Heterophily, Stable Matching, and Intergenerational Transmission in Cultural Evolution

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Abstract

We demonstrate that marital preferences, the marriage market, and intergenerational transmission mechanisms must be jointly considered to gain a more complete picture of cultural evolution. We characterize different cultural processes in settings with different combinations of (i) homophilic and heterophilic marital preferences, (ii) men-optimal or women-optimal stable matching scheme, and (iii) familial, societal, and rational forces of intergenerational transmission. First, with perfect vertical transmission in homogamies and oblique transmission in heterogamies, the presence of even a small fraction of heterophilic proposers leads to complete cultural homogeneity; cultural heterogeneity arises only when proposers are all homophilic. Notably, a stable matching scheme that is optimal for a gender in the short run can lead to suboptimal outcomes for them in the long run. Second, when transmission in heterogamies takes the form of an imitative process, persistent or temporary cycles between cultural homogeneity and heterogeneity may arise. Third, with imperfect vertical transmission in homogamies, heterophilic preferences and heterogamies play a significant role in the determination of cultural distribution; cultural substitutability is neither sufficient nor necessary for cultural heterogeneity. Finally, we discuss our model's implications for matriarchal and patriarchal societies, the evolution of gender roles as well as cultural assimilation and identity formation of minorities and immigrants.

Keywords: cultural evolution, marital preferences, stable matching, intergenerational cultural transmission, imitative dynamics, evolutionary game theory

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1 Introduction

Culture plays a crucial role in many economic choices and outcomes on both the individual and national level (Landes, 1998; Guiso et al., 2006; Fernández, 2008, 2011; Alesina and Giuliano, 2015). A growing literature seeks to better understand the mechanisms that drive the evolution of culture across generations (Bisin and Verdier, 2011). Family is the primary environment in which children are socialized, and parents are the main agents through which cultural traits are transmitted from one generation to another. Hence, taking into account how families are formed (i.e., how parents are matched)—in addition to how families transmit their preferences—is of prime importance for our understanding of cultural evolution. Whereas most of the literature sidesteps this issue or assumes a specific matching, we propose a cultural transmission model with heterogeneous marital preferences and equilibrium matching.

This paper considers the evolution of cultural traits under stable matching with heterogeneous marital preferences (Gale and Shapley, 1962).[1] The primitives of the matching model are a set of women, a set of men, and a preference ordering of each woman (man) over men (women). In our model, these elements are determined in the following way. At the beginning of each discrete period, a mass of women and a mass of men become adult. Each adult wants to match with an adult of the opposite sex to form a family. Each participant in the matching market has a cultural trait, acquired during childhood, and preferences over the cultural trait of their partner. In addition to the usually considered *homophilic* preferences (individuals prefer partners of the same trait the most), we consider the possibility that the preferences of some men and women are *heterophilic*: Individuals prefer partners of a different cultural trait. Stable matching depends on the distribution of cultural traits and the distribution of preferences in both populations such that no positive mass of individuals of opposite sexes would both rather have each other than their current mates. As is well known, in such a matching market, stable matching exists but is not necessarily unique. In the case of multiplicity, one matching is men-optimal stable matching (MOSM) and another women-optimal stable matching (WOSM), with MOSM and WOSM being the stable matching the most preferred by men and women, respectively. Gale and Shapley (1962) propose a procedure—the so-called deferred acceptance (DA) algorithm—that attains either MOSM or WOSM, and Baïou and Balinski (2002) generalize it to the setting with a continuum of agents. In this procedure, individuals on one side of the market are proposers and those on the other side are receivers. The DA algorithm returns MOSM when men are proposers, and returns WOSM when women are proposers. Hence, for a joint distribution of cultural traits and preferences, in the case of multiplicity, stable matching depends on which side of the market plays the role of proposers.

Once matched, the two spouses have two children, one son and one daughter. During childhood they acquire the cultural trait they will retain when they become adults. Through the cultural transmission process, stable matching in one period will determine the joint distribution of cultural traits among populations of both men and women in the next period. Since homogamies (in which both spouses have the

[1]As discussed by Pollak (2019), the Gale and Shapley (1962) matching model is the appropriate framework for analyzing marriage market equilibrium under the assumption that bargaining in marriage determines allocation within marriage; ample empirical evidence supports this assumption. The Gale-Shapley model is especially appropriate and also essential when agents may have heterogeneous preferences—e.g., preferences other than homophily—as considered in the paper.

same cultural trait) have a well-defined cultural model to transmit, they have a more efficient socialization technology than mixed families. We consider two transmission technologies in homogamies. As a benchmark, we assume *perfect vertical transmission*: If a man and a woman with the same trait are married, they transmit this trait to their children with probability one. Then, we consider *imperfect vertical transmission* with *cultural substitutability* (Bisin and Verdier, 2000, 2001, 2011): Parents from a homogamous family transmit their culture with a probability that is decreasing in the proportion of that trait in the population. Since children from mixed families do not have a well-defined familial model to follow, we assume they are socialized by the society at large. Here again, we consider two transmission technologies. As a benchmark, and as is standard in the cultural transmission literature, we assume *oblique transmission*: A child from a mixed family adopts the trait of a randomly chosen adult role model with probability one. Then, we consider *imitative logit transmission* (Weibull, 1995; Björnerstedt and Weibull, 1996; Sandholm, 2010): A child adopts the trait of a randomly chosen role model with a probability that depends on (i) the distribution of traits in the populations, (ii) the comparison of the expected utilities associated with different traits, and (iii) noise. When the noise level approaches infinity, it becomes oblique transmission; when the noise level is zero, the transmission can be considered as purely guided by rational choices, and we call it *Darwinian transmission.*

We analyze how the interactions between the determinants of stable matching (the distribution of marital preferences and the side of the market favored by the matching procedure) and the characteristics of transmission (perfect or imperfect vertical transmission in homogamies and oblique, imitative logit, or Darwinian transmission in heterogamies) influence cultural evolution. To those extents, we provide a unified and generalizable model to consider different forms of matching and intergenerational transmission in the evolution of cultural traits. We can characterize the conditions under which cultural heterogeneity is sustainable in the long run and compare and contrast with previous results (Cavalli-Sforza and Feldman, 1981; Boyd and Richerson, 1985; Bisin and Verdier, 2000, 2001; Wu and Zhang, 2021).

Table 1 summarizes the different cultural evolutionary outcomes under the different combinations of preferences of proposers and receivers (homophilic only, heterophilic only, or mixed) and intergenerational transmission of homogamies (perfect or imperfect vertical) and heterogamies (oblique, imitative logit, or Darwinian).

We show that when all proposers are homophilic, the cultural distribution converges to a steady state characterized by cultural diversity; this holds regardless of the distribution of preferences among receivers and the cultural transmission technology we envisage. In contrast, when all proposers have heterophilic preferences, the cultural distribution converges to a steady state characterized by cultural homogeneity; this also holds regardless of the distribution of preferences among receivers and the cultural transmission technology (except Darwinian transmission). According to these results, marital preferences are crucial determinants for the long-run evolution of culture. Moreover, in case of multiple stable matchings—which arise when some men and women have antagonistic preferences—the matching institution, which determines the side of the market that will play the role of proposers, also crucially matters. However, we show that even if, from a static point of view, MOSM is the matching men prefer, this is not necessarily true from a dynamic point of view: The cultural evolution driven under MOSM might lead to a path considered to

Table 1: Stable steady states under different types of transmission and preference distributions

Section Prop	Transmission in homogamies	Transmission in heterogamies	Preference distributions Proposers'	Preference distributions Receivers'	Stable set or stable steady states	Cultural category
§3.1.1 Prop 1	perfect vertical	oblique	homophilic	any	$\{(r, r) : r \in (0, 1)\}$	heterogeneous
§3.1 & §3.2 Prop 2–4	perfect vertical	oblique	nonhomophilic	any	$(0, 0)$ or $(1, 1)$	homogeneous
§4.1 Prop 5	perfect vertical	imitative logit	homophilic	any	$\{(r, r) : r \in (0, 1)\}$	heterogeneous
§4.2 Prop 6	perfect vertical	Darwinian	heterophilic	any	$(0, p^0 + q^0)$ or $(1, p^0 + q^0 - 1)$	one-sided homogeneous
§4.2 Prop 6	perfect vertical	imitative logit	heterophilic	any	$(0, 0)$ or $(1, 1)$	homogeneous
§4.3 Prop 7	perfect vertical	Darwinian	mixed	homophilic	$\emptyset$	cycles
§4.3 Prop 8–9	perfect vertical	imitative logit	mixed	homophilic	$(0, 0)$ or $(1, 1)$	homogeneous
§4.3 Prop 8–9	perfect vertical	imitative logit	mixed	homophilic	$(0, 0)$ or $(1, 1)$ or $(1/2, 1/2)$	homogeneous or heterogeneous
§5.1 Prop 10	imperfect vertical	oblique	homophilic	any	$(1/2, 1/2)$	symmetrically heterogeneous
§5.2 Prop 11	imperfect vertical	oblique	heterophilic	any	$(0, 0)$ or $(1, 1)$	homogeneous
§5.3 Prop 12	imperfect vertical	oblique	mixed	heterophilic	$(0, 0)$ or $(1, 1)$	homogeneous
§5.3 Prop 12	imperfect vertical	oblique	sufficiently homophilic	nonheterophilic	$(1/2, 1/2)$	symmetrically heterogeneous
§5.3 Prop 12	imperfect vertical	oblique	sufficiently heterophilic	nonheterophilic	(r, r) or $(1 - r, 1 - r)$ $0 < r < 1/2$	asymmetrically heterogeneous

Note. A preference distribution is mixed if masses of homophilic and heterophilic individuals are both positive; is nonhomophilic if the mass of heterophilic individuals is positive; and is nonheterophilic if the mass of homophilic individuals is positive. We assume that imperfect vertical transmission and oblique transmission are culturally substitutable.

be suboptimal by men; symmetric reasoning applies to WOSM.

The results are more elaborate when marital preferences are heterogeneous. With a mixture of heterophilic and homophilic individuals within each population, the long-run distribution of traits crucially depends on transmission technology. In the benchmark model, with perfect vertical transmission in homogamies and oblique transmission in heterogamies, cultural homogeneity is the generic long-run outcome. Indeed, a fraction—even arbitrarily small—of heterophilic proposers is sufficient to cause the dynamics to converge to cultural homogeneity. Hence, in this benchmark setting, cultural diversity is sustainable in the long run if and only if all proposers are homophilic.[2] When we introduce rational forces in oblique transmission in heterogamies, the level of noise—the relative weight of imitation versus rational choices—plays an important role. The larger the level of the noise, the closer we are to the benchmark model. For a lower level of noise—when rational choices matter more—new cyclic configurations may arise. With the extremely rational Darwinian transmission, these cycles are permanent. With imitative

[2] We show in an extension that, still in the benchmark setting, cultural diversity is also the long-run outcome if all individuals (both women and men) who belong to a cultural group are homophilic.

logit transmission, they are temporary; in general, the cultural evolution may end up with a locally stable culturally heterogeneous steady state or an asymptotically stable homogeneous one. When we replace perfect vertical transmission in homogamies with culturally substitutable imperfect vertical transmission, either a locally stable equilibrium with symmetric cultural diversity or locally stable equilibria with asymmetric cultural diversity—majority and minority cultures in an otherwise symmetric environment—may arise.

Finally, we connect our theoretical results to historical observations. First, we discuss the possibility that the distinction between evolutionary outcomes under MOSM and WOSM may help explain gender differences in behavior in patriarchal and matriarchal societies (Gneezy et al., 2009; Andersen et al., 2013). Second, we extend our model to gender imbalance. Using the fact that a slight sex imbalance can have a stark effect on stable matching (Ashlagi et al., 2017), we show how temporary gender imbalance can shift the matching from MOSM to WOSM and have lasting impacts on cultural evolution (Grosjean and Khattar, 2019; Baranov et al., 2021). Third, we apply our results to connect intermarriages with cultural assimilation and preservation, and provide examples of how heterophilic individuals lead to cultural homogeneity and homophilic individuals lead to cultural heterogeneity.

After a brief literature review, the rest of the paper is organized as follows. Section 2 provides the general setup of the model. Section 3 presents the benchmark model in which children in homogamies are influenced only by their parents and children in heterogamies are influenced only by the society. Section 4 presents the imitative logit model in which children in heterogamies are influenced by both societal impacts and rational choices through imitative logit learning. Section 5 presents the model in which children in homogamies are influenced by their parents and the society. Section 6 presents the implications of the model, and Section 7 concludes. Appendices collect omitted details and proofs.

1.1 Literature review

Most of the cultural evolution literature (Cavalli-Sforza and Feldman, 1981; Boyd and Richerson, 1985; Bisin and Verdier, 2001; Cheung and Wu, 2018) considers asexual reproduction models in which cultural transmission is the result of vertical parental socialization and oblique socialization by the society. Since a child is socialized by one unique parent, the process of couple formation does not play a role in cultural evolution. We depart from these foundational models along two main lines. First, we consider a two-sex cultural transmission model in which marital preferences are heterogeneous and the matching between spouses is endogenous; second, we consider different types of cultural transmission technologies.

Several papers consider that a child might be socialized by two parents. Bisin and Verdier (2000) propose a cultural transmission model with a marriage market. Individuals might be one of two types and prefer that their children have their own trait. Women and men must enter a frictional marriage market to marry and produce offspring. The marriage market consists of two restricted matching pools exclusive to the two types, respectively, and a common matching pool. Entering a restricted matching pool is costly. The authors assume that homogamous parents enjoy a more efficient socialization technology for their shared type than heterogamous parents. As a result, individuals prefer to and do marry their own type (homophily and homogamy). They also assume that daughters and sons are socialized in the same way, such that the cultural distribution is the same in populations of both women and men. In contrast, we propose a two-sex cultural transmission model and allow for heterophilic preferences. These two features separate the consideration of the socialization of cultural traits and marital preferences, and allow the consideration of joint cultural evolution in populations of both women and men.

Recently, some two-sex cultural evolution models have been developed. Hiller and Baudin (2016) and Baudin and Hiller (2019) propose models in which parents may socialize their sons and daughters differently. However, their analysis considers random matching; the effects of table matching on the evolution of preferences are not considered. Wu and Zhang (2021) allow for random matching and assortative matching of spouses, but implicitly assume homophily (because heterohpilic individuals are not distinct from homophilic ones when stable matching was not considered). In contrast, we consider stable matching with heterophilic individuals so that there could exist multiple stable matches; as a result, multiple cultural equilibria may arise.

Other related works on the formation of cultural preferences also consider marriage markets. Robson (1996) considers risk-taking in an assortative marriage market without frictions. Fernández et al. (2004) consider female labor force participation in a random marriage market. Mailath and Postlewaite (2006) demonstrate the social value of unproductive heritable traits in a stable matching model with intergenerational transmission. Bisin and Tura (2020) study cultural integration in a model of an assortative marriage market and collective household decisions on fertility and cultural socialization. Cigno et al. (2021) consider the effect of different two-sided matching technologies on the evolution of taste for filial attention and its implications for family rules.

