\cdot\frac{1}{n^2}\cdot\left[1-H_*^n\left(\rho_*\right)+H_*^n\left(\rho_*\right)\cdot\log\left(H_*^n\left(\rho_*\right)\right)\right].
\end{aligned}$$

The two terms in the square brackets in the LHS are both negative as shown in the previous proof (first by Assumption 2 and second by Assumption 4). Therefore $\frac{d\rho_*}{dn}$ has the same sign as that of RHS. Since $H_*^n\left(\rho_*\right)<1$,

$$H_*^n\left(\rho_*\right)\cdot\left(1-\log\left(H_*^n\left(\rho_*\right)\right)\right)-1=(1+x)/\exp(x)-1<0$$

for $x=-\log\left(H_*^n\left(\rho_*\right)\right)>0$ as $\exp(x)>1+x$ by Taylor expansion. Differentiation of Eq. 24 yields the same result as with s, so it is Eq. 28 with n replacing s, so $dH_*\left(\rho_*\right)/dn<0$. Furthermore,

$$\begin{aligned}
d\left(1-H_*^n\left(\rho_*\right)\right)/dn&=-H_*^n\left(\rho_*\right)\cdot\log\left(H_*\left(\rho_*\right)\right)-n\cdot H_*^{n-1}\left(\rho_*\right)\cdot\frac{dH_*\left(\rho_*\right)}{dn} \\
&=-H_*^{n-1}\left(\rho_*\right)\cdot\left(H_*\left(\rho_*\right)\cdot\log\left(H_*\left(\rho_*\right)\right)+n\cdot\frac{dH_*\left(\rho_*\right)}{dn}\right) \\
&>0. \qquad (30)
\end{aligned}$$

In summary thus far, $d\rho_*/dn>0$, $dH_*\left(\rho_*\right)/dn<0$, and $d\left(1-H_*^n\left(\rho_*\right)\right)/dn>0$.

Next, I show that $dH_*(v)/dn<0$ for all v. First, $1+\frac{s}{1-s}\cdot\frac{1}{n}\cdot\left(1-H_*^n\left(\rho_*\right)\right)$ is decreasing because its change equals

$$\left(\frac{1}{F\left(\rho_*\right)+\rho_*\cdot f\left(\rho_*\right)}\right)'\cdot\frac{d\rho_*}{dn},$$

which is negative, as the first term is negative and the second term is positive. Therefore, $H_*(v)$ decreases for all $v<\rho_*$ as n increases. Next, differentiate Eq. 29 with respect to n, the LHS equals

$$\left[1+\frac{s}{1-s}\cdot H_*^{n-1}(v)\right]\cdot\frac{dx}{dn}-\frac{s}{1-s}\cdot\frac{1}{n^2}\cdot\left(H_*^n(v)-H_*^n(v)\cdot\log\left(H_*^n(v)\right)\right),$$

where $x\equiv H_*(v)$. The RHS equals the derivative of

$$\frac{s}{1-s}\cdot\frac{1}{n}-(1-F(v))\cdot\left[1-\frac{s}{1-s}\cdot\frac{1}{n}\cdot\left(1-H_*^n(\rho_*)\right)\right],$$

which is

$$\frac{s}{1-s}\cdot\left[-\frac{1}{n^2}+(1-F(v))\cdot d\left(\frac{1}{n}\cdot\left(1-H_*^n(\rho_*)\right)\right)/dn\right]$$

Rearrange the terms:

$$\begin{aligned}&\left[1+\frac{s}{1-s}\cdot H_*^{n-1}(v)\right]\frac{dx}{dn}\\&\quad=-\frac{s}{1-s}\cdot\frac{1}{n^2}\cdot\left[1-H_*^n(v)+H_*^n(v)\cdot\log\left(H_*^n(v)\right)\right]\\&\qquad+\frac{s}{1-s}\cdot(1-F(v))\cdot d\left(\frac{1}{n}\cdot\left(1-H_*^n(\rho_*)\right)\right)/dn<0,\end{aligned}$$

where the first term being negative follows from Eq. 30 and the second term being negative follows from $\left[1+\frac{s}{1-s}\cdot\frac{1}{n}\cdot\left(1-H_*^n(\rho_*)\right)\right]$ decreasing in n.

Finally, buyers' utility decreases and sellers' revenue increases because the WTP increases for all buyers of different values. □

Proof of Proposition 8 A change in δ does not affect the equilibrium reserve price, so the equilibrium probability of sale and stationary value distribution are not affected either. Buyers' utility increases because the subsequent market becomes more buyer-friendly: directly by Eq. 19, increase in δ increases the integral. The seller's revenue decreases as each buyer's WTP decreases and the buyer composition does not change. The reduction in WTP is by Eq. 16,

$$w_*(v)=v-\mathbf{1}_{v\geq\rho_*}\cdot\int_{\rho_*}^{v}\frac{H_*^{n-1}(z)}{\left(\frac{1}{\delta\cdot s}-1\right)+H_*^{n-1}(z)}dz.$$

□

Proof of Proposition 9 There is a closed form expression of ρ_* when $F(v)=v^k$. Namely, equilibrium reserve type (and price) is

$$\rho_*=\left[(k+1)\cdot\left(1+\frac{s}{1-s}\cdot\frac{1}{n}\cdot\left(1-(k+1)^{-n}\right)\right)\right]^{-1/k}.$$