As is standard in the cultural transmission literature, children are first socialized by their parents (vertical transmission) and then by the society (oblique transmission). In the oblique socialization stage, it is usually assumed that the child picks the trait of a role model randomly chosen from the whole population. Under those assumptions, cultural substitutability between vertical and oblique transmission is often viewed as a sufficient and necessary condition for rendering cultural heterogeneity sustainable (Bisin and Verdier, 2000, 2001, 2011).[3] In contrast, we show that for some matching structures, cultural heterogeneity may arise even in the absence of cultural substitutability, and cultural homogeneity might be the long-run outcome even in the presence of cultural substitutability.[4]

Inspired by a growing literature that argues that cultural evolution can be Darwinian, in the sense that evolution is at least partly driven by cultural fitness success (Besley, 2017, 2020; Besley and Persson, 2017, 2019), we propose a generalization of the oblique transmission technology. Specifically, we consider an imitative logit model (Weibull, 1995; Björnerstedt and Weibull, 1996; Sandholm, 2010) in which the probability that a child adopts the trait of a randomly chosen role model increases with the fitness associated with that trait. Whereas in the above-mentioned papers the Darwinian forces push toward cultural homogeneity, in our setup, vertical transmission—and in particular the role played by the matching structure

[3] In Bisin and Verdier (2000), when the proportion of a type of individuals decreases in the population, individuals of that type have a stronger incentive to enter the restricted marriage pool and exert a higher effort to socialize their type to their children, such that the cultural substitutability property is satisfied.

[4] Della Lena and Panebianco (2021) also show the possibility of long-run cultural homogeneity even under cultural substitutability in a setting that generalizes Bisin and Verdier (2001) to incorporate incomplete information.

in vertical transmission—might act as a counterforce. It might result in cyclical evolution or in situations in which cultural heterogeneity is sustained.

Note that in our model, people achieve stable matching quickly and preferences evolve slowly; and in extreme cases, the preferences of the two genders evolve differently. Sandholm (2001) proposes a two-speed dynamic model in which individual behavior in some games evolves quickly (in the extreme case, reaches equilibrium instantly) and their preferences evolve slowly. Kuran and Sandholm (2008) elaborate on the idea of the two-speed dynamic to study cultural integration and preservation. People are facing conforming pressure, so they adjust their behavior accordingly, and preferences evolve slowly in the direction of their behavior. In their model, policies promoting diversity and multiculturalism inevitably lead to assimilation and cultural homogeneity, similar to the effects of heterophilic individuals in our model on achieving cultural homogeneity.

Our work is also related to the matching literature, which usually assumes that individuals' attributes or preferences are fixed and thus the matching mechanism only affects ex post allocations. Some papers depart from this assumption by considering that individuals engage in pre-match investments that might change their attributes and the matching outcome (Nöldeke and Samuelson, 2015; Bhaskar and Hopkins, 2016; Zhang, 2020, 2021). Here we take another avenue by considering that the matching obtained at one date influences the distribution of traits, and in turn the matching outcome in the future. Wu (2021) attempts to build a bridge between matching and cultural evolution, but focuses on cultural evolution of one population without considering intergenerational transmission.

2 General structure of the model

There is a unit mass of men (m) and a unit mass of women (w) in every discrete period $t \in \{0, 1, \dots\}$.[5] Each person lives for two periods: *childhood* and *adulthood*. Each adult has either trait/type a or trait b. Let p^t denote the mass of type-a men in period t, and q^t the mass of type-a women in period t.

2.1 Marital preferences

Let $U_{\theta_m \theta_w}$ denote a type-θ_m man's utility from marrying a type-θ_w woman and let $V_{\theta_w \theta_m}$ denote a type-θ_w woman's utility from marrying a type-θ_m man. We assume that, for any θ and θ', $U_{\theta\theta'} > 0$ and $V_{\theta\theta'} > 0$ and we normalize the utility from remaining single to 0. This implies that each woman considers each man as an acceptable match and vice versa. Each individual belongs to one of two preference groups: *homophilic* or *heterophilic*. For any θ and $\theta' \neq \theta$, a type-θ man has a *homophilic preference* if $U_{\theta\theta} > U_{\theta\theta'}$, and a *heterophilic preference* if $U_{\theta\theta} < U_{\theta\theta'}$. Women's preferences are similarly defined. We assume that each adult of gender $g \in \{m, w\}$ has homophilic preferences with an independent probability denoted by $h_g \in [0, 1]$. In particular, the probability that an individual has homophilic preferences is independent of their cultural trait.[6] For a given time t, we define $M^t_{\theta_i}$ as the set of type-θ men having group $i = 1$ (homophilic) or group $i = 2$ (heterophilic) preferences; $W^t_{\theta_i}$ is similarly defined for women. Since transmission probabilities

[5]We relax the assumption of equal population size in an application (see Section 6.1.2).

[6]We relax the independence assumption in an application (see Section 6.2.2).

depend on expected utility differences in the imitative logit transmission we consider, we also assume that the utility functions are von Neumann-Morgenstein utility functions.

2.2 Stable matching

In each period, men and women match to form marriage pairs. A couple with a type-θ_m husband and a type-θ_w wife will be called a $\theta_m\theta_w$ couple. Moreover, when $\theta_m = \theta_w$, a $\theta_m\theta_w$ couple is a *homogamous couple* and when $\theta_m \neq \theta_w$, the couple will be referred to as a *heterogamous couple*. Let $\mu^t_{\theta_m\theta_w}$ denote the mass of $\theta_m\theta_w$ couples in period t. A matching is a 2-by-2 matrix $\mathsf{M}^t = (\mu^t_{\theta_m\theta_w})$ such that $\mu^t_{\theta_m\theta_w} \in \mathbf{R}_+$ and $\mu^t_{aa} + \mu^t_{ab} = p^t$, $\mu^t_{ba} + \mu^t_{bb} = 1 - p^t$, $\mu^t_{aa} + \mu^t_{ba} = q^t$, $\mu^t_{ab} + \mu^t_{bb} = 1 - q^t$.[7]

Assume stable matching. Since we are looking at continuous populations with finite types, we adopt the definition of stable matching for aggregate matching from Echenique et al. (2013). A pair of types (θ_m, θ_w) is a blocking pair for M^t if $\theta'_m \neq \theta_m$ and $\theta'_w \neq \theta_w$ satisfy that $U_{\theta_m\theta_w} > U_{\theta_m\theta'_w}$, $V_{\theta_w\theta_m} > V_{\theta_w\theta'_m}$, $\mu^t_{\theta_m\theta'_w} > 0$ and $\mu^t_{\theta'_m\theta_w} > 0$. A matching M^t is stable if there are no blocking pairs for M^t.[8]

We further assume that men and women match according to either *men-optimal stable matching* (*MOSM*)—the stable matching most preferred by men—or *women-optimal stable matching* (*WOSM*)—Baïou and Balinski (2002) show the existence of these two matchings for continuous populations with finite types. MOSM (resp., WOSM) can be achieved by the generalized Gale-Shapley deferred acceptance algorithm if men (resp., women) are proposers. Hence, we sometimes refer to the side that has implemented their most preferred stable matching as proposing side, and to the opposite side as receiving side.

2.3 Intergenerational transmission

All men and women pair up and each pair reproduces two children, one son and one daughter; equivalently, each child is a male or a female with equal probabilities. Children are born without a well-defined culture and will acquire, during childhood, a cultural trait they will hold during their entire adulthood. As is usual in the intergenerational cultural transmission literature (Cavalli-Sforza and Feldman, 1981; Boyd and Richerson, 1985; Bisin and Verdier, 2001, 2011), the cultural transmission process entails two steps. Children are first socialized by their parents. If this stage of *vertical transmission* fails, children are socialized by the society at large in the second stage of *oblique transmission*. We assume that only parents from homogamies can directly transmit their culture during the vertical transmission stage. Or, stated differently, for heterogamous couples the probability of vertical transmission is zero. Since heterogamous couples do not have a well-defined cultural type to transmit, it is natural to assume that a homogamy has

[7] If there are any singles, the masses of singles who possess different cultural traits can be backed out from the matching matrix. Note that since we are interested in the joint evolution of p^t and q^t, and since—as will become clear—this evolution depends on the composition of couples in terms of cultural types, we define a matching in terms of only those types rather than in terms of both cultural types and preference groups.

[8] A stable matching must also be individually rational, meaning that no individual should prefer remaining single to retaining their match. This is always the case given our earlier assumption, since the utility derived from celibacy is normalized to 0 and the utility of a match is strictly positive. This assumption, along with the fact that there are as many women as men, implies that at any stable matching, all individuals are matched. In Section 6.1.2 we propose an extension in which the two populations are not balanced, so that some individuals remain single.

a more superior transmission technology than does a heterogamy. This is considered in Bisin and Verdier (2000) and Hiller and Baudin (2016), for example; see Dohmen et al. (2012) for empirical support. Hence, the probability that a child adopts a particular culture depends on the cultural types of both parents. We let $P^t_{\theta\theta'}$ and $Q^t_{\theta\theta'}$ denote the probability that the son and daughter of trait-θ father and trait-θ' mother possess trait a at date $t+1$, respectively. Their expressions depend on the assumptions we make regarding the oblique transmission process and the vertical transmission process in homogamies. We describe below the different assumptions we will consider.

Benchmark transmission: Perfect vertical transmission in homogamies and oblique transmission in heterogamies. As a benchmark, we consider that parents from homogamous couples transmit their cultural type to their children with probability one (*perfect vertical transmission*). As we said, this probability is zero for heterogamous couples.[9] Hence, children from heterogamous couples must acquire their culture during the oblique socialization stage. During this stage, we assume that each child randomly searches for a role model in their respective gender. Moreover, as a benchmark, we assume that they adopt the cultural type of this role model with probability one (*oblique transmission*).[10] This assumption involves a simple conformist component in cultural evolution. Under those assumptions, transmission probabilities can be written as

$$P^t_{aa} = Q^t_{aa} = 1, \quad P^t_{bb} = Q^t_{bb} = 0, \quad P^t_{ab} = P^t_{ba} = p^t, \quad Q^t_{ab} = Q^t_{ba} = q^t.$$

Imitative logit and Darwinian transmission in heterogamies. While we still consider perfect vertical transmission in homogamies, we generalize the oblique transmission. We incorporate the insights from a growing literature that argues that cultural evolution can be Darwinian, in the sense that the evolution is at least partly driven by cultural fitness, with the cultural fitness of a type's being defined as the expected utility of that type (Besley, 2017, 2020; Besley and Persson, 2017, 2019). Hence, how the frequency of a type evolves in the population is determined by its relative expected utility compared with the other type.[11]

The aforementioned literature does not reconcile the conformist component and the Darwinian component, which we believe are both important for capturing the gist of intergenerational transmission. Hence, an approach that can incorporate both components and allow one to assess the relative importance of each is desired. Here we consider the following more general forms of transmission in heterogamous families. Let $P^t \in [0,1]$ and $Q^t \in [0,1]$ denote the probability that a boy and a girl adopt type a, respectively:

$$P^t = \frac{p^t(\exp U^t_a/\delta)}{p^t \exp(U^t_a/\delta) + (1-p^t)\exp(U^t_b/\delta)}, \tag{1}$$

[9]Our benchmark setting can be related to the assumption made in the first formal theoretical contributions to the modeling of cultural transmission (Cavalli-Sforza and Feldman, 1981; Boyd and Richerson, 1985). In those models, the vertical transmission probability is exogenous and constant. Instead, we consider that this probability depends on the homogamous or heterogamous type of couple—but, for a given type of couple, it is exogenous and constant.

[10]This corresponds to the benchmark assumption in the cultural transmission literature (Cavalli-Sforza and Feldman, 1981; Boyd and Richerson, 1985; Bisin and Verdier, 2001, 2011).

[11]As argued by Besley and Persson (2017), to use such a model, there must be a capacity for intra-personal comparisons of utility between the two types. For example, we need to assess the gain and loss in psychological well-being of adopting a certain type in addition to its material consequence. Note that all of these papers adopt evolutionary dynamics from Sandholm (2010).

where U_θ^t denotes the expected utility of a man with type $\theta \in \{a, b\}$, and $\delta > 0$ denotes the level of noise. Similarly,

$$Q^t = \frac{q^t \exp(V_a^t/\delta)}{q^t \exp(V_a^t/\delta) + (1-q^t)\exp(V_b^t/\delta)}, \tag{2}$$

where V_θ^t denotes the expected utility of a woman with type $\theta \in \{a, b\}$. The transmission probabilities specified here can be derived from a noisy repeated sampling process, which is called *imitative logit*, proposed by Weibull (1995) and Björnerstedt and Weibull (1996). We show the derivation in Appendix B.1. It captures the importance of the population composition and the utilities associated with different traits and noises.

Note that the two forces that drive the probability that a child in a heterogamy adopts one trait—the conformist and the Darwinian forces—can be separated by the log likelihood ratio of adoption of the two traits (use a boy as an example):

$$\log \frac{P^t}{1-P^t} = \log \frac{p^t}{1-p^t} + \frac{U_a^t - U_b^t}{\delta}.$$

As previously, the boy is influenced by the composition of types in all men in the society (the conformist component). Indeed, the proportion of type-a men increases the probability that a boy picks a type-a role model and adopts his type; P^t is increasing in p^t. However, an additional evolutionary force is at play. The probability for a boy to adopt the trait of a type-a role model increases in the difference between the expected utility associated with trait a and the expected utility associated with trait b (the Darwinian component) for men; P^t is increasing in $U_a^t - U_b^t$. The parameter δ measures the relative strength of these two forces. The higher δ, the larger the societal influence compared with the evolutionary component.[12]

Note that when $\delta \to \infty$, children in heterogamies are influenced only by the society: $\lim_{\delta\to\infty} P^t = p^t$ and $\lim_{\delta\to\infty} Q^t = q^t$. This corresponds to the benchmark oblique transmission. At the other extreme, when $\delta \to 0$, only the rational choice is in play:

$$P_0^t := \lim_{\delta\to 0} P^t = \begin{cases} 1 & \text{if } U_a^t > U_b^t, \\ \frac{1}{2} & \text{if } U_a^t = U_b^t, \\ 0 & \text{if } U_a^t < U_b^t, \end{cases} \text{ and } Q_0^t := \lim_{\delta\to 0} Q^t = \begin{cases} 1 & \text{if } V_a^t > V_b^t, \\ \frac{1}{2} & \text{if } V_a^t = V_b^t, \\ 0 & \text{if } V_a^t < V_b^t. \end{cases} \tag{3}$$

We refer to this case as *Darwinian transmission.* To sum up, transmission probabilities can be written as

$$P_{aa}^t = Q_{aa}^t = 1, \quad P_{bb}^t = Q_{bb}^t = 0, \quad P_{ab}^t = P_{ba}^t = P^t, \quad Q_{ab}^t = Q_{ba}^t = Q^t,$$

where the expressions of P^t and Q^t are respectively given by equations (1) and (2) in imitative logit transmission and by equation (3) in Darwinian transmission.

[12]In Besley (2017), the probabilities of adopting different types for children in heterogamous families are modeled as logit functions. Hence, there is no societal influence and δ measures the strength of the noise.

Imperfect vertical transmission in homogamies. In the last setup, we again assume oblique transmission in heterogamies as in the benchmark setup (children who are not directly socialized by their parents adopt the trait of a randomly chosen role model with probability one). We relax the perfect vertical transmission assumptions in homogamous couples such that for these couples, the probability of direct transmission of the parental trait is not always equal to one. Let this vertical transmission probability be $d(r)$, where r is the mass of individuals of the same gender and type, and $1-d(r)$ is the probability that the transmission is oblique. We assume that vertical and oblique transmission in homogamies satisfy *cultural substitutability* (Bisin and Verdier, 2001): $d(r)$ is continuous and strictly decreasing in r, and $d(1) = 0$. That is, the vertical transmission probability of a trait is higher if there is a lower proportion of the trait, and the vertical transmission probability is 0 if an individual of the same gender in the previous generation has the trait with probability one. This cultural substitutability property might be rationalized in a model in which homogamous parents have the possibility to make costly efforts to transmit their traits and exhibit a form of cultural intolerance as in Bisin and Verdier (2001). We propose a simple derivation of this result in Appendix C.1. Under those assumptions, transmission probabilities can be written as

$$P^t_{aa} = d(p^t),\ Q^t_{aa} = d(q^t),\ P^t_{bb} = 1 - d(1-p^t),\ Q^t_{bb} = 1 - d(1-q^t),\ P^t_{ab} = P^t_{ba} = p^t,\ Q^t_{ab} = Q^t_{ba} = q^t.$$

2.4 Cultural evolution and steady states

The distribution of traits in a period depends on the proportion of families of different pairs of traits and the intergenerational transmission in different types of families. Generally, cultural evolution is characterized by the following system of equations:

$$p^{t+1} = \mu^t_{aa}P^t_{aa} + \mu^t_{ab}P^t_{ab} + \mu^t_{ba}P^t_{ba} + \mu^t_{bb}P^t_{bb}; \tag{4}$$

$$q^{t+1} = \mu^t_{aa}Q^t_{aa} + \mu^t_{ab}Q^t_{ab} + \mu^t_{ba}Q^t_{ba} + \mu^t_{bb}Q^t_{bb}. \tag{5}$$

This system clarifies that cultural evolution is determined by intergenerational transmission ($P^t_{\theta\theta'}$ and $Q^t_{\theta\theta'}$) and matching ($\mu^t_{\theta\theta'}$), which depend on underlying preference and trait distributions as well as the stable matching scheme.

We consider the evolutionary outcome from any interior initial state $(p^0, q^0) \in (0,1)^2$ as $t \to \infty$. When the limits exist, we denote the steady-state masses of type-a men and women by $p^* = \lim_{t\to\infty} p^t$ and $q^* = \lim_{t\to\infty} q^t$, respectively. The steady state may depend on the initial distributions. We say that a steady state (p^*, q^*) is *stable* if for all $\varepsilon > 0$, there exists $\delta_\varepsilon > 0$ such that $|(p^0, q^0), (p^*, q^*)| < \delta_\varepsilon$ implies $|(p^t, q^t), (p^*, q^*)| < \varepsilon$ for all $t > 0$. A steady state is *unstable* if it is not stable. (p^*, q^*) is *attracting* if there exists $\eta > 0$, such that $|(p^0, q^0), (p^*, q^*)| < \eta$ implies $\lim_{t\to\infty}(p^t, q^t) = (p^*, q^*)$. A steady state is *asymptotically stable* if it is stable and attracting. We say a steady state is *globally asymptotically stable* if it is stable and globally attracting; when such a steady state exists, it is the unique steady state. A *stable set* $\mathcal{S}$ of steady states is a non-singleton connected set of steady states such that there exists an open neighborhood of the set, $\mathcal{N} \supset \mathcal{S}$, such that for any initial state $(p, q) \in \mathcal{N}$, the steady state reached is in the set $\mathcal{S}$.

3 Benchmark transmission: Perfect vertical transmission in homogamies and oblique transmission in heterogamies

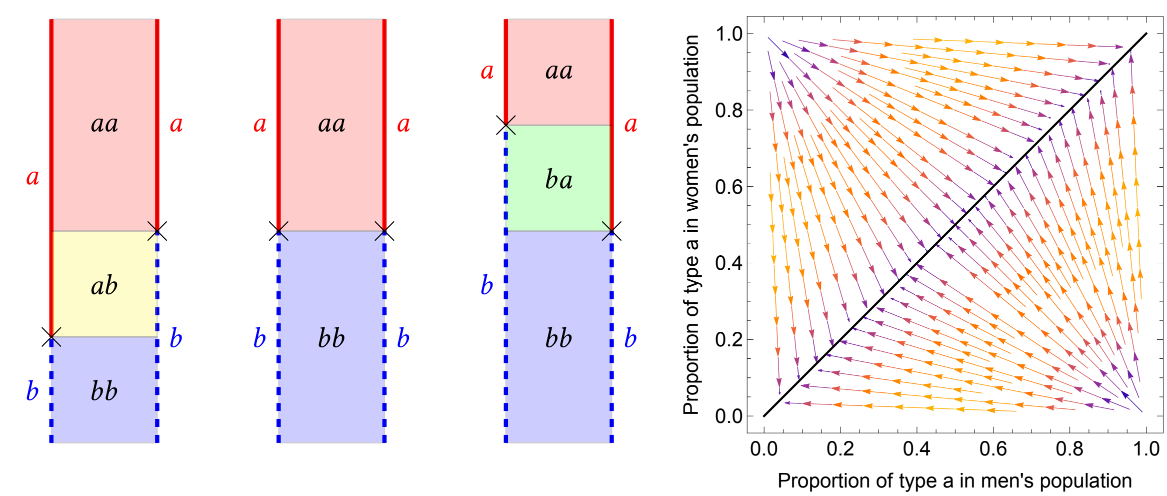
In this section, we consider the benchmark transmission technology and examine the long-run behavior of the joint distribution of cultural traits for different assumptions about marital preferences (i.e., for different values of h_m and h_w).

3.1 Homophilic versus heterophilic proposers

As a first step, we will consider that there is no heterogeneity in preferences among proposers. Without loss of generality, we will assume that men are proposers and we will successively address the case in which all proposers are homophilic ($h_m = 1$) in Section 3.1.1 and the case in which all proposers are heterophilic ($h_m = 0$) in Section 3.1.2. Note that we do not make any assumption on the distribution of preferences among receivers ($h_w \in [0, 1]$). Finally, in Section 3.1.3 we briefly discuss the optimality of MOSM and WOSM from a dynamic point of view.

3.1.1 Homophilic proposers lead to cultural heterogeneity

Suppose all men have homophilic preferences: $U_{\theta\theta} > U_{\theta\theta'}$ for any θ and $\theta' \neq \theta$ and consider MOSM (i.e. men are the proposers in the stable matching algorithm).[13] This is independent of women's preference distribution, as long as they find every man to be acceptable. MOSM is given by mass q^t of aa couples, mass $1 - p^t$ of bb couples, and mass $p^t - q^t$ of ab couples if $p^t \geqslant q^t$ (Figure 1a); mass p^t of aa couples and mass $1 - p^t$ of bb couples if $p^t = q^t$ (Figure 1b); and mass p^t of aa couples, mass $1 - q^t$ of bb couples, and mass $q^t - p^t$ of ba couples if $p^t < q^t$ (Figure 1c).



(a) $p^t > q^t$ (b) $p^t = q^t$ (c) $p^t < q^t$ (d) Evolution

Figure 1: Matching and evolution with homophilic proposers under benchmark transmission

[13]Symmetrically, we could assume that women have homophilic preferences and are the proposers without any consequences for our results.

Replacing the values of $\mu^t_{\theta\theta'}$ corresponding to this stable matching and the values of $P^t_{\theta\theta'}$ and $Q^t_{\theta\theta'}$ corresponding to the benchmark transmission into equations (4) and (5), we get that the cultural evolution when $p^t \geqslant q^t$ is characterized by the following dynamic system:

$$p^{t+1} = q^t + (p^t - q^t)p^t; \tag{6}$$

$$q^{t+1} = q^t + (p^t - q^t)q^t. \tag{7}$$

Here are three observations of the dynamic system. First, from equation (6), $p^{t+1} \leqslant p^t$ for any $p^t \in (0, 1]$. The equality holds only when either $p^t = 1$ or $p^t = q^t$ (or both). Second, from equation (7), $q^{t+1} \geqslant q^t$ for any $q^t \in [0, 1)$. The equality holds only when either $q^t = 0$ or $p^t = q^t$ (or both). Third, from equations (6) and (7), $p^{t+1} \geqslant q^{t+1}$ for any $0 \leqslant q^t \leqslant p^t \leqslant 1$. The equality holds only when $p^t = q^t$. Hence, for any initial condition (p^0, q^0) that satisfies $0 \leqslant q^0 \leqslant p^0 \leqslant 1$ with either the first or the last inequality being strict or both (the southeast triangle in Figure 1d without the point $(1, 0)$), $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t = r$, for some $r \in (0, 1)$. The dynamic system in the case of $p^t < q^t$ is symmetric to that in the case of $p^t > q^t$ (the northwest triangle in Figure 1d without the point $(0, 1)$). Note that $\{(r, r)|r \in (0, 1)\}$ constitutes a stable set of steady states in the sense that perturbations from a steady state in this set may result in a different steady state, but the new steady state falls in the set of steady states.

Proposition 1. *Suppose transmission is perfect vertical in homogamies and oblique in heterogamies. With homophilic proposers, for any interior initial state, (p^t, q^t) converges to (r, r) for some $r \in (0, 1)$; $\{(r, r)|r \in (0, 1)\}$ forms a stable set.*

Proposition 1 demonstrates that the dynamic system converges to steady states in which both types coexist in both genders and the distributions of types are balanced across genders. Hence, we have cultural diversity (coexistence of both types) as the most frequent long-run outcome.

3.1.2 Heterophilic proposers lead to cultural homogeneity

Suppose men have heterophilic preferences that favor heterogamies: $U_{\theta\theta} < U_{\theta\theta'}$ for any θ and $\theta' \neq \theta$. Again, we consider MOSM, which is again independent of women's preference distribution. Stable matching is given by mass $1 - p^t$ of *ba* couples, mass $1 - q^t$ of *ab* couples, and mass $p^t + q^t - 1$ of *aa* couples if $p^t + q^t > 1$ (Figure 2a); mass p^t of *ab* couples and mass q^t of *ba* couples if $p^t + q^t = 1$ (Figure 2b); and mass p^t of *ab* couples, mass q^t of *ba* couples, and mass $1 - p^t - q^t$ of *bb* couples (Figure 2c).

Figure 2d illustrates cultural evolution. When $p^t + q^t > 1$, it is characterized by

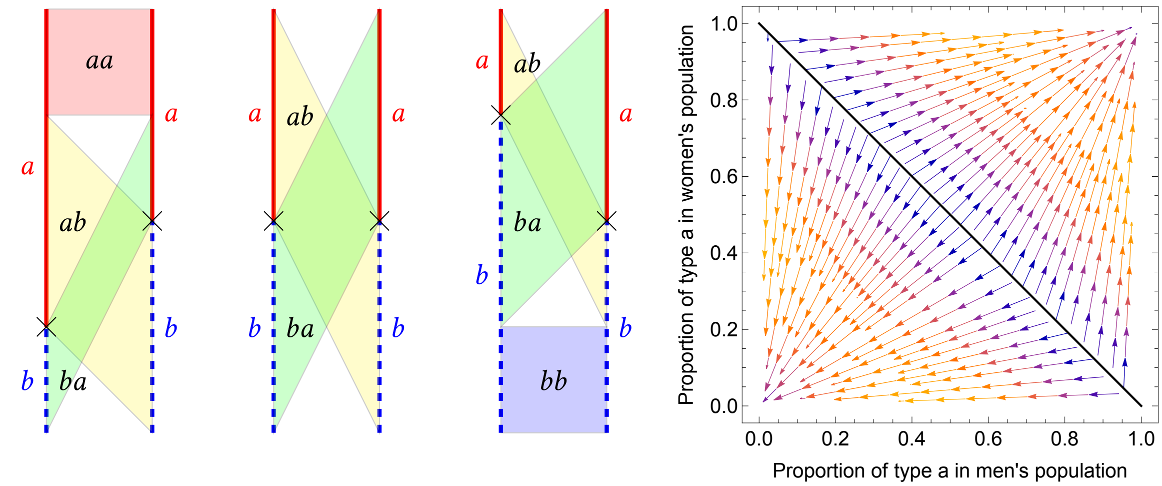
$$p^{t+1} = (p^t + q^t - 1) + (2 - p^t - q^t)p^t; \tag{8}$$

$$q^{t+1} = (p^t + q^t - 1) + (2 - p^t - q^t)q^t. \tag{9}$$

Rearrange the equations:

$$p^{t+1} = p^t + (1 - p^t - q^t)(p^t - 1); \tag{10}$$

$$q^{t+1} = q^t + (1 - p^t - q^t)(q^t - 1). \tag{11}$$



(a) $p^t + q^t > 1$ (b) $p^t + q^t = 1$ (c) $p^t + q^t < 1$ (d) Evolution

Figure 2: Matching and evolution with heterophilic proposers under benchmark transmission

Observe that (i) $p^{t+1} \geqslant p^t$, and the equality holds only when $p^t = 1$; and (ii) $q^{t+1} \geqslant q^t$, and the equality holds only when $q^t = 1$. Hence, $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t = 1$ for any initial condition (p^0, q^0) that satisfies $p^0 + q^0 > 1$. When $p^t + q^t = 1$, the dynamic is always in a steady state. The dynamic system in the case of $p^t + q^t < 1$ (the southwestern triangle in Figure 2d) is given by

$$p^{t+1} = (p^t + q^t)p^t; \tag{12}$$

$$q^{t+1} = (p^t + q^t)q^t. \tag{13}$$

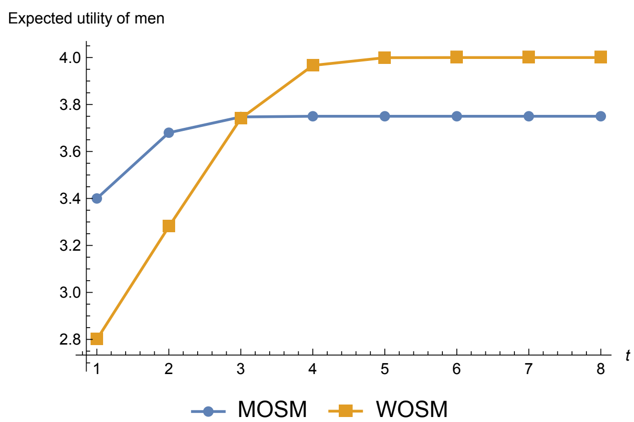
It converges to $(0, 0)$.

Proposition 2. *Suppose transmission is perfect vertical in homogamies and oblique in heterogamies. With heterophilic proposers,* (p^t, q^t) *converges to* $(0, 0)$ *if* $p^0+q^0 < 1$, (p^0, q^0) *if* $p^0+q^0 = 1$, *and* $(1, 1)$ *if* $p^0+q^0 > 1$.