The expression is increasing in k. The probability of sale, $1 - H_*^n(\rho_*)$, has the opposite change as $H_*(\rho_*)$, and

$$H_*(\rho_*) = F(\rho_*)/[F(\rho_*) + \rho_* \cdot f(\rho_*)] = 1/(k+1).$$

Therefore, as k increases, the probability of sale increases. A value v buyer's expected utility is

$$\begin{aligned} \underline{u}_*(v) &= \mathbf{1}_{v \geq \rho_*} \cdot \int_{\rho_*}^{v} \frac{H_*^{n-1}(z)}{1 - s \cdot \delta \cdot \left(1 - H_*^{n-1}(z)\right)} dz \\ &= \mathbf{1}_{v \geq \rho_*} \cdot \int_{\rho_*}^{v} \frac{1}{(1 - s \cdot \delta) \cdot k^{n-1} + s \cdot \delta} dz. \end{aligned}$$

Since ρ_* increases in k and the integrand decreases in k, the expected utility decreases in k for all v. Sellers' revenue increases and the stationary value distribution increases first-order stochastically by the fact that the newborn value distribution increases first-order stochastically. □

Proof of Proposition 10 The difference between the competitive marginal revenues is

$$\begin{aligned} \mathrm{MR}_{\Lambda}^{\mathrm{A}}(v) - \mathrm{MR}_{\Lambda}^{\mathrm{P}}(v) &= \left[w_{\Lambda}(v) - \eta\left(F^n(v)\right) \cdot w_{\Lambda}'(v)\right] \\ &\quad - \left[w_{\Lambda}(v) - \eta\left(F^n(v)\right) \cdot w_{\Lambda}'(v)\right] \\ &= \left[\eta\left(F^n(v)\right) - \eta(F(v))\right](1 - \delta \cdot \Lambda(v)). \end{aligned}$$

The term in the square brackets is positive, so when $\tilde{\Lambda}(v) \geq \Lambda(v)$,

$$\mathrm{MR}_{\tilde{\Lambda}}^{\mathrm{A}}(v) - \mathrm{MR}_{\tilde{\Lambda}}^{\mathrm{P}}(v) \leq \mathrm{MR}_{\Lambda}^{\mathrm{A}}(v) - \mathrm{MR}_{\Lambda}^{\mathrm{P}}(v).$$

Also, when the discount factor becomes larger, the difference becomes strictly smaller as well as $\Lambda(v) > 0$. Take any critical type τ and the difference between the revenues in the presence of Λ is

$$r(\mathrm{A}(\tau)) - r(\mathrm{P}(\tau)) = \int_{\tau}^{1} \left[\mathrm{MR}_{\Lambda}^{\mathrm{A}}(v) - \mathrm{MR}_{\Lambda}^{\mathrm{P}}(v)\right] dF^n(v).$$

□

B The Monopolist's problem with asymmetric buyers

I characterize the revenue-maximizing mechanism for n buyers with possibly asymmetric value distributions. Each buyer i has independent private value drawn distribution F_i on support $\left[\underline{v}_i, \overline{v}_i\right]$ such that $(1 - F_i(v))/f_i(v)$ is weakly decreasing. The subsequent market is Λ, which is fixed throughout the section, so I do not include subscript. Let $\mathbf{v} = (v_1, \ldots, v_n)$ be the vector of values and $F(\mathbf{v}) = \prod_{i=1}^{n} F_i(v_i)$ be the value distribution.

A DSM $(P(\cdot), C(\cdot)) \equiv (P_i(\cdot), C_i(\cdot))_{i=1}^{n}$ consists of a collection of probability assignment functions and cost functions. The probability assignment functions take reports of the buyers to assign each buyer a probability of obtaining the item, with the properties that $0 \leq P_i(z_1, \ldots, z_n) \leq 1$ and $\sum_{i=1}^{n} P_i(z_1, \ldots, z_n) \leq 1$ for each buyer i. The cost function $C_i(z_1, \ldots, z_n)$ specifies the transfer from buyer i to the monopolist given all buyers' reports. Define the expected probability assignment and expected cost function as

$$\overline{P}_i(z_i) \equiv \int_{\prod_{j\neq i} z_j \in \prod_{j\neq i}\left[\underline{v}_j, \overline{v}_j\right]} P_i(z_1, \ldots, z_n) \prod_{j\neq i} dF_j\left(z_j\right),$$

$$\overline{C}_i(z_i) \equiv \int_{\prod_{j\neq i} z_j \in \prod_{j\neq i}\left[\underline{v}_j, \overline{v}_j\right]} C_i(z_1, \ldots, z_n) \prod_{j\neq i} dF_j\left(z_j\right).$$

Restrict attention to incentive-compatible (IC) mechanisms. Buyer i's expected utility of reporting z_i in the mechanism is the expected utility when he gets the object from the mechanism plus the expected utility if she does not get and waits until the next period,

$$u(z_i|v_i) = \overline{P}_i(z_i) \cdot v_i - \overline{C}_i(z_i) + \left(1 - \overline{P}_i(z_i)\right) \cdot \underline{u}(v_i).$$

Proposition 11 $(P_i(\cdot), C_i(\cdot))_{i=1}^{n}$ *is incentive-compatible if and only if for every buyer* i,

1. $\overline{P}_i(v_i)$ *is non-decreasing in* v_i, *and*
2. $\overline{C}_i(v_i) = \overline{C}_i\left(\underline{v}_i\right) - \overline{P}_i\left(\underline{v}_i\right) \cdot w\left(\underline{v}_i\right) + \overline{P}_i(v_i) \cdot w(v_i) - \int_{\underline{v}_i}^{v_i} \overline{P}_i(x)\, dw(x)$ *for all* $v_i \in \left[\underline{v}_i, \overline{v}_i\right]$.