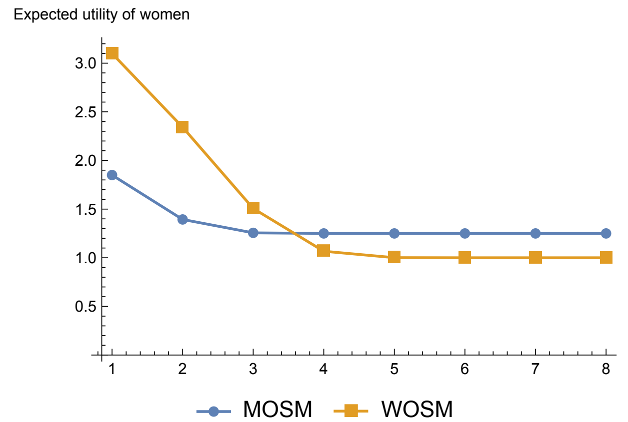
Proposition 2 shows that with heterophilic proposers, in the long run, the entire society consists of only one type. Hence, cultural integration is the robust long-run phenomenon.

3.1.3 Short-run optimal stable matching may lead to long-run loss

The analyses in Section 3.1.1 and 3.1.2 demonstrate that cultural evolution with homophilic proposers leads to cultural heterogeneity, but cultural evolution with heterophilic proposers leads to cultural homogeneity. The rationale is as follows. With homophilic proposers, the proportion of homogamies in each trait is determined by the short side of the marriage market. This creates a tendency toward a balanced sex ratio, which guarantees that people will maintain their legacies through homogamies. However, there is no tension between the two types in cultural transmission. Therefore, the dynamic can reach any steady state with a balanced sex ratio in traits. With heterophilic proposers, the minority proposers enjoy the full benefits of complementarity between types through matching within each generation. However, heterogamies make it hard for them to maintain their legacies. In the long run, the majority drives out the



(a) Men are worse off in MOSM than in WOSM



(b) Women are worse off in WOSM than in MOSM

Figure 3: Expected payoffs of homophilic men and heterophilic women under MOSM and WOSM

Note. For the illustrations, we use $U_{aa} = 4$, $U_{bb} = 3$, $U_{ab} = U_{ba} = 2$, $V_{ab} = V_{ba} = 4$, $V_{bb} = 2$, $V_{aa} = 1$, $p^0 = 0.6$, and $q^0 = 0.8$. Both men and women are strictly worse off in the long run in the stable matching scheme that is better for them in the short run.

minority because the majority type manages to keep a fraction of homogamies due to sheer population size. Note that a type is categorized as the majority type if proposers of this type and receivers of the opposite type outnumber proposers of the other type and receivers of the opposite type.

Since men are proposers under MOSM and women are proposers under WOSM, Propositions 1 and 2 also imply that when men and women have opposite preferences (one side is homophilic and the other side is heterophilic), the long-run distribution of cultural traits depends on the selected stable matching. For instance, when men are homophilic and women are heterophilic, MOSM leads to cultural heterogeneity and WOSM leads to cultural homogeneity. This result compels us to reassess the optimality of MOSM and WOSM from a dynamic point of view. By definition, the distribution of cultural traits being given, all men prefer MOSM to WOSM and the reverse is true for women. However, the cultural transformation induced by the choice of MOSM might lead to a situation in which men get a lower expected utility than if WOSM had been chosen (while the reverse might be true for women).

Remark 1. *Suppose homophilic men and heterophilic women under perfect vertical transmission in homogamies and oblique transmission in heterogamies. When $p^0 + q^0 > 1$, men are strictly better off (resp., worse off) in the long run under MOSM than under WOSM if and only if $U_{aa} < U_{bb}$ (resp., $U_{aa} > U_{bb}$); and women are strictly better off (resp., worse off) in the long run under WOSM than under MOSM if and only if $V_{aa} > V_{bb}$ (resp., $V_{aa} < V_{bb}$). When $p^0 + q^0 < 1$, the strict inequality signs in the necessary and sufficient conditions are reversed.*[14]

[14]When $p^0 + q^0 = 1$, the steady state under WOSM is $(p^w, q^w) = (p^0, q^0)$, so men's and women's average payoffs involve U_{ab}, U_{ba}, V_{ab}, and V_{ba}; the steady state under MOSM is (p^m, q^m) such that $p^m = q^m$, but there is no analytical formula to relate (p^m, q^m) to (p^0, q^0). All in all, when $p^0 + q^0 = 1$, there is no clean condition to characterize when a gender is strictly better off (or worse off) under MOSM than under WOSM in the long run.

Figure 3 illustrates a case in which men under MOSM and women under WOSM are strictly better off in the short run, but are strictly worse off in the long run due to the induced evolution of the distribution of traits.

3.2 Mixture of homophilic and heterophilic proposers

In this section, we relax the assumption of preference homogeneity within the population of proposers. We still assume—without loss of generality—that men are proposers. We consider that $h_m \in (0,1)$ so that homophilic and heterophilic men coexist. In Section 3.2.1 we consider the case in which all receivers are homophilic ($h_w = 1$). In Section 3.2.2, we consider the case in which there might be a mixture of homophilic and heterophilic receivers ($h_w \in [0,1]$).

3.2.1 Some heterophilic proposers sufficient for cultural integration

Now we consider a mixture of homophilic and heterophilic men who are proposers and homophilic women who are receivers. Recall that preference group 1 corresponds to homophilic preferences while preference group 2 corresponds to heterophilic preferences. Then, denoted by $\pi^t_{\theta_g}$ the mass of men of type $\theta \in \{a,b\}$ who belong to preference group g at time t, we have $\pi^t_{a_1} = p^t h_m$, $\pi^t_{b_1} = (1-p^t)h_m$, $\pi^t_{a_2} = p^t(1-h_m)$, and $\pi^t_{b_2} = (1-p^t)(1-h_m)$; to obtain these proportions we use the fact that the probability h_m that a man is homophilic is independent of his trait. In order to analyze cultural evolution, we first characterize the stable matching.

Stable matching. Lemma 1 and Figure 4 summarize men-optimal stable matching. When q is sufficiently small, all type-b men, including those having heterophilic preferences, are matched with type-b women (Figure 4a); and when q is sufficiently large, all type-a men, including those who are heterophilic, are matched with type-a women (Figure 4e). In these cases, there is a unique stable matching that exactly corresponds to the matching obtained when all proposers are homophilic (see Section 3.1.1). When q is in the intermediate range, all homophilic men get to match with women of the same type, and as many heterophilic men as possible match with women of the opposite type (Figures 4b, 4c, and 4d). In those configurations, ab couples coexist with ba couples.

Lemma 1. *Define the following sets:*

$$\begin{aligned}
\Omega_1 &:= \left\{(p,q) \in (0,1)^2 : q \in (0, \pi_{a_1}) = (0, ph_m)\right\};\\
\Omega_2 &:= \left\{(p,q) \in (0,1)^2 : q \in (\pi_{a_1}, \pi_{a_1} + \pi_{b_2}) = (ph_m, ph_m + (1-p)(1-h_m))\right\};\\
\Omega_3 &:= \left\{(p,q) \in (0,1)^2 : q \in (\pi_{a_1} + \pi_{b_2}, 1-\pi_{b_1}) = (ph_m + (1-p)(1-h_m), 1-(1-p)h_m)\right\};\\
\Omega_4 &:= \left\{(p,q) \in (0,1)^2 : q \in (1-\pi_{b_1}, 1) = (1-(1-p)h_m, 1)\right\}.
\end{aligned}$$

Let $\overline{\Omega}$ denote the closure of set Ω. In the market with mass p of type-a men and mass q of type-a women,

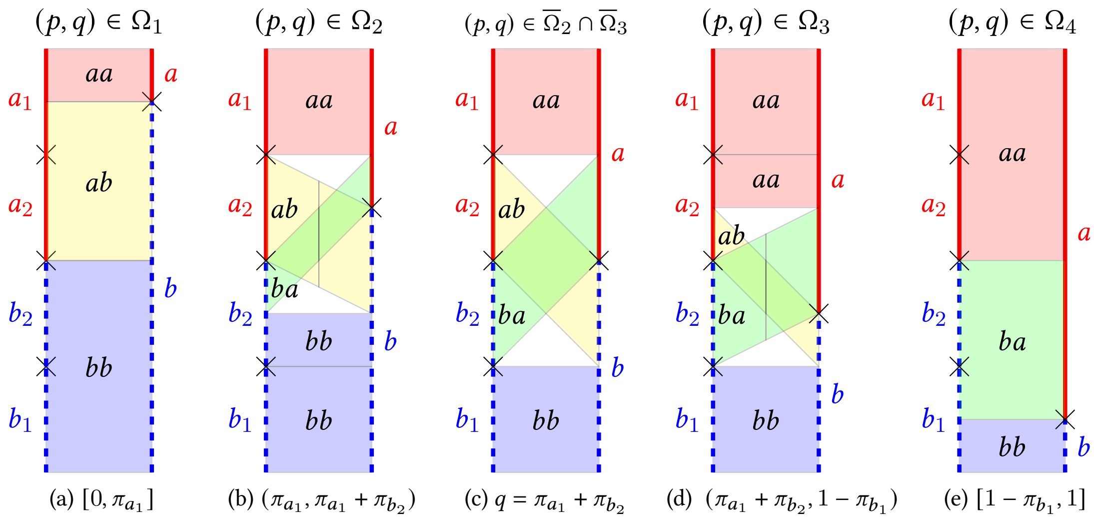


Figure 4: Stable matching with mixed proposing men and homophilic receiving women, by q

MOSM $\mu = (\mu_{aa}, \mu_{ab}, \mu_{ba}, \mu_{bb})$ with mixed proposing men and homophilic receiving women is

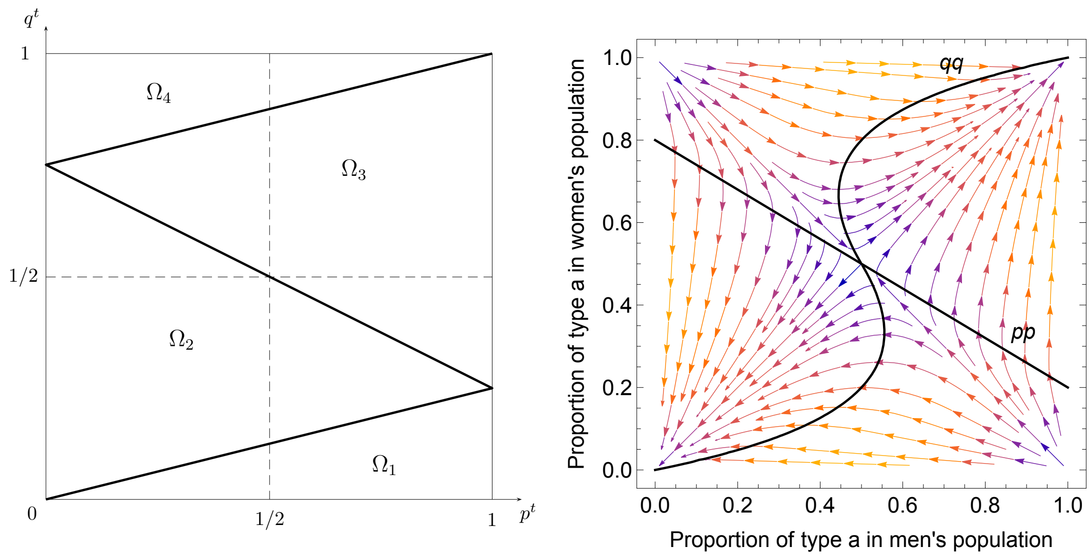
$$\begin{cases} (q, p-q, 0, 1-p) & \text{if} \quad (p,q) \in \overline{\Omega}_1, \\ (ph_m, p(1-h_m), q - ph_m, 1 - q - p(1-h_m)) & \text{if} \quad (p,q) \in \overline{\Omega}_2, \\ (q - (1-p)(1-h_m), 1 - q - (1-p)h_m, (1-p)(1-h_m), (1-p)h_m) & \text{if} \quad (p,q) \in \overline{\Omega}_3, \\ (p, 0, q-p, 1-q) & \text{if} \quad (p,q) \in \overline{\Omega}_4. \end{cases}$$

Figure 5a depicts the four sets defined in Lemma 1. The boundary between Ω_1 and Ω_2 is $q = ph_m$, which connects $(0,0)$ and $(1, h_m)$; the boundary between Ω_2 and Ω_3 —i.e., line pp—is $q = ph_m + (1-p)(1-h_m)$, which connects $(1, h_m)$ and $(0, 1-h_m)$; and the boundary between Ω_3 and Ω_4 is $q = 1 - (1-p)h_m$, which connects $(0, 1-h_m)$ and $(1,1)$. When $h_m = 0$, all men have heterophilic preferences so that regions Ω_1 and Ω_4—in which all men of one type are matched with women of the same type—disappear. These regions widen as h_m increases. When $h_m = 1$, all men have homophilic preferences, so that regions Ω_2 and Ω_3—in which *ab* couples coexist with *ba* couples—disappear.

Using Lemma 1 as well as the transmission probabilities associated with benchmark transmission, we can rewrite the two-dimensional dynamic system in equations (4) and (5) as

$$(p^{t+1}, q^{t+1}) = \begin{cases} (q^t + (p^t - q^t)p^t, q^t + (p^t - q^t)q^t) & \text{if} \quad (p^t, q^t) \in \overline{\Omega}_1, \\ (p^t h_m + [q^t + p^t - 2p^t h_m]p^t, p^t h_m + [q^t + p^t - 2p^t h_m]q^t) & \text{if} \quad (p^t, q^t) \in \overline{\Omega}_2, \\ (q^t - (1-p^t)(1-h_m) + [2 - q^t - p^t - 2(1-p^t)h_m]p^t, & \\ q^t - (1-p^t)(1-h_m) + [2 - q^t - p^t - 2(1-p^t)h_m]q^t) & \text{if} \quad (p^t, q^t) \in \overline{\Omega}_3, \\ (p^t + (q^t - p^t)p^t, p^t + (q^t - p^t)q^t) & \text{if} \quad (p^t, q^t) \in \overline{\Omega}_4. \end{cases}$$

If $h_m = 1$, the preferences of men are fully aligned with the preferences of women: they are all homophilic. In this case, the cultural dynamics is characterized by cultural diversity (Proposition 1). However, as stated in Proposition 3 below, if h_m is strictly lower than 1—even if it is arbitrarily close to 1—the



(a) Matching-outcome state space partition

(b) Evolution

Figure 5: Matching-outcome state space partition and evolution with mixed proposers and homophilic receivers under benchmark transmission

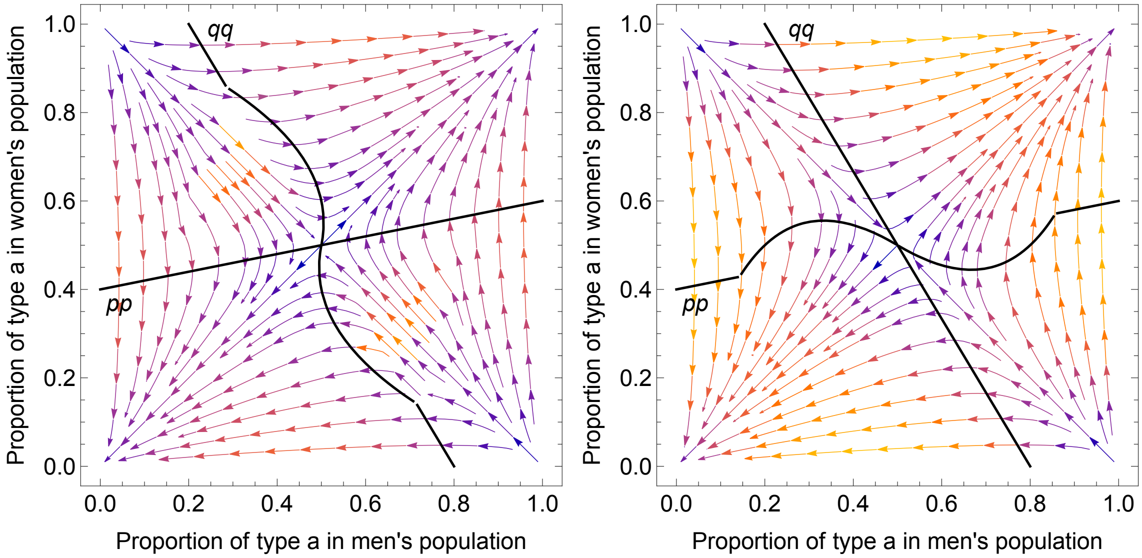
cultural dynamics is characterized by three steady states: stable states $(0,0)$ and $(1,1)$ and unstable state $(1/2,1/2)$. Hence, the long-run behavior of the cultural distribution is qualitatively similar to that obtained with heterophilic proposers (Proposition 2).