Proof of Proposition 11 First I show that if the mechanism is IC, then both conditions 1 and 2 hold. Define the benefit of lying,

$$\psi_i(z|v_i) \equiv u_i(z_i|v_i) - u_i(v_i|v_i).$$

IC implies that for any v_i, there is no benefit of lying, so $\psi_i(z_i|v_i) \leq 0$ for all z_i, v_i, so

$$\begin{aligned}
0 &\geq \psi_i(z_i|v_i) + \psi_i(v_i|z_i) \\
&= \left[\overline{P}_i(z_i) \cdot w(v_i) - \overline{C}_i(z_i) + (v_i - w(v_i)) \right.\\
&\quad \left. - \left(\overline{P}_i(v_i) \cdot w(v_i) - \overline{C}_i(v_i) + (v_i - w(v_i))\right)\right] \\
&\quad - \left[\overline{P}_i(v_i) \cdot w(z_i) - \overline{C}_i(v_i) + (z_i - w(z_i)) \right.\\
&\quad \left. - \left(\overline{P}_i(z_i) \cdot w(z_i) - \overline{C}_i(z_i) + (z_i - w(z_i))\right)\right] \\
&= \left[\overline{P}_i(z_i) \cdot w(v_i) - \overline{C}_i(z_i) - \left(\overline{P}_i(v_i) \cdot w(v_i) - \overline{C}_i(v_i)\right)\right] \\
&\quad - \left[\overline{P}_i(v_i) \cdot w(z_i) - \overline{C}_i(v_i) - \left(\overline{P}_i(z_i) \cdot w(z_i) - \overline{C}_i(z_i)\right)\right] \\
&= \left(\overline{P}_i(z_i) - \overline{P}_i(v_i)\right) \cdot (w(v_i) - w(z_i)).
\end{aligned}$$

Since $w(v_i)$ is non-decreasing in v_i, $\overline{P}_i(v_i)$ is non-decreasing in v_i. Furthermore, IC implies that $u_i(z_i|v_i)$ is maximized at $z_i = v_i$, then by envelope theorem,

$$\left.\frac{\partial u_i(z_i|v_i)}{\partial z_i}\right|_{z_i=v_i} = \overline{P}_i'(v_i) \cdot w(v_i) - \overline{C}_i'(v_i) = 0.$$

By fundamental theorem of calculus,

$$\overline{C}_i(v_i) = \overline{C}_i\left(\underline{v}_i\right) + \int_{\underline{v}_i}^{v_i} \overline{P}_i'(v_i) \cdot w(v_i)\, dv_i.$$

Integration by parts yields condition 2 as desired. The converse is shown directly by the definition of incentive-compatibility:

$$\begin{aligned}
\psi_i(z_i|v_i) &= \left[\overline{P}_i(z_i) \cdot w(v_i) - \overline{C}_i(z_i) + (v_i - w(v_i))\right] \\
&\quad - \left[\overline{P}_i(v_i) \cdot w(v_i) - \overline{C}_i(v_i) + (v_i - w(v_i))\right] \\
&= \left[\overline{P}_i(z_i) - \overline{P}_i(v_i)\right] \cdot w(v_i) + \overline{C}_i(v_i) - \overline{C}_i(z_i).
\end{aligned}$$

Plugging in condition 2,

$$\begin{aligned}
\psi_i(z_i|v_i) &= \left[\overline{P}_i(z_i) - \overline{P}_i(v_i)\right] \cdot w(v_i) + \overline{P}_i(v_i) \cdot w(v_i) \\
&\quad - \int_{z_i}^{v_i} \overline{P}_i(x)\, dw(x) - \overline{P}_i(z_i) \cdot w(z_i) \\
&= \overline{P}_i(z_i) \cdot [w(v_i) - w(z_i)] - \int_{z_i}^{v_i} \overline{P}_i(x)\, dw(x) \\
&= \int_{z_i}^{v_i} \left(\overline{P}_i(z_i) - \overline{P}_i(x)\right) dw(x).
\end{aligned}$$

Then by condition 1, the expression is nonpositive. □

Because the expected probability assignment function pins down the expected cost function, if the expected cost function of the buyer of the lowest value is the same and the expected probability assignment is the same, then the expected revenue is the same for the seller, and all buyers are indifferent between the incentive-compatible mechanisms the seller runs.