In Figure 5b we have drawn the phase diagram associated with the dynamics of (p^t, q^t) for $h_m < 1/2$ (the results are unchanged when $h_m \geqslant 1/2$). Construction of the phase diagram is formally presented in Appendix A.5. In that figure, the qq curve corresponds to the stationary locus of q^t, $qq := \left\{(p^t, q^t) \in [0,1]^2 : q^{t+1} = q^t\right\}$, and the pp line corresponds to the stationary locus of p^t, $pp := \left\{(p^t, q^t) \in [0,1]^2 : p^{t+1} = p^t\right\}$. The latter corresponds to the boundary between regions Ω_2 and Ω_3. As shown in this diagram, when there is a strong imbalance in the sex ratio in types (near the northwest corner in Ω_4 and the southeast corner in Ω_1), within each type, there is a tendency toward a more balanced sex ratio similar to the case of homophilic proposers in Section 3.1.1; those are configurations in which as many homogamous couples as possible are formed. However, as the distribution of types becomes increasingly balanced between sex, more and more heterogamous couples are formed and the majority eventually drives out the minority, as in the case of heterophilic proposers in Section 3.1.2.

Proposition 3. *Suppose transmission is perfect vertical in homogamies and oblique in heterogamies. With mixed proposing men* $(0 < h_m < 1)$ *and homophilic receiving women* $(h_w = 1)$*,* (p^t, q^t) *converges to* $(0,0)$ *if* $p^0 + q^0 < 1$*,* $(1,1)$ *if* $p^0 + q^0 > 1$*, and* $(1/2, 1/2)$ *if* $p^0 + q^0 = 1$.

3.2.2 Mixtures of heterophilic and homophilic proposers and receivers

In this section, we provide the most general setup by allowing both the populations of men and women to be mixed with homophilic and heterophilic preferences. Without loss of generality, we assume $h_m \geqslant h_w$.



(a) MOSM

(b) WOSM

Figure 6: Evolution with mixed proposers and receivers under benchmark transmission

Note. $h_m = 0.6$ and $h_w = 0.2$

We will focus our discussion on the implications for long-run cultural evolution, and the characterization of stable matching (into nine cases) is relegated to Appendix A.6.

Figures 6a and 6b illustrate the phase diagrams for the cultural evolution under MOSM and WOSM, respectively. In Appendix A.7, we provide the formal construction of the phase diagrams. In these figures, the pp and qq curves, respectively, correspond to the stationary locus of p^t and q^t. As shown in these figures—and Proposition 4 states that this is a general result—under either matching scheme, we have two stable steady states $(0, 0)$ and $(1, 1)$, and one unstable steady state $(1/2, 1/2)$. By extension, even if we allow for a selection of stable matching between MOSM and WOSM (for example, by median stable matching), cultural evolution retains the same steady states.

Proposition 4. *Suppose transmission is perfect vertical in homogamies and oblique in heterogamies. With mixed proposing men* $(0 < h_m < 1)$ *and mixed receiving women* $(0 < h_w < 1)$*,* (p^t, q^t) *converges to* $(0, 0)$ *if* $p^0 + q^0 < 1$*,* $(1, 1)$ *if* $p^0 + q^0 > 1$*, and* $(1/2, 1/2)$ *if* $p^0 + q^0 = 1$*.*

This result implies that Proposition 3 might be generalized to the presence of mixed receivers. Hence, in the presence of mixed proposers, regardless of the preferences of receivers, the long-run distribution of preferences is fully homogeneous. Put differently, as long as there is a tiny mass of heterophilic men and women, regardless of the stable matching scheme, generically, complete cultural integration is the long-run stable outcome. Even more strikingly, the respective basin of attraction of the $(0, 0)$ and $(1, 1)$ steady states are exactly the same under WOSM and MOSM even if, for some (p^t, q^t), the two mechanisms do not result in the same matching.

3.3 Summary

In summary, we would have culturally diverse stable equilibria only with homophilic proposers. Cultural homogeneity is the generic long-run outcome when some individuals are heterophilic, vertical transmission is perfect and the learning for children in heterogamies is entirely societal. When either the vertical transmission becomes imperfect or the oblique transmission incorporates the Darwinian forces, which we will explore in the two next sections, cultural homogeneity is not necessarily the long-run outcome, because evolutionary fitness consideration or cultural substitution acts as a counterforce.

4 Imitative logit and Darwinian transmission in heterogamies

So far, we have assumed that when a man and a woman with different traits are married, each of their children randomly searches for a role model in their respective gender and adopts the cultural type of the role model ("cultural parent") with probability one. In this section, we consider that the socialization of children in heterogamies is described by the imitative logit model described in Section 2.3. We will simplify the exposition by assuming that $U_{ab} = U_{ba}$ and $V_{ab} = V_{ba}$.

4.1 Homophilic proposers

Consider homophilic proposing men. Cultural evolution is characterized by the following dynamic system:

$$\begin{aligned} p^{t+1} &= \min\{p^t, q^t\} + \left[\max\{p^t, q^t\} - \min\{p^t, q^t\}\right] P^t; \\ q^{t+1} &= \min\{p^t, q^t\} + \left[\max\{p^t, q^t\} - \min\{p^t, q^t\}\right] Q^t. \end{aligned}$$

We have $p^{t+1} < p^t$ and $q^{t+1} > q^t$ when $p^t > q^t$, and $p^{t+1} > p^t$ and $q^{t+1} < q^t$ when $p^t < q^t$. The dynamic system converges to a steady state (p^*, q^*) on the line $p^* = q^*$, which is the stable set, exactly as in the benchmark model. Hence, homophilic proposers lead to cultural heterogeneity even with the additional Darwinian component of intergenerational transmission.

Proposition 5. *Suppose transmission is perfect vertical in homogamies and imitative logit in heterogamies. With homophilic proposers, from any interior initial state, (p^t, q^t) converges to (r, r) for some $r \in (0, 1)$; $\{(r, r) | r \in (0, 1)\}$ forms a stable set.*

Furthermore, note that this result holds regardless of the intergenerational transmission technologies in heterogamies. Therefore, our result that homophilic proposers lead to cultural heterogeneity (Proposition 1) is a robust prediction independent of the distribution of traits and preferences of the receivers and independent of the intergenerational transmission technologies in heterogamies, when transmission is perfect vertical in homogamies.

4.2 Heterophilic proposers

Now consider WOSM with homophilic receiving men and heterophilic proposing women. When $p^t + q^t > 1$, cultural evolution is characterized by the following dynamic system:

$$p^{t+1} = (p^t + q^t - 1) + (2 - p^t - q^t)P^t; \tag{14}$$
$$q^{t+1} = (p^t + q^t - 1) + (2 - p^t - q^t)Q^t. \tag{15}$$

The expected payoffs are

$$U_a^t = \frac{p^t + q^t - 1}{p^t} U_{aa} + \frac{1 - q^t}{p^t} U_{ab}, \text{ and } U_b^t = U_{ba} = U_{ab};$$
$$V_a^t = \frac{p^t + q^t - 1}{q^t} V_{aa} + \frac{1 - p^t}{q^t} V_{ab}, \text{ and } V_b^t = V_{ba} = V_{ab}.$$

The expected payoff differences are

$$U_a^t - U_b^t = \frac{p^t + q^t - 1}{p^t} (U_{aa} - U_{ab}) > 0;$$
$$V_a^t - V_b^t = \frac{p^t + q^t - 1}{q^t} (V_{aa} - V_{ab}) < 0.$$

Hence, the Darwinian force favors the diffusion of trait a within the population of men ($P^t > p^t$) and the diffusion of trait b within the population of women ($Q^t < q^t$). There is a positive mass of aa pairs (see Figure 2a) such that possessing trait a offers the chance to be part of a homophilic match. This generates a higher expected payoff for men, who are homophilic, but a lower expected payoff for women, who are heterophilic.

The dynamic system in the case of $p^t + q^t < 1$ is given by

$$p^{t+1} = (p^t + q^t)P^t; \tag{16}$$
$$q^{t+1} = (p^t + q^t)Q^t. \tag{17}$$

The expected payoffs are

$$U_a^t = U_{ab} = U_{ba}, \text{ and } U_b^t = \frac{q^t}{1 - p^t} U_{ba} + \frac{1 - p^t - q^t}{1 - p^t} U_{bb};$$
$$V_a^t = V_{ab} = V_{ba}, \text{ and } V_b^t = \frac{p^t}{1 - q^t} V_{ba} + \frac{1 - p^t - q^t}{1 - q^t} V_{bb}.$$

The expected payoff differences are

$$U_a^t - U_b^t = \frac{1 - p^t - q^t}{1 - p^t} (U_{ba} - U_{bb}) < 0;$$
$$V_a^t - V_b^t = \frac{1 - p^t - q^t}{1 - q^t} (V_{ba} - V_{bb}) > 0.$$

Hence, in this case, the Darwinian force favors the diffusion of trait b within the population of men and the diffusion of trait a within the population of women.

It appears that men and women are heading in opposite directions according to the Darwinian force, but we show that generically, cultural homogeneity arises. The consequences in terms of the long-run cultural composition of the society are summarized in the following proposition.

Proposition 6. *Suppose WOSM with homophilic men and heterophilic women under perfect vertical transmission in homogamies. Under imitative logit transmission in heterogamies, only* $(0,0)$, $(1,1)$ *and all states* (p^*, q^*) *on the line* $p^* + q^* = 1$ *are steady states, with* $(0,0)$ *and* $(1,1)$ *asymptotically stable and others unstable. Under Darwinian transmission in heterogamies,* (p^t, q^t) *converges in one period to* $(1, p^0 + q^0 - 1)$ *if* $p^0 + q^0 > 1$, *and to* $(0, p^0 + q^0)$ *if* $p^0 + q^0 < 1$.

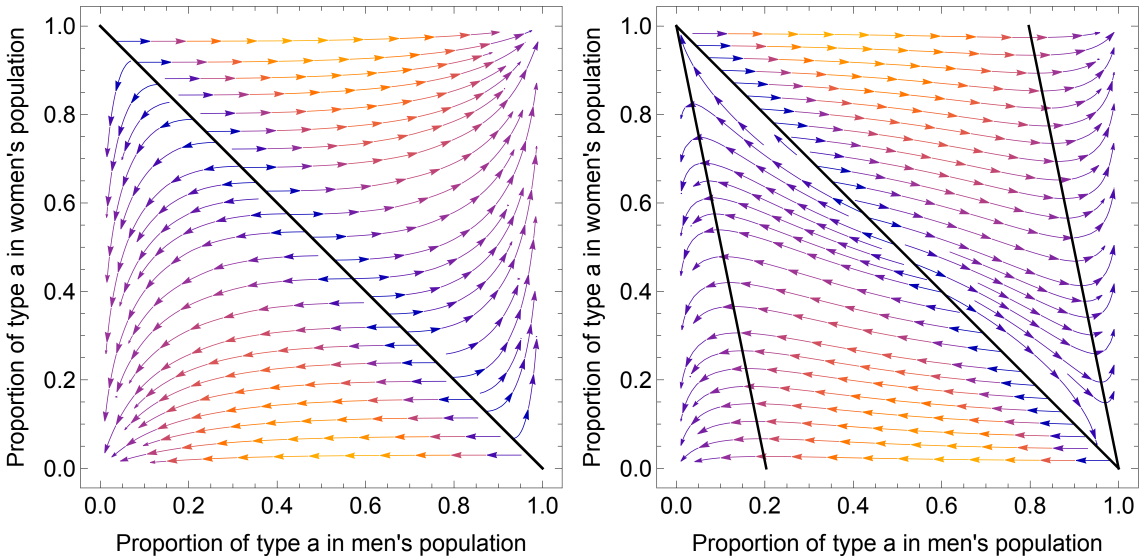
According to Proposition 6, regardless of the value of δ—provided that it is positive—cultural heterogeneity disappears in the long run. This is true even if, within the population of women, the Darwinian force acts as a counterforce. As previously discussed, when $p^t + q^t > 1$ (resp., $p^t + q^t < 1$), it favors the diffusion of trait b (resp., a) among women. As shown in Figure 7a, when δ is relatively large, this counterforce is never sufficient to overcome the effect of the rapid increase in p^t that sustains the diffusion of trait a within both populations through its positive impact on the proportion of homogamous aa couples.[15] Hence, in that case, the cultural distribution monotonically converges to $(0,0)$ or $(1,1)$. As illustrated in Figure 7b, for lower values of δ, the counterforce might lead to a temporary decrease (resp., increase) of q^t when $p^t + q^t > 1$ (resp., $p^t + q^t < 1$). It might even push the cultural distribution to a steady state on the locus $q^t = 1 - p^t$. However, any steady state on the locus is unstable in the sense that a small perturbation may result in the convergence to $(0,0)$ or $(1,1)$.

As shown in Figure 8, the value of δ also has an important impact on the speed of convergence to the stable steady states. Figure 8a shows that the convergence of p^t to 1 is very fast. It comes from the fact that when $p^t + q^t > 1$, the Darwinian component accelerates the spread of trait a within the population of men. In contrast, among women, this component acts as a counterforce that slows the diffusion of trait a. When δ is low (i.e., when the weight of the rational component relative to the societal component is high) this might lead to a temporary decrease in q^t. However, as p^t becomes big, q^t will eventually increase and converge to one. Nevertheless, as illustrated by the case $\delta = 0.2$, this convergence might be very slow (after 25 periods, q^t equals 0.2206 while it was equal to 0.2204 after four periods). Hence, the addition of an evolutionary component might lead to long-lasting discrepancies between the cultural composition of the two populations. Finally, as claimed in Proposition 6, under Darwinian transmission, q^t converges to a stationary value that is different from p^* such that these discrepancies last forever.

4.3 Mixed proposers

We now consider a mixture of homophilic and heterophilic proposing men and a unit mass of homophilic receiving women. This extension corresponds to what we have done in Section 3.2.1 for the benchmark model. An important difference with the benchmark model is that here, transmission in heterogamous

[15] See Appendix B.3 for a formal derivation of the phase diagrams shown in Figure 7.