Corollary 1 (Buyer indifference and revenue equivalence) *If two incentive-compatible mechanisms have (i) the same expected probability assignment functions, and (ii) the same expected costs for the buyer of the lowest value, then all the agents are indifferent between the two mechanisms, as they yield the same expected revenue, and the same expected payoffs for the buyers of the same value.*

Proof of Corollary 1 Let $M^I = \left(\overline{P}_i^I(\cdot), \overline{C}_i^I(\cdot)\right)_{i=1}^n$ and $M^{II} = \left(\overline{P}_i^{II}(\cdot), \overline{C}_i^{II}(\cdot)\right)_{i=1}^n$ denote two mechanisms. In any IC mechanism $M = \left(\overline{P}_i(\cdot), \overline{C}_i(\cdot)\right)_{i=1}^n$, the expected cost function is

$$\overline{C}_i(v_i) = \overline{C}_i\left(\underline{v}_i\right) - \overline{P}_i\left(\underline{v}_i\right) \cdot \tilde{v}\left(\underline{v}_i\right) + \overline{P}_i(v_i) \cdot w(v_i) - \int_{\underline{v}_i}^{v_i} \overline{P}_i(x)\, dw(x).$$

Because $\overline{P}_i^I(\cdot) = \overline{P}_i^{II}(\cdot)$ for all i by (i) and $\overline{C}_i^I\left(\underline{v}_i\right) = \overline{C}_i^{II}\left(\underline{v}_i\right)$ by (ii), $\overline{C}_i^I(v_i) = \overline{C}_i^{II}(v_i)$ for all v_i for all i. Expected utility of v_i in mechanism M is

$$u_i^M(v_i|v_i) = \overline{P}_i(v_i) \cdot w(v_i) - \overline{C}_i(v_i) + v_i - w(v_i).$$

$u_i^{M^I}(v_i|v_i) = u_i^{M^{II}}(v_i|v_i)$ for all v_i for i. Revenue is determined by

$$r(M) = \sum_{i=1}^n \int_{\underline{v}}^{\overline{v}} \overline{C}^M(v_i)\, dF_i(v_i).$$

Then as shown that $\overline{C}_i^I(v_i) = \overline{C}_i^{II}(v_i)\ \forall v_i\ \forall i$, $r\left(M^I\right) = r\left(M^{II}\right)$. □

The seller maximizes expected revenue subject to the incentive compatibility and individual rationality constraints.

Definition 3 A revenue-maximizing IC, IR DSM is a mechanism $M = (P_i(\cdot), C_i(\cdot))_{i=1}^n$ that maximizes $r = \sum_{i=1}^n \int_{\underline{v}_i}^{\overline{v}_i} \overline{C}_i(v_i)\, dF_i(v_i)$ subject to $\forall i$: $\tilde{u}_\Lambda(v_i|v_i) \geq \tilde{u}_\Lambda(z_i|v_i)\ \forall z_i \neq v_i$ and $\tilde{u}_\Lambda(v_i|v_i) \geq 0\ \forall v_i$.

Proposition 12 *The revenue-maximizing IC, IR DSM* $(P_i^*(\cdot), C_i^*(\cdot))_{i=1}^n$ *is that* $P_i^*(\mathbf{v}) = 1$ *if* $\tilde{v}(v_i) - \frac{1-F_i(v_i)}{f_i(v_i)} \cdot w'(v_i) > \max\left\{\tilde{v}(v) - \frac{1-F_j(v_j)}{F_j(v_j)} \cdot w'(v_i), 0\right\}$ *and* $P_i^*(\mathbf{v}) = 0$ *otherwise, and*

$$\overline{C}_i^*(v_i) = \overline{C}_i^*\left(\underline{v}_i\right) - \overline{P}_i^*\left(\underline{v}_i\right) \cdot \tilde{v}\left(\underline{v}_i\right) + \overline{P}_i^*(v_i) \cdot \tilde{v}(v_i) - \int_{\underline{v}_i}^{v_i} \overline{P}_i^*(z)\, d\tilde{v}(z).$$

The three key steps in the proof are (i) characterizing the individual rationality constraint, (ii) exchanging integrals, and (iii) expanding expected probability assignment functions to obtain a weighted average of probability assignment functions.

Proof of Proposition 12 The constraints are

1. IC1: $\overline{P}_i(v_i) \geq \overline{P}_i(z_i)\ \forall v_i \geq z_i$,
2. IC2: $\overline{C}_i(v_i) = \overline{C}_i(\underline{v}_i) + \int_{\underline{v}_i}^{v_i} \overline{P}_i'(v_i) \cdot w(v_i)\, dv_i\ \forall v_i$, and
3. IR: $u_i(v_i|v_i) \geq \underline{u}_i(v_i)$.

In particular, The IR implies that $\overline{C}_i(\underline{v}_i) - \overline{P}_i(\underline{v}_i)\, w(\underline{v}_i) \leq 0$.

$$
\begin{aligned}
r =& \sum_{i=1}^{n} \int_{\underline{v}_i}^{\overline{v}_i} \left[\overline{P}_i(v_i) \cdot w(v_i) - \int_{\underline{v}_i}^{v_i} \overline{P}_i(z)\, dw(z) \right] dF_i(v_i) \\
&+ \sum_{i=1}^{n} \left[\overline{C}_i(\underline{v}_i) - \overline{P}_i(\underline{v}_i) \cdot w(\underline{v}_i) \right],
\end{aligned}
$$