(a) $\delta = 2$

(b) $\delta = 1$

Figure 7: Evolution with WOSM with homophilic men and heterophilic women under perfect vertical transmission in homogamies and imitative logit transmission in heterogamies

Note. $U_{aa} = U_{bb} = 4$, $U_{ab} = U_{ba} = 2$, $V_{ab} = V_{ba} = 4$, and $V_{aa} = V_{bb} = 2$.

families depends on the expected payoff associated with each trait, while a person's payoff depends on their preference. As described in Section 2.1, each man is randomly selected into a preference group once he becomes an adult. Hence, the preference group a man belongs to is decided after he has adopted a cultural trait and before the matching process takes place. Under this assumption,

$$U_\theta^t = h_m U_{\theta_1}^t + (1-h_m)U_{\theta_2}^t$$

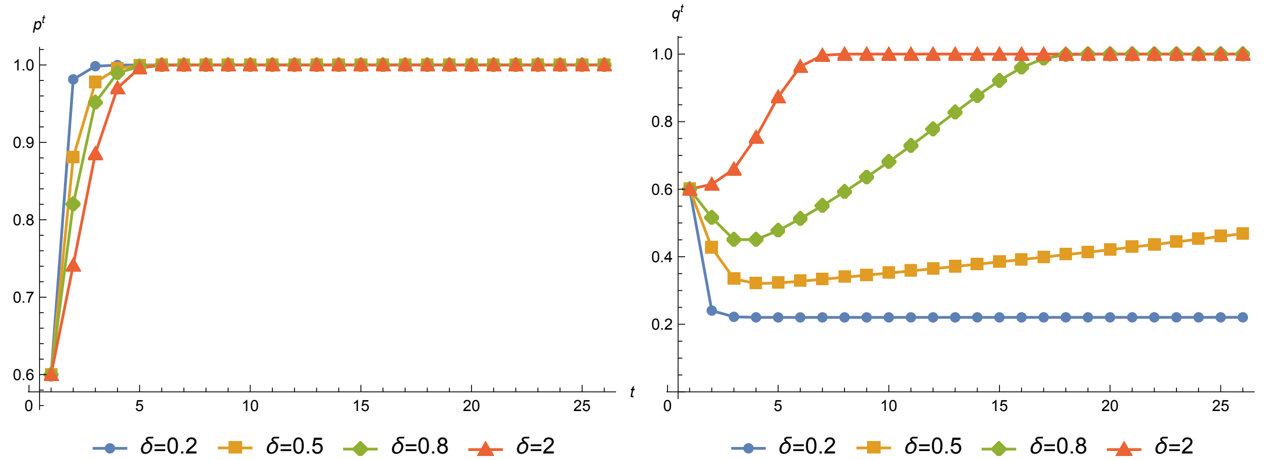
for all $\theta \in \{a, b\}$, where

$$U_{\theta_g}^t = \mu_{\theta_g a} U_{\theta_g a} + \mu_{\theta_g b} U_{\theta_g b}$$

for all $\theta \in \{a, b\}$ and $g \in \{1, 2\}$ and where $\mu_{\theta_g \theta'}$ is the mass of type-θ_g men matched with a type-θ' woman.

The stable matching is given in Lemma 1, and the evolution is given by

$$\begin{aligned} p^{t+1} &= \mu_{aa}^t + (\mu_{ab}^t + \mu_{ba}^t)P^t; \\ q^{t+1} &= \mu_{aa}^t + (\mu_{ab}^t + \mu_{ba}^t)Q^t. \end{aligned}$$



(a) Evolution of p^t (b) Evolution of q^t

Figure 8: Evolution of p^t and q^t with WOSM with homophilic men and heterophilic women under perfect vertical transmission in homogamies and imitative logit transmission in heterogamies
Note. $U_{aa} = U_{bb} = 4$, $U_{ab} = U_{ba} = 2$, $V_{ab} = V_{ba} = 4$, $V_{aa} = V_{bb} = 2$, and $p^0 = q^0 = 0.6$.

Explicitly,

$$(p^{t+1}, q^{t+1}) = \begin{cases} (q^t + (p^t - q^t)P^t, q^t + (p^t - q^t)Q^t) & \text{if} \quad (p^t, q^t) \in \overline{\Omega}_1 \\ (p^t h_m + [q^t + p^t - 2p^t h_m]P^t, p^t h_m + [q^t + p^t - 2p^t h_m]Q^t) & \text{if} \quad (p^t, q^t) \in \overline{\Omega}_2 \\ (q^t - (1-p^t)(1-h_m) + [2 - q^t - p^t - 2(1-p^t)h_m]P^t, & \\ q^t - (1-p^t)(1-h_m) + [2 - q^t - p^t - 2(1-p^t)h_m]Q^t) & \text{if} \quad (p^t, q^t) \in \overline{\Omega}_3 \\ (p^t + (q^t - p^t)P^t, p^t + (q^t - p^t)Q^t) & \text{if} \quad (p^t, q^t) \in \overline{\Omega}_4 \end{cases} \tag{18}$$

To keep exposition simple, we assume that $\overline{U} := U_{a_2b} = U_{b_2a} = U_{a_1a} = U_{b_1b} > U_{a_1b} = U_{b_1a} = U_{a_2a} = U_{b_2b} =: \underline{U}$ and $\overline{V} := V_{aa} = V_{bb} > V_{ab} = V_{ba} =: \underline{V}$. Moreover, we define $\Delta U := \overline{U} - \underline{U} > 0$ and $\Delta V := \overline{V} - \underline{V} > 0$. Also, let us consider $0 < h_m < 1$ because we analyzed the case of $h_m = 1$ in Section 4.1 and the case of $h_m = 0$ in Section 4.2. Let us first consider Darwinian transmission, in which children in heterogamies rationally choose their cultural traits.

Proposition 7. *Suppose a mixture of homophilic and heterophilic proposers and homophilic receivers under perfect vertical transmission in homogamies and Darwinian transmission in heterogamies. There is an unstable steady state* $(1/2, 1/2)$*, and cultural evolution exhibits cycles.*

The cyclic evolution is illustrated in Figure 9 in which the boundaries between the regions Ω_1, Ω_2, Ω_3, Ω_4, and the 45° line are in black, and the evolution of (p^t, q^t) from the starting point $(0.9, 0.1) \in \Omega_1$ is in blue. We can observe that, (p^t, q^t) first joins the first diagonal and then goes back and forth between Ω_4 and the first diagonal while progressively shifting to the right. The process lasts until the distribution of types reaches a point on the first diagonal that is below $(1/2, 1/2)$ and then, in the next period, (p^t, q^t) goes back to Ω_4.

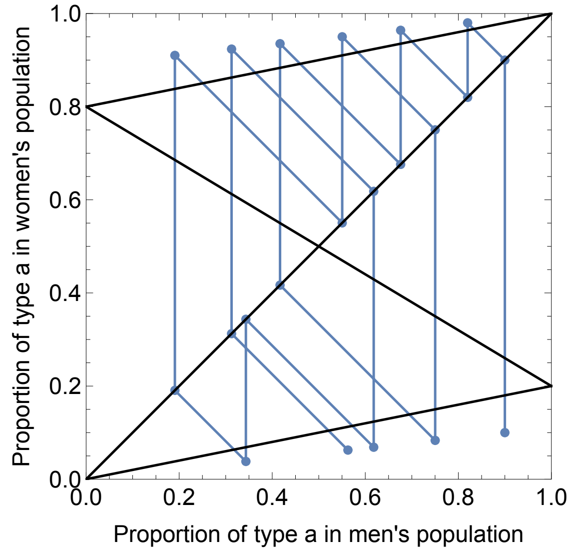


Figure 9: Evolution with heterophilic proposers under perfect vertical transmission in homogamies and Darwinian transmission in heterogamies

Note. $\overline{U} = \overline{V} = 4$, $\underline{U} = \underline{V} = 2$, $h_m = 0.3$ and $(p^0, q^0) = (0.9, 0.1)$.

To have a better understanding of the mechanisms behind this cyclical behavior, let us focus on the case $h_m < 1/2$ (depicted in Figure 9) and first consider the configuration in which $(p^t, q^t) \in \Omega_1$ (symmetric reasoning would apply to $(p^t, q^t) \in \Omega_4$). In that case, there are fewer type-a women than homophilic type-a men, such that all type-a women are matched with type-a men and some type-b women must take part in heterogamies. Since women are homophilic, they prefer to adopt trait a such that $Q^t = 1$. Now, since q^t is relatively small, this evolutionary force pushes it upward (according to equation (18), we must have $q^{t+1} = p^t$). Let us now examine the evolution of p^t, according to Lemma 1, when $q^t < p^t h_m$, all trait-b men are matched with a trait-b woman while only a proportion $h_m < 1/2$ want this. Moreover, all heterophilic men of type-a are matched with a trait-b woman—which is their preferred option—and not all homophilic trait-a men can be matched with a trait-b woman. Hence, men are, in expectation, better off when adopting trait a such that $P^t = 1$. However, since p^t is already relatively large, the evolutionary force does not induce further increases in p^t (according to equation (18), we have $p^{t+1} = p^t$). Hence, (p^{t+1}, q^{t+1}) joins the first diagonal and belongs to Ω_2 if $p^t < 1/2$ or Ω_3 if $p^t > 1/2$. So, let us now consider the configuration $(p^t, q^t) \in \Omega_3$. We know from Lemma 1 that in this case, all type-b men get their preferred matching. In contrast, some heterophilic men of type-a must be matched with a type-a women. Hence, in expectation, men are better off when adopting the trait b such that $P^t = 0$. Since p^t is relatively high (higher than $1/2$), this implies that $p^{t+1} < p^t$. We can also observe that the number of bb couples equals the number of homophilic type-b men and the number of aa couples is higher than the number of homophilic type-a men. Moreover, since $p^t > 1/2$, there are more homophilic type-a than homophilic type-b men. Hence, in the end, women have a higher chance of taking part in homogamies when they are type-a such that $Q^t = 1$ and $q^{t+1} > q^t$. Plugging $P^t = 0$ and $Q^t = 1$ into equation (18), we get $(q^{t+1}, p^{t+1}) \in \Omega_4$.

Let us now assess how the results are modified when $\delta \neq 0$. As claimed in Proposition 8, the steady states $(0,0)$ and $(1,1)$ are now locally stable, even for an arbitrarily small value of δ. Moreover, for some configurations of the parameters, those steady states might coexist with another stable one, $(1/2,1/2)$.

Proposition 8. *Suppose mixed proposing men ($0 < h_m < 1$) and homophilic receiving women ($h_w = 1$) under perfect vertical transmission in homogamies and imitative logit transmission in heterogamies.*

- *If $\delta \geqslant (1-h_m)\Delta U$, there are three steady states, $(0,0)$, $(1,1)$ and $(1/2,1/2)$; $(0,0)$ and $(1,1)$ are asymptotically stable, while $(1/2,1/2)$ is unstable.*
- *If $\delta < (1-h_m)\Delta U$, there are five steady states, $(0,0)$, $(1,1)$, $(1/2,1/2)$ and two others; $(0,0)$ and $(1,1)$ are asymptotically stable, while $(1/2,1/2)$ is either stable or unstable.*

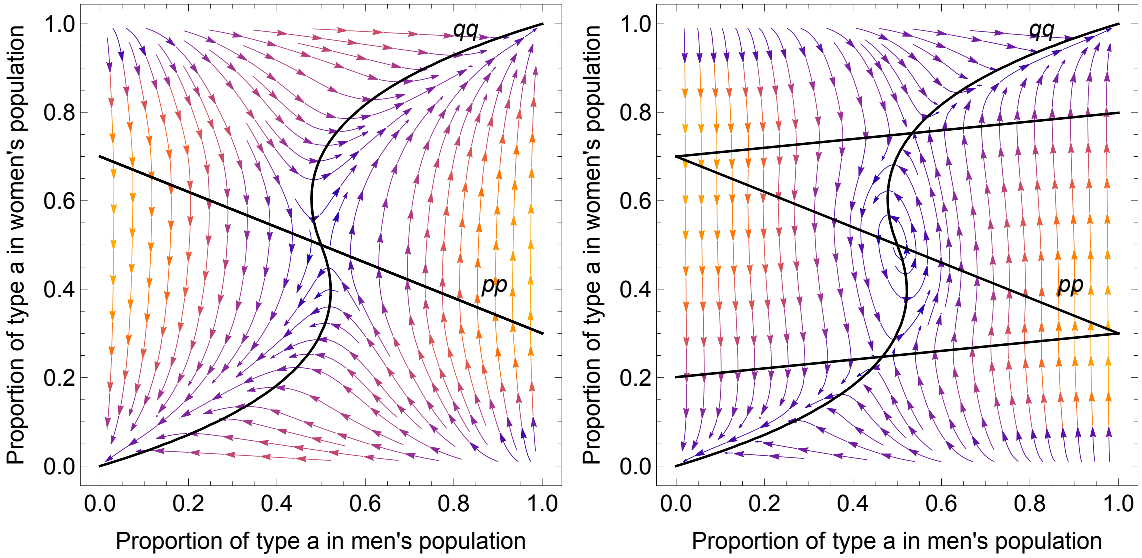
We have seen that under Darwinian transmission, evolutionary forces push the cultural distribution away from the homogeneous steady states $(0,0)$ and $(1,1)$ such that those steady states turn unstable. According to Proposition 8, when $\delta \neq 0$, imitation acts as a powerful counterforce. In particular, even for small values of δ, when there is a vast majority of type-b men and women, it moves (p^t, q^t) closer to $(0,0)$. Hence, $(0,0)$ (and symmetrically $(1,1)$) are locally asymptotically stable. For more mixed values of (p^t, q^t), cultural evolution crucially depends on parameter values. Proposition 9 more precisely characterizes the local dynamics in the neighborhood of $(1/2,1/2)$. The parametric restrictions assumed in Proposition 9 are mainly made for ease of presentation.

Proposition 9. *Suppose mixed proposing men ($0 < h_m < 1$) and homophilic receiving women ($h_w = 1$) under perfect vertical transmission in homogamies and imitative logit transmission in heterogamies. Assume $\Delta V \in \left[\Delta U, \Delta U + \frac{4\delta}{3h_m - 1}\right]$.*

- *If $h_m < 1/2$, $(1/2,1/2)$ is a saddle if $\delta \geqslant (1-h_m)\Delta U$ and a spiral source otherwise.*
- *If $h_m \geqslant 1/2$, there exists a threshold $\delta_m < (1-h_m)\Delta U$ such that $(1/2,1/2)$ is a saddle when $\delta \geqslant (1-h_m)\Delta U$; $(1/2,1/2)$ is a spiral sink when $\delta \in [\delta_m, (1-h_m)\Delta U)$; and $(1/2,1/2)$ is a spiral source when $\delta < \delta_m$.*

When $h_m < 1/2$, $(1/2,1/2)$ is either a saddle (when δ is sufficiently large) or a source. The phase diagrams that illustrate each of these situations are depicted in Figure 10. In both cases, $(0,0)$ and $(1,1)$ are the unique asymptotically stable steady states. In the first case (Figure 10a), the relative weight of imitation forces is high, such that the dynamics behavior is very close to what we had in the benchmark model (Section 3.2.1). However, in the second case (Figure 10b), when (p^0, q^0) is close to $(1/2,1/2)$, the imitation forces are counterbalanced by evolutionary forces and the cultural dynamics exhibit cycles. It is also characterized by a local indeterminacy. Indeed, the basins of attraction of $(0,0)$ and $(1,1)$ are not clearly defined and, starting from the neighborhood of $(1/2,1/2)$, the cultural distribution may indifferently converge to one or the other stable steady state.