where

$$
\begin{aligned}
&\int_{\underline{v}_i}^{\overline{v}_i} \left[\overline{P}_i(v_i) \cdot w(v_i) - \int_{\underline{v}_i}^{v_i} \overline{P}_i(x)\, dw(x) \right] dF_i(v_i) \\
&= \int_{\underline{v}_i}^{\overline{v}_i} \left[\overline{P}_i(v_i) \cdot w(v_i) \cdot f_i(v_i) \right] dv_i \\
&\quad - \int_{\underline{v}_i}^{\overline{v}_i} \left[\int_{x}^{\overline{v}_i} \left[\overline{P}_i(x) \cdot w'(x) \cdot f_i(v_i) \right] dv_i \right] dx \\
&= \int_{\underline{v}_i}^{\overline{v}_i} \left[\overline{P}_i(v_i) \cdot w(v_i) \cdot f_i(v_i) \right] dv_i - \int_{\underline{v}_i}^{\overline{v}_i} \left[\overline{P}_i(x)\ cdot w'(x) \cdot (1 - F_i(x)) \right] dx \\
&= \int_{\underline{v}_i}^{\overline{v}_i} \left[\overline{P}_i(v_i) \cdot \left[w(v_i) - \frac{1 - F_i(v_i)}{f_i(v_i)} \cdot w'(v_i) \right] \cdot f_i(v_i) \right] dv_i \\
&= \int_{\underline{v}_1}^{\overline{v}_1} \cdots \int_{\underline{v}_n}^{\overline{v}_n} \left[P_i(\mathbf{v}) \cdot \left[w(v_i) - \frac{1 - F_i(v_i)}{f_i(v_i)} \cdot w'(v_i) \right] \right] dF(\mathbf{v}).
\end{aligned}
$$

Therefore,

$$
r \leq \int_{\underline{v}_1}^{\overline{v}_1} \cdots \int_{\underline{v}_n}^{\overline{v}_n} \left[\sum_{i=1}^{n} P_i(\mathbf{v}) \left[w(v_i) - \frac{1 - F_i(v_i)}{f_i(v_i)} w'(v_i) \right] \right] dF(\mathbf{v}),
$$

and any mechanism that implements the specified probability assignment function will achieve the upper bound. $\square$

C Omitted details in Sect. 6

C.1 Optimal posted price

In this section, I first study determination of the optimal posted price, and then draw parallels with determination of the optimal reserve price to compare the revenues between an auction and a posted price in the presence of subsequent purchasing opportunities for buyers.

Let's consider the posted price mechanism and its relation to the reserve-price auction. A posted price $\mathrm{P}(\phi)$ is the mechanism whereby a seller posts price $w_\Lambda(\phi)$ and buyers who are willing to buy at the price enter the lottery to be picked as a winner with equal chances. The posted price mechanism is an incentive-compatible direct-revelation mechanism: Only buyers with willingness to pay $w_\Lambda(\phi)$—or equivalently, values above ϕ—enter the lottery.[11]

All buyers with value v above the posted type ϕ have the same probability of winning and the same expected payment, so their expected utility is

$$u_\Lambda(v|\mathrm{P}(\phi)) = \frac{1}{n} \cdot \frac{1 - F^n(\phi)}{1 - F(\phi)} \cdot [v - w_\Lambda(\phi)],$$

and the seller's expected revenue is

$$r(\mathrm{P}(\phi)) = \left(1 - F^n(\phi)\right) \cdot w_\Lambda(\phi).$$

Define the posted price marginal revenue in the presence of subsequent market Λ to be

$$\mathrm{MR}^{\mathrm{P}}_\Lambda(v) = w_\Lambda(v) - \eta\left(F^n(v)\right) \cdot w'_\Lambda(v). \tag{31}$$

Then the revenue-maximizing posted price mechanism in the presence of subsequent market Λ is $\mathrm{P}^* = \mathrm{P}(\phi^*)$, where ϕ^* is the unique solution to $\mathrm{MR}^{\mathrm{P}}_\Lambda(\phi^*) = 0$.

The posted price and the reserve-price auction have many similarities. Both the reserve type and posted type are determined by equating the marginal revenue to zero, and the types calculated specify the set of willing participants. Together, I call the reserve-price auction and the posted price critical type mechanisms.

Definition 4 A critical type mechanism (CTM) $M(\tau)$ is either a reserve-price auction $\mathrm{A}(\rho)$ or a posted price $\mathrm{P}(\phi)$. Its revenue is

$$r(M(\tau)) = \int_\tau^1 \mathrm{MR}^M_\Lambda(v)\, dF^n(v),$$

[11] The lottery seems unrealistic, but it is equivalent to the following natural selling process in which buyers arrive stochastically. The seller posts price $w_\Lambda(\phi)$ and n buyers uniformly randomly arrive within a time interval. The first buyer with a willingness to pay greater than $w_\Lambda(\phi)$ takes the item and pays the posted price.

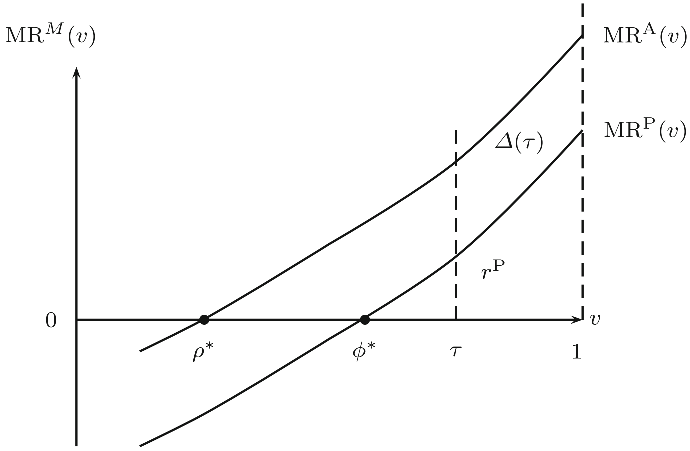

Fig. 1 Comparison of optimal auction and optimal price

and the optimal CTM is $M(\tau^*)$, where $\mathrm{MR}_\Lambda^M(\tau^*) = 0$. The probability of sale is $1 - F^n(\tau)$.