When $h_m > 1/2$, when δ is large $(1/2,1/2)$ is a saddle. Then, as δ reduces to $(1-h_m)\Delta U$, the dynamic system undergoes a flip bifurcation, and $(1/2,1/2)$ becomes a sink. In that case, three locally stable steady states coexist: $(0,0)$, $(1,1)$, and $(1/2,1/2)$. As δ further reduces to δ_m, the dynamics undergoes a Hopf



(a) $\delta = 2$ (b) $\delta = 0.8$

Figure 10: Evolution with mixed proposers under perfect vertical transmission in homogamies and imitative logit transmission in heterogamies
Note: $\overline{U} = \overline{V} = 4$, $\underline{U} = \underline{V} = 2$, and $h_m = 0.3$.

bifurcation as $(1/2, 1/2)$ becomes a source. In the latter case, $(1/2, 1/2)$ is no longer stable, but the dynamic system around this point is cyclical. Hence, for intermediate values of δ, cultural homogeneity is locally stable, but there may also exist a locally unstable steady state $(1/2, 1/2)$.

With a mixture of homophilic and heterophilic proposers and homophilic receivers, permanent cycles arise with Darwinian transmission (Proposition 7) and cultural homogeneity and heterogeneity may both be stable with imitative logit transmission (Propositions 7 and 8). These results will continue to hold when there is also a mixture of homophilic and heterophilic receivers.

5 Imperfect vertical transmission in homogamies

We now consider imperfect vertical transmission in homogamies as described in Section 2.3. Given the transmission technology and equations (4)) and (5), cultural evolution is characterized by

$$p^{t+1} = p^t + \mu_{aa}^t d(p^t)(1 - p^t) - \mu_{bb}^t d(1 - p^t)p^t;$$

$$q^{t+1} = q^t + \mu_{aa}^t d(q^t)(1 - q^t) - \mu_{bb}^t d(1 - q^t)q^t.$$

An interpretation of the transition is as follows. Each man is subject to societal impact through oblique transmission and possesses trait a with probability p^t, except that the probability increases by $d(p^t)(1-p^t)$ when vertical transmission (of trait a) is successful in aa homogamies, and the probability decreases by $d(1 - p^t)p^t$ when vertical transmission (of trait b) is successful in bb homogamies.

5.1 Homophilic proposers

Start with homophilic proposers. Suppose $p^t \geqslant q^t$. Using the properties of the stable matching (see Figure 1a), cultural evolution is characterized by

$$\begin{aligned} p^{t+1} - p^t &= q^t d(p^t)(1 - p^t) - (1 - p^t) d(1 - p^t) p^t; \\ q^{t+1} - q^t &= q^t d(q^t)(1 - q^t) - (1 - p^t) d(1 - q^t) q^t. \end{aligned}$$

We show that only (p^*, q^*) such that $p^* = q^* := r$ can be a steady state, and in addition, r satisfies

$$r(1 - r)[d(r) - d(1 - r)] = 0.$$

Hence, when $d(\cdot)$ is strictly decreasing, the solutions are 0, 1/2, and 1. This equation also clarifies that when homogamies have perfect vertical transmission, $d(r) = 1$ for all r, any (r, r) is a steady state (Propositions 1 and 5). To establish the global asymptotical stability of $(1/2, 1/2)$, we use cultural substitutability and find a Lyapunov function.

Proposition 10. *Suppose homophilic proposers under culturally substitutable imperfect vertical transmission in homogamies and oblique transmission in heterogamies. Let $d(\cdot)$ be differentiable. There is a unique globally asymptotically stable steady state $(1/2, 1/2)$.*

5.2 Heterophilic proposers

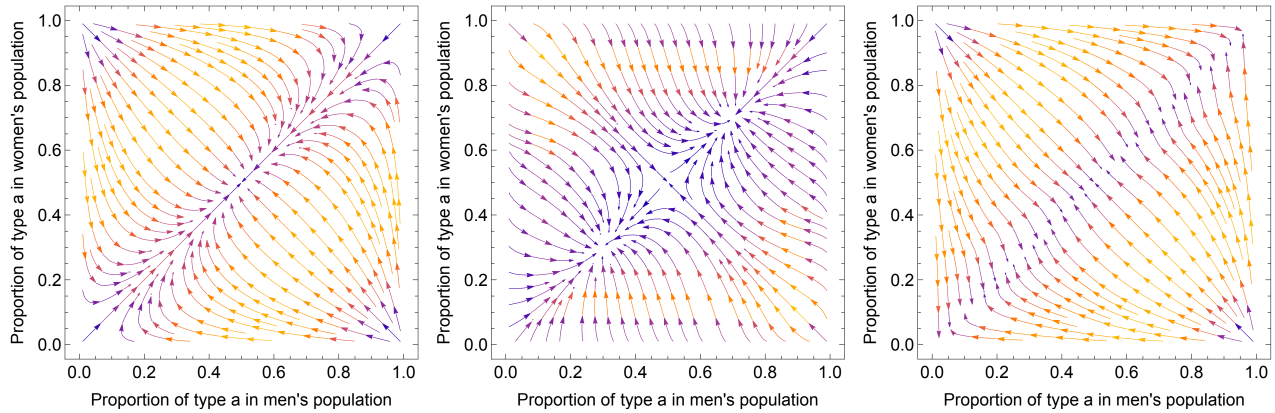
Consider heterophilic proposers. Regardless of p^t and q^t, given the properties of the stable matching described in Figure 2, cultural evolution is characterized by

$$\begin{aligned} p^{t+1} - p^t &= (p^t + q^t - 1) d(1 - p^t) p^t; \\ q^{t+1} - q^t &= (p^t + q^t - 1) d(1 - q^t) q^t. \end{aligned}$$

From the equations above, we can see that the system reaches a steady state only when $p^t + q^t = 1$ or when $p^t = q^t = 0$ or $p^t = q^t = 1$. The system moves toward $(0, 0)$ when $p^t + q^t < 1$ and toward $(1, 1)$ when $p^t + q^t > 1$.

Proposition 11. *Suppose heterophilic proposers under culturally substitutable imperfect vertical transmission in homogamies and oblique transmission in heterogamies. (p^t, q^t) converges to $(1, 1)$ if $p^0 + q^0 > 1$, $(0, 0)$ if $p^0 + q^0 < 1$, and (p^0, q^0) if $p^0 + q^0 = 1$.*

Only culturally homogeneous states $(0, 0)$ and $(1, 1)$ are stable, even under imperfect vertical transmission that satisfies cultural substitutability. This result implies that a positive mass of homophilic individuals in each period is needed to avoid complete cultural homogeneity in the long run.



(a) $(1/2, 1/2)$ stable (b) $(1/2, 1/2)$ saddle point (c) $\{(r, r) : r \in (0, 1)\}$ stable set

Figure 11: Evolution with a mixture of homophilic and heterophilic proposers under imperfect vertical transmission in homogamies and oblique transmission in heterogamies

5.3 Mixed proposers

Finally, consider the case in which there is a mixture of homophilic and heterophilic proposers. Characterization of cultural evolution depends on stable matching.

Proposition 12. *Suppose mass $h_m \in (0, 1)$ of homophilic proposers under culturally substitutable imperfect vertical transmission in homogamies and oblique transmission in heterogamies. Let $d(\cdot)$ be differentiable. $(1/2, 1/2)$ is an asymptotically stable steady state if and only if $h_m > d(1/2)/[d(1/2) - d'(1/2)/2]$. When there is mass $h_w \in (0, 1]$ of homophilic receivers, only (r, r), where $0 < r < 1$, can be a steady state; when receivers are all heterophilic, $(0, 0)$ and $(1, 1)$ are asymptotically stable steady states.*

When the condition for h_m in the proposition does not hold, $(1/2, 1/2)$ is a saddle point. It can be a saddle point or a stable point (instead of an unstable point) regardless of the environment, because the system with an initial state (p^0, q^0) such that $p^0 + q^0 = 1$ always converges to $(1/2, 1/2)$ (mathematically, there is a negative eigenvalue for the Jacobian matrix of the evolution equations).

We can show that only a state with equal masses of type-a men and women, (r, r), can be a steady state. By symmetry, $(1-r, 1-r)$ is a steady state when so is (r, r), and they also share the stability property. The number of stable steady states will depend on the shape of $d(\cdot)$. More specifically, the set of steady states (r, r) and $(1 - r, 1 - r)$, $r < 1/2$, will be the set of solutions to

$$d(1 - r) = \frac{h_m(1 - r)}{1 - 2r + rh_m} d(r), \quad r \in (0, 1/2).$$

There could be one stable steady state $(1/2, 1/2)$ (Figure 11a). There could be two stable steady states (r, r) and $(1 - r, 1 - r)$, where $r < 1/2$ (Figure 11b). There could be many stable steady states. In the extreme case, in which the equation holds for all $r \in (0, 1/2)$, $\{(r, r) : 0 < r < 1\}$ forms a stable set (Figure 11c). Depending on the functional form of $d(\cdot)$, even when $(1/2, 1/2)$ is a stable steady state, there can be additional stable steady states.

Though any (r, r), where $0 < r < 1$, is a culturally heterogeneous steady state, $(1/2, 1/2)$ can be thought of as a special case—the state in which there is no dominant or dominated trait. In other states, there is asymmetry that results in the identification of majority and minority groups. The necessary condition for the resilience of the minority group is also clarified in the propositions: It is necessary to have positive measures of homophilic proposers *and* receivers of a minority group for the group not to be extinct. The existence of homogamies *and* minority groups' higher direct socialization efforts and probabilities are both important in preserving a cultural trait.

6 Implications

6.1 Implications for gender norms

6.1.1 Patriarchal versus matriarchal societies

A growing literature highlights the existence of systematic cultural differences between patriarchal and matriarchal societies, especially in terms of gender differences in preferences for competition and risk attitudes (Andersen et al., 2008; Gneezy et al., 2009; Gong and Yang, 2012; Andersen et al., 2013; Gong et al., 2015; Pondorfer et al., 2017; Liu and Zuo, 2019; Brulé and Gaikwad, 2021). The two types of societies widely differ in their social structure, especially in dimensions related to the specific role of men and women in terms of land ownership, power over a household's monetary decisions, lineage norms, and household residence location (patrilocality versus matrilocality). All of these differences must have an impact on the respective role of men and women in the marriage market. For instance, in patriarchal societies, ownership over resources and patrilocality confer to men a dominant position in the marriage market, while the reverse might be true in matriarchal societies.[16] Then, we can wonder whether this difference alone—abstracting from other discrepancies in the social structure—might be the source of divergent cultural compositions or explain the persistence of a gender gap in attitudes.

From a conceptual point of view, the relative positions of men and women in the marriage market might be captured by the choice between MOSM and WOSM. If we consider that only stable matchings are feasible and if men are in a position to choose a specific stable matching, they will opt for MOSM. In contrast, women would choose WOSM. In our benchmark model we show that when one side of the market has homogeneous preferences and the other side is heterogeneous, the choice of MOSM or WOSM indeed influences the long-run cultural distribution. However, in both configurations the stationary distribution is the same within the populations of men and women ($p^* = q^*$), so we cannot talk about gender differences in cultural attitudes. Moreover, in the general model in which we introduce a heterogeneity of preferences within both populations, the long-run cultural distribution is no longer affected by the choice of a specific stable matching. Nevertheless, in the imitative logit model presented in Section 4 (with homophilic preferences on one side of the market and heterophilic preferences on the other), we have shown that when the

[16] According to Gneezy et al. (2009), the age difference at marriage is significantly higher among patriarchal Massai than among matriarchal Khasi (i.e., Massai women marry older men on average). This might be viewed as a consequence of the difference in the status of men and women in the marriage market in the two societies.

strength of the evolutionary force is sufficiently high, the cultural distribution among women might differ from the cultural distribution among men for a very long period (see Figure 8). This finding offers a way to rationalize a possible impact of the relative position of men and women in the marriage market—which could stem from the deeply rooted structure of the society—on the gender gap in attitudes.

6.1.2 Gender imbalance

A growing empirical literature emphasizes the impact of historical sex ratios on contemporaneous cultural values, especially those related to the definition of gender roles.[17] In this section we introduce a skewed sex ratio in our benchmark model. We show how it might influence equilibrium matching and, in turn, the cultural transmission process. Finally, we provide an illustration of how these mechanisms might rationalize the long-lasting impact of a temporary shock on sex ratio on the cultural composition of a society.

Let us first introduce gender imbalance in the model. Regarding the intergenerational transmission process, we make the same assumptions as in the benchmark model (Section 3). In addition, we assume that at the beginning of each time t, and before matching takes place, a mass λ of adult males arrive in the economy. Those incoming men do not have a well defined culture. They randomly pick a cultural model within the population of adult males already present in the economy and adopt the trait of this model with probability one. Hence, after this arrival, there is a mass $p^t(1+\lambda)$ (resp. $(1-p^t)(1+\lambda)$) of type-a men (resp. type-b men) in the economy. To make things interesting, we assume that men and women have antagonistic preferences. Without loss of generality, we consider that men have homophilic and women have heterophilic preferences. In the next point, we describe the stable matching obtained in this setting.

Ashlagi et al. (2017) show that even the slightest imbalance between the number of individuals on either side of the market can yield a unique stable matching that is favorable to individuals on the short side. The following lemma establishes that this is indeed the case in our setting. More specifically, even for an arbitrarily small λ, there exists a unique stable matching at which as many heterogamous couples as possible are formed. Hence, compared with the benchmark case ($\lambda = 0$), this matching is close to WOSM and in sharp contrast to MOSM.

Lemma 2. *Define the following functions: $\phi_1(p) := 1-(1+\lambda)p$ and $\phi_2(p) := (1+\lambda)(1-p)$. In the market with a mass $p(1+\lambda)$ of type-a men, a mass $(1-p)(1+\lambda)$ of type-b men, a mass q of type-a women and a mass $(1-q)$ of type-b women, the unique stable matching $\mu = (\mu_{aa}, \mu_{ab}, \mu_{ba}, \mu_{bb})$ with homophilic men and heterophilic women is*

$$\begin{cases} (0, p(1+\lambda), q, 1-q-p(1+\lambda)) & \text{if} \quad q < \phi_1(p), \\ (0, 1-q, q, 0) & \text{if} \quad q \in [\phi_1(p), \phi_2(p)], \\ (q-(1+\lambda)(1-p), 1-q, (1+\lambda)(1-p), 0) & \text{if} \quad q > \phi_2(p). \end{cases}$$

[17] See Gay (2019); Grosjean and Khattar (2019); Teso (2019); Alix-Garcia et al. (2020); Baranov et al. (2021).