$\mathrm{MR}^{\mathrm{A}}(v) \geq \mathrm{MR}^{\mathrm{P}}(v)$ for all v, because $\eta(F^n(v)) > \eta(F(v))$.[12] Therefore, the auction MR curve always dominates the posted price MR curve and the optimal posted type ϕ_Λ^* is always greater than the optimal reserve type ρ_Λ^*. The difference in revenues between the auction and the posted price of the same critical type is positive, and I call it the auction premium, $\Delta(\tau) \equiv r(\mathrm{A}(\tau)) - r(\mathrm{P}(\tau))$. Figure 1 illustrates the similarities between the two mechanisms in optimal price determination and revenue determination. Whenever the subsequent market becomes more buyer-friendly, the revenue difference between the optimal auction and the posted price decreases.[13]

It is also worth mentioning that the change in the revenue difference between the optimal auction and the optimal price with respect to the number of buyers is nonmonotonic. When there is only one buyer, the seller essentially faces a bargaining problem: A buyer either accepts or rejects the price the seller proposes, so the optimal auction and the optimal posted price yield the same revenue. When there are several buyers, the auction offers the extra benefit of buyer competition that drives up the transaction price. However, when the number of buyers approaches infinity, the two revenues are again equal. There is a high probability that a buyer has an arbitrarily high value, so a high revenue is guaranteed by posting a high price. What the subsequent market limits is the maximum revenue any mechanism obtains, which is the willingness to pay of the highest value buyer, $w_\Lambda(1)$.

Figure 2 shows the ratio of the two revenues with respect to different numbers of buyers, when the buyers' values are drawn from uniform distribution ($r\left(\mathrm{P}_\Lambda^*\right)/r\left(\mathrm{A}_\Lambda^*\right)$,

[12] The expression $\eta\left(F^n(v)\right) = \eta(F(v))\left(1 + F(v) + \cdots + F^{n-1}(v)\right)/\left(n \cdot F^{n-1}(v)\right)$ is greater than $\eta(F(v))$ and is decreasing when F has decreasing inverse hazard rate.

[13] Relatedly, Kultti (1999), realizing the disadvantages of an auction facing subsequent competition, shows that in the limit when the market is infinitely competitive, an auction and a posted price are equivalent, generating the same revenue.

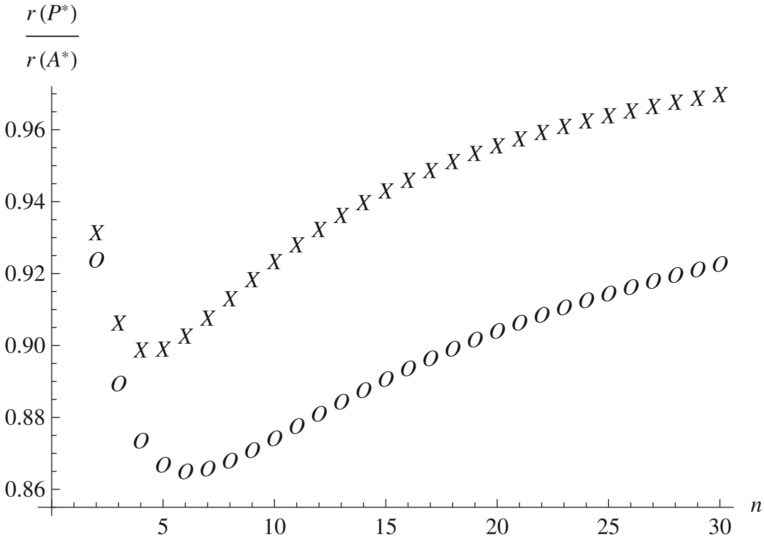

Fig. 2 Ratio of optimal auction revenue to optimal price revenue

$F(v) = v$). “O” shows the ratio in a pure monopoly setting, and “X” shows the ratio when the subsequent market is $\Lambda(x) = x$—i.e., each buyer expects to receive a price offer randomly drawn from the uniform distribution. As the plot illustrates, the percentage of expected revenue the optimal posted price attains with respect to the optimal auction increases when there is a subsequent market. For example, when there are five buyers, the seller can get 86% of the optimal revenue from a posted price as a monopoly, but almost 90% of the optimal revenue when a subsequent market is present. The absolute revenue difference exhibits a similar pattern: As the market gets more buyer-friendly, the optimal auction premium is likely to decrease.

C.2 Definition and characterization of equilibrium

A SSSE $(M_*(\cdot), \sigma_*(\cdot,\cdot), H_*(\cdot), \mu_*(\cdot))$ in this setting only differs from Definition 1 by that each cost c seller chooses a possibly different mechanism $M_*(c)$. In equilibrium, there is a cutoff cost c_* such that any seller with cost $c > c_*$ uses posts price mechanism $\mathrm{P}(\phi_*)$ and any other with cost $c < c_*$ runs a reserve-price auction $\mathrm{A}(\rho_*)$. Cost c_* seller is indifferent between the two mechanisms because he yields the same profit,

$$\int_{\rho_*}^{1} \mathrm{MR}_*^{\mathrm{A}}(v)\, dH_*^n(v) - c_* = \int_{\phi_*}^{1} \mathrm{MR}_*^{\mathrm{P}}(v)\, dH_*^n(v).$$

The buyers with the highest values above exit slower than in the previous setting because all the participating buyers have the same probability of winning in a posted price. Furthermore, a posted price makes the sale probability smaller, resulting in more lower value buyers crowding the market, bringing down the stationary value distribution. In particular, such an equilibrium exists and is unique under the same assumptions as in the previous section. The derivations and characterizations of the SSSE are as below.

In the equilibrium, for a cost c seller facing n symmetric buyers whose WTP is determined by $w_*(\cdot)$, the profit-maximizing auction is $\mathrm{A}(\rho_*)$ where ρ_* is determined by

$$\rho_* - \frac{1 - H_*(\rho_*)}{h_*(\rho_*)} = 0$$

and the expected profit from it is $\pi(\mathrm{A}(\rho_*)) = r(\mathrm{A}(\rho_*)) - c$, where

$$r(\mathrm{A}(\rho_*)) = \int_{\rho_*}^{1} \left[w_*(v) - \frac{1 - H_*(v)}{h_*(v)} \cdot w_*'(v) \right] dH_*^n(v).$$

The profit-maximizing posted price is $\mathrm{P}(\phi_*)$ where ϕ_* is determined by

$$w_*(\phi_*) - \frac{1 - H_*^n(\phi_*)}{\left(H_*^n(\phi_*)\right)'} \cdot w_*'(\phi_*) = 0$$

and the expected profit and revenue from it is

$$\pi(\mathrm{P}(\phi_*)) = r(\mathrm{P}(\phi_*)) = \left(1 - H_*^n(\phi_*)\right) w_*(\phi_*).$$

Since a seller can only run an auction or post a price, he essentially makes a choice between $\mathrm{A}(\rho_*)$ and $\mathrm{P}(\phi_*)$, and he will choose the auction if and only if the auction generates higher expected profit, or equivalent, the equilibrium auction premium is greater than the cost,

$$\pi(\mathrm{A}(\rho_*)) \geq \pi(\mathrm{P}(\phi_*)) \Leftrightarrow \Delta_* = r(\mathrm{A}(\rho_*)) - r(\mathrm{P}(\phi_*)) \geq c,$$

and posts the optimal price otherwise. The cost $c_* \equiv \Delta_*$ seller is indifferent between the two mechanisms, and I call c_* the equilibrium cutoff cost such that measure $p_* = G(c_*)$ sellers with cost lower than c_* runs $\mathrm{A}(\rho_*)$ and measure $1 - p_*$ of the sellers with cost higher than c_* chooses $\mathrm{P}(\phi_*)$.

The total discounted payoff is the expected utilities from participation in auctions and posted prices,

$$\underline{u}_*(v) = \mathbf{1}_{v \geq \rho_*} \cdot G(c_*) \cdot u(v|\mathrm{A}(\rho_*)) + \mathbf{1}_{v \geq \phi_*} \cdot (1 - G(c_*)) \cdot u(v|\mathrm{P}(\phi_*)),$$

where the payoffs in the auction and the posted price are respectively,

$$u_*(v|\mathrm{A}(\rho_*)) = \mathbf{1}_{v \geq \rho_*} \cdot \left[\int_{\rho_*}^{v} H_*^{n-1}(z)\, dw(z) + \delta \cdot s \cdot \underline{u}(v) \right],$$

and

$$\begin{aligned}&u_*\left(v|\mathrm{P}\left(\phi_*\right)\right)\\&=\mathbf{1}_{v\geq\phi_*}\cdot\left[\frac{1}{n}\cdot\frac{1-H_*^n\left(\phi_*\right)}{1-H_*\left(\phi_*\right)}\cdot\left(v-w\left(\phi_*\right)\right)+\left(1-\frac{1}{n}\cdot\frac{1-H_*^n\left(\phi_*\right)}{1-H_*\left(\phi_*\right)}\right)\cdot\delta\cdot s\cdot\underline{u}\left(v\right)\right]\\&=\mathbf{1}_{v\geq\phi_*}\left[\frac{1}{n}\cdot\frac{1-H_*^n\left(\phi_*\right)}{1-H_*\left(\phi_*\right)}\cdot\left(w\left(v\right)-w\left(\rho\right)\right)+\delta\cdot s\cdot\underline{u}\left(v\right)\right],\end{aligned}$$

respectively. For value $v\in\left(\rho_*,\phi_*\right)$, the derivation is similar to the previous section and only differs by the extra $G\left(c_*\right)$ term.

$$u_*\left(v|\mathrm{A}\left(\rho_*\right)\right)=\frac{1}{1-G\left(c_*\right)\cdot\delta\cdot s}\int_{\rho_*}^{v}H_*^{n-1}\left(z\right)dw\left(z\right)$$

and

$$w_*'\left(v\right)=\frac{1-s\cdot\delta\cdot G\left(c_*\right)}{1-s\cdot\delta\cdot G\left(c_*\right)+s\cdot\delta\cdot G\left(c_*\right)\cdot H_*^{n-1}\left(v\right)}.$$

For $v\geq\phi_*$, the calculation is more convoluted,

$$\left(1-\delta\cdot s\right)\cdot\underline{u}\left(v\right)=\int_{\rho_*}^{v}H_*^{n-1}\left(z\right)dw_*\left(z\right)+\frac{1}{n}\cdot\frac{1-H_*^n\left(\phi_*\right)}{1-H_*\left(\phi_*\right)}\cdot\left(w_*\left(v\right)-w_*\left(\phi_*\right)\right).$$

In summary, an equilibrium $\left(M_*\left(\cdot\right),\sigma_*\left(\cdot,\cdot\right),H_*\left(\cdot\right),\mu_*\left(\cdot\right)\right)$ is characterized by c_*, the cutoff cost, ϕ_*, the optimal posted type, ρ_*, the optimal reserve type, and the stationary value distribution H_*, where

1. Mass p_* of sellers with cost $c\leq c_*$ run the same optimal reserve-price auction $\mathrm{A}\left(\rho_*\right)$ and the other mass $1-p_*$ of sellers with cost $c>c_*$ uses the same optimal posted price mechanism $\mathrm{P}\left(\phi_*\right)$, where ρ_* and ϕ_* are determined by $\mathrm{MR}_*^{\mathrm{A}}\left(\rho_*\right)=\mathrm{MR}_*^{\mathrm{P}}\left(\phi_*\right)=0$.
2. Value v buyer's equilibrium WTP is

$$\begin{aligned}&w_*\left(v\right)\\&=v-\frac{\delta\cdot s\cdot\left[\mathbf{1}_{v\geq\rho_*}\cdot p_*\cdot u\left(v|\mathrm{A}\left(\rho_*\right)\right)+\mathbf{1}_{v\geq\phi_*}\cdot\left(1-p_*\right)\cdot u\left(v|\mathrm{P}\left(\phi_*\right)\right)\right]}{1-\delta\cdot s\cdot\left[1-\mathbf{1}_{v\geq\rho_*}\cdot p_*\cdot H_*^{n-1}\left(v\right)-\mathbf{1}_{v\geq\phi_*}\cdot\left(1-p_*\right)\cdot\frac{1}{n}\cdot\frac{1-H_*^n\left(\phi_*\right)}{1-H_*\left(\phi_*\right)}\right]}.\end{aligned}$$

3. Stationary value distributions h_* and H_* are characterized by Eqs. 3 and 4 with the expected losing probability function

$$l_*\left(v\right)=\begin{cases}1 & v\leq\rho_*\\ p_*\cdot\left(1-H_*^{n-1}\left(v\right)\right) & v\in\left(\rho_*,\phi_*\right]\\ p_*\cdot\left(1-H_*^{n-1}\left(v\right)\right)+\left(1-p_*\right)\cdot\left(1-\frac{1}{n}\cdot\frac{1-H_*^n\left(\phi_*\right)}{1-H_*\left(\phi_*\right)}\right) & v\in\left(\phi_*,1\right].\end{cases}$$

4. The equilibrium belief μ_* is

$$\mu_* \left(H_* (\cdot), (p_* \circ \mathrm{A}(\rho_*)), (1-p_*) \circ \mathrm{P}(\phi_*)), \sigma_* (\cdot, \cdot)\right) = 1.$$

Proposition 13 *Suppose Assumption 4 holds. When sellers have heterogeneous auction costs, as posted prices are introduced, allocative inefficiency increases but the change in sale efficiency is ambiguous.*

Proof of Proposition 13 Similar to the previous model, the equilibrium is

$$\begin{aligned}
H_* (\rho_*) &= \left[1 + \frac{s}{1-s} \cdot \frac{1}{n} \cdot \left[\left(1 - H_*^n (\rho_*)\right) \cdot G (c_*)\right.\right.\\
&\quad \left.\left. + \left(1 - H_*^n (\phi_*)\right) \cdot (1 - G (c_*))\right]\right] \cdot F (\rho_*)\\
&= \left[1 + \frac{s}{1-s} \cdot \frac{1}{n} \cdot \left(1 - H_*^n (\rho_*)\right) + \frac{s}{1-s} \cdot \frac{1}{n} \cdot \left(H_*^n (\rho_*) - H_*^n (\phi_*)\right)\right.\\
&\quad \left. \cdot (1 - G (c_*))\right] \cdot F (\rho_*)\\
&\equiv \left[1 + \frac{s}{1-s} \cdot \frac{1}{n} \cdot \left(1 - H_*^n (\rho_*)\right) - \epsilon (\rho_*)\right] \cdot F (\rho_*),
\end{aligned}$$

where $\epsilon (\rho_*)$ is strictly positive. As before, substitute in the equilibrium condition

$$\begin{aligned}
H_* (\rho_*) &= F (\rho_*) / (F (\rho_*) + \rho_* \cdot f (\rho_*)),\\
\frac{1}{F (\rho_*) + \rho_* \cdot f (\rho_*)} &= 1 + \frac{s}{1-s} \cdot \frac{1}{n} \cdot \left(1 - \left(\frac{F (\rho_*)}{F (\rho_*) + \rho_* \cdot f (\rho_*)}\right)^n\right)\\
&\quad - \epsilon (\rho_*). \qquad (32)
\end{aligned}$$

Compared with Eq. 27, the first two terms of RHS are increasing, but with the last term, the curve shifts down and intersects the LHS at a bigger ρ_* than before. However, bigger ρ_* means smaller $H_* (\rho_*)$ and bigger $1 - H_*^n (\rho_*)$. Therefore, sale probability and efficiency increases for the auctions, but it increases when there are sellers switching to posted prices which have sale efficiency $1 - H_*^n (\phi_*)$, so the total effect is ambiguous. However, more posted price mechanisms bring more allocative inefficiency. □

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