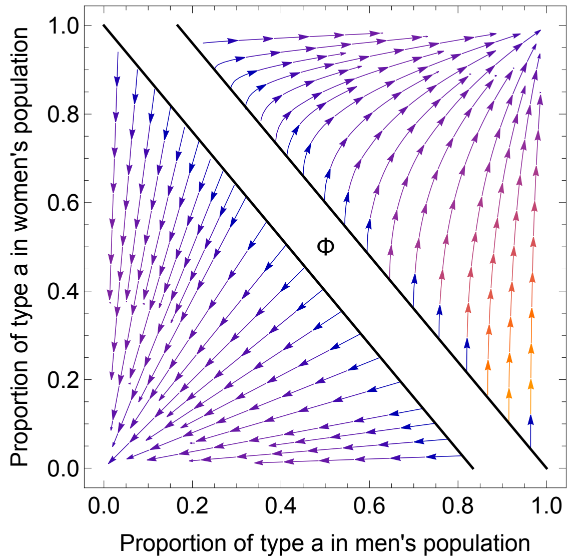


Figure 12: Evolution under benchmark transmission and gender imbalance

Note. $\lambda = 0.2$.

We assess the consequences of gender imbalance on cultural evolution. From Lemma 2, we get that

$$p^{t+1} = \begin{cases} p^t\left[q^t + p^t(1+\lambda)\right] & \text{if} \quad q^t < \phi_1(p^t) \\ p^t & \text{if} \quad q^t \in \left[\phi_1(p^t), \phi_2(p^t)\right] \\ p^t + (1-p^t)\left[q^t - (1+\lambda)(1-p^t)\right] & \text{if} \quad q^t > \phi_2(p^t) \end{cases} \tag{19}$$

$$q^{t+1} = \begin{cases} q^t\left[q^t + p^t(1+\lambda)\right] & \text{if} \quad q^t < \phi_1(p^t) \\ q^t & \text{if} \quad q^t \in \left[\phi_1(p^t), \phi_2(p^t)\right] \\ q^t + (1-q^t)\left[1 - (1+\lambda)(1-p^t)\right] & \text{if} \quad q^t > \phi_2(p^t) \end{cases} \tag{20}$$

Cultural evolution is summarized in Proposition 13 and illustrated by Figure 12.[18] In the figure, the region Φ is defined as

$$\Phi := \left\{(p^t, q^t) \in [0,1]^2 : q^t \in \left[\phi_1(p^t), \phi_2(p^t)\right]\right\}.$$

In that region, there are more type-a men than type-b women and more type-b men than type-a women. Hence, all women can be matched with a man of the opposite type. As a consequence, there is no homogamous couple and the initial distribution of traits perpetuates over time. Outside this region, $(0,0)$ and $(1,1)$ are stable. Specifically, for any (p^0, q^0) such that $q^0 < \phi_1(p^0)$ (resp., $q^0 > \phi_2(p^0)$), the distribution of cultural traits converges toward $(0,0)$ (resp., $(1,1)$).

Proposition 13. *Suppose homophilic proposers and heterophilic receivers under benchmark transmission. Φ is a stable set. Moreover, (p^t, q^t) converges to $(1,1)$ if $q^0 < \phi_1(p^0)$ and $(0,0)$ if $q^0 > \phi_2(p^0)$.*

Note that even for arbitrarily small values of λ, cultural evolution is radically different from the $\lambda = 0$ case. In particular, under MOSM, when $\lambda = 0$ the steady states are characterized by cultural diversity (see

[18]The proof of Proposition 13 is omitted, since it can be directly deduced from inspection of equations (19) and (20).

Proposition 1); when λ is positive but very close to 0, the steady states exactly correspond to the ones obtained under WOSM (see Proposition 2) and are characterized by cultural homogeneity. As discussed below, these features offer a rationale for why an unbalanced sex ratio in the distant past could influence the contemporaneous cultural composition of a society.

Consider two countries that only differ in terms of sex ratio. The initial cultural distribution (p^0, q^0) is the same in both countries. There is no gender imbalance in Country 1 ($\lambda_1 = 0$), but there are more men than women in Country 2 ($\lambda_2 > 0$, such that the set Φ_2 is not empty). Assume also that in both countries, MOSM is selected and that $(p^0, q^0) \notin \Phi_2$. Suppose (p^0, q^0) is located below the region Φ_2. Under these assumptions, in Country 1, the cultural distribution converges to a point on the first diagonal $(p_1^*, q_1^*) = (r^*, r^*)$; in Country 2, it converges to $(p_2^*, q_2^*) = (0, 0)$. Now, if later in time, gender imbalance disappears in Country 2 such that $\lambda_2 = \lambda_1 = 0$, since $(0, 0)$ is on the first diagonal, it would not have any consequences for the cultural composition of Country 2. In the end, even if initial conditions were the same in the two countries, temporary differences in sex ratio have a long-lasting impact on the cultural composition of each country.

As already noted, this path dependence property might be related in an interesting way to recent empirical findings. In particular, Grosjean and Khattar (2019) and Baranov et al. (2021) show that male-biased sex ratio—which originated in the British policy of sending convicts to Australia—had persistent effects on the culture, and in particular on gender role attitudes or the extent of masculinity norms, even though gender balance was restored after the transportation of convicts stopped.[19] We propose a new channel of persistence of historical gender imbalance on culture. In our model, the sex ratio influences the matching pattern (Ashlagi et al., 2017), which, in turn, impacts the intergenerational transmission process and finally the long-run distribution of cultural traits.

6.2 Cultural integration and preservation

6.2.1 Effects of government

In the last 2,000 years, many northern nomadic groups—e.g., Xianbeis, Mongolians, and Manchurians—conquered and governed, for an extended time period, the heartland of China (*Zhongyuan*) inhabited by Han Chinese. Their population sizes were similarly small compared with the Han Chinese, but they differed in their governing policies toward ethnic intermarriage and integration. Xianbeis have been genetically and culturally integrated with the Han and other ethnicities due to their intermarriage policies promoted and practiced by their governing bodies, and the Mongolians and Manchurians have preserved their cultural traditions and identities partly due to the governing dynasties' policies against intermarrying with Han Chinese.

Xianbeis' integration with the Han Chinese is an example of how cultural integration can be achieved with a small group of heterophilic elites. The Northern Wei dynasty established by the Tuoba clan of the Xianbei ethnic group, who were originally from northern Mongolia and Siberia, ruled northern China

[19] Gay (2019); Teso (2019); and Alix-Garcia et al. (2020) reach a similar conclusion with respect to the impact of a female-biased sex ratio caused by the transatlantic slave trade in Sub-Saharan Africa, the War of the Triple Alliance in Paraguay, and World War I in France, respectively.

from 385 to 535 AD (Liu, 2020).[20] As the Northern Wei unified northern China around 439, the emperors' desire for Han Chinese institutions and cultures grew. The Northern Wei started to arrange for local Han Chinese elites to marry daughters of the Xianbei Tuoba royal family in the 480s (Watson, 1991).[21] More than 50% of Tuoba Xianbei princesses of the Northern Wei were married to southern Han Chinese men from the imperial families and aristocrats from southern China of the southern dynasties who defected and moved north to join the Northern Wei. The Sinicization was thorough: The royal families moved to central China and adopted Chinese surnames, all Xianbei officials were forced to speak and write Chinese, and Xianbei family and imperial traditions were abandoned for Chinese traditions. Other nomadic groups of the time—e.g., Qiang, Xiongnu (Huns), and Rouran—also joined the ethnic integration. With the rise of nomadic groups and the collapse of weak Han Chinese rule in northern China, due to politically encouraged ethnic intermarriages, this was one of the biggest—if not the biggest—periods of cultural integration in Chinese history. Nowadays, these groups do not have separate cultural identities, but they infused the genetic makeup of northern Chinese (Dien and Knapp, 2020).

In contrast, Mongolian and Manchurian cultural and ethnic identities have been preserved by politically motivated homophilic policies of their governing elites. Mongolians' Yuan dynasty (1271 to 1368) and Machurians' Qing dynasty (1644 to 1912) conquered China and governed from Beijing. Both dynasties adopted ethnic tier systems in which the Han Chinese were treated as inferior to the governing ethnicities in terms of political and economic rights (Franke and Twitchett, 1994; Peterson, 2002).[22] Intermarriage was also not encouraged as a result of these tiers, if not completely banned. The governing body maintained their non-Han blood (though the Manchurians intermarried with Mongolians during the Qing dynasty). As a result, they are officially recognized ethnic minorities in modern China. Admittedly, many other factors have contributed to their cultural preservation (Mongolians have an independent nation-state and a large autonomous region within China, and Manchurians' governance is so recent that its longer-term cultural implications are still evolving). Nonetheless, intermarriage policies and political and economic rules that affected intermarrying incentives steered them away from Xianbei-style cultural integration.

6.2.2 Effects of religion

Religion is a frequent barrier to intermarriage. It frequently serves as an important base for mate selection (Marcson, 1951). Profound values are attached to religious group membership, and such membership exercises strong control over marital behavior, which renders religious endogamy prescriptive. For example, Orthodox Judaism upholds historic Jewish attitudes toward intermarriage and discourages intermarriage. Intermarriage is considered to be a deliberate rejection of Judaism, and consequently an intermarried person is often cut off from the Orthodox Jewish community; see Bisin and Verdier (2000) for a discussion. As a result, the Orthodox Jews are able to preserve their distinctive culture.

[20] *Mulan* is believed to be based on a Northern Wei Xianbei heroine who joined the army for her father.

[21] The subsequent royal families governing Sui and Tang dynasties (581 to 907 AD) were from this elite group (the so-called *Guanlong elites*, named after the region they governed from). Both the Sui dynasty's Yang royal family and the Tang dynasty's Li royal family had maternal Xianbei lineages.

[22] Yuan's priority order was Mongolians, ethnic groups in western China, northern Chinese, and southern Chinese. Qing politically and economically favored Manchurian and Mongolian elites (Eight Banners).

Many parts of Southeast Asia have been continuously settled by Chinese for several centuries. In Indonesia and Malaysia, the practice of Islam has been an important expression of ethnic and national identity for natives and it forms a strong obstacle to intermarriage between natives and Chinese (Silcock, 1963; Murray, 1968; Edmonds, 1968). As a result, Chinese still retain their names and languages, and continue to identify, generation after generation, as Chinese in Indonesia and Malaysia. In contrast, in Thailand, Buddhism is the main religion, which is arguably more permissive and tolerant of intermarriage (Skinner, 2008). Chinese minorities assimilated to the host culture by adopting Thai language and names.

These are examples in which preference for homophily is not independently distributed among cultural groups. Let us look at the implications of the model in which the two cultural groups differ in their homophilic preferences. To align with previous examples, assume that all trait-a men and women are homophilic, but we do not make any assumption regarding the distribution of preferences among trait-b individuals. It is easy to deduce that the unique stable matching corresponds to the one obtained when all proposers are homophilic (see Section 3.1.1). To see this, it is sufficient to note that since all trait-a men and women are homophilic, in any stable matching, as many aa couples as possible must be formed such that the mass of aa couples must be $\min\{p^t, q^t\}$. We know that regardless of the transmission we consider, the long-run steady state will be characterized by cultural diversity (see Propositions 1, 5, and 10). These results are consistent with the evidence presented whereby 1) the religious or ethnic minority groups that are less permissive to intermarriage are more likely to preserve their cultures, and 2) the religious or ethnic majority group that is less permissive to intermarriage helps the minorities preserve their cultures.

7 Conclusion

We demonstrate that joint consideration of marital preferences, matching markets, and intergenerational transmission technologies is required for a more complete understanding of cultural evolution. Under perfect familial transmission in homogamies, with the presence of a small mass of heterophilic individuals, cultural homogeneity is the generic long-run outcome. With the addition of rational evolutionary considerations in cultural transmission in heterogamies, there may exist cycles or spirals of acculturation and preservation that sustain extended periods of cultural heterogeneity. When familial transmission in homogamies is not perfect, cultural heterogeneity arises but is sustained only when not all individuals are heterophilic; the resilience of cultural traits relies on both the socialization efforts of the minority families to pass on their traits and the homophilic marital preferences of some agents of both genders. Additional transmission technologies and matching mechanisms can be considered in our framework, and they may generate additional insights into the evolution of cultural traits.

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Online appendices

A Omitted details with benchmark transmission

A.1 Proof of Proposition 1

Proof of Proposition 1. Suppose $p^t \geqslant q^t$. Cultural evolution is characterized by equations (6) and (7). When $p^0 = q^0$, we have $p^t = q^t$ for any t. Consider (p^0, q^0) that satisfies $0 \leqslant q^0 < p^0 \leqslant 1$ with either the first or the last inequality being strict or both. By subtracting equation (7) from equation (6), we have $p^{t+1} - q^{t+1} = (p^t - q^t)^2$. Since $0 < p^0 - q^0 < 1$, we have $\lim_{t\to\infty}(p^t - q^t) = 0$. In other words, $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t = r^*$, for some $r^* \in [0, 1]$.

By the same logic, we can prove similar results for the case $p^t < q^t$. For (p^0, q^0) that satisfies $0 \leqslant p^0 < q^0 \leqslant 1$ with either the first or the last inequality being strict or both, we have $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t = r^*$, or some $r^* \in [0, 1]$.

The steady state $(1, 0)$ is unstable because if $(p^0, q^0) = (1, \varepsilon)$ for some arbitrarily small $\varepsilon > 0$, $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t = 1$. Similarly, $(0, 1)$ is unstable.

Now consider (r^*, r^*) for some $r^* \in [0, 1]$. For any arbitrarily small $\varepsilon > 0$ and $\delta > 0$, if $(p^0, q^0) = (r^* + \varepsilon, r^* + \delta)$, then (p^0, q^0) cannot be either $(0, 1)$ or $(1, 0)$. According to the above analysis, $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t$. Hence, $\{(r^*, r^*)|r^* \in [0, 1]\}$ constitutes a stable set of steady states. □

A.2 Proof of Proposition 2

Proof of Proposition 2. Let us focus on the case $p^t + q^t > 1$. The dynamic system is characterized by equation (8) and equation (9), which imply that $p^{t+1} > p^t$, $q^{t+1} > q^t$. Consider the region T_ε in which $p^t + q^t \geqslant 1 - \varepsilon$, for some arbitrarily small $\varepsilon > 0$. T_ε is a compact set since it is closed and bounded. For any $(p^t, q^t) \in T_\varepsilon$, we have $|(p^t, q^t), (1, 1)| > |(p^{t+1}, q^{t+1}), (1, 1)|$. Hence, the dynamics is a contraction mapping in the compact set T_ε, and by the contraction mapping theorem, (p^t, q^t) converges to $(1, 1)$ as time approaches infinity. Hence, $(1, 1)$ is attracting. Also, since the distance between (p^t, q^t) and $(1, 1)$ is monotonically decreasing in t, $(1, 1)$ must be stable. Therefore, it is asymptotically stable. Since ε is arbitrarily small, we can say that for any (p^0, q^0) such that $p^0 + q^0 > 1$, $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t = 1$.

By using the same logic, we can prove that for any (p^0, q^0) that satisfies $p^0 + q^0 < 1$, $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t = 0$. Also, $(0, 0)$ is asymptotically stable.

Now consider a steady state $(r^*, 1 - r^*)$ for some $r^* \in [0, 1]$. For any arbitrarily small $\varepsilon > 0$, if $(p^0, q^0) = (r^* + \varepsilon, 1 - r^*)$, $p^0 + q^0 > 1$. According to the above analysis, $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t = 1$. Hence, any steady state $(r^*, 1 - r^*)$ for $r^* \in [0, 1]$ is unstable. □

A.3 Proof of Remark 1

Proof of Remark 1. Suppose $p^0 + q^0 > 1$. Under MOSM, the steady state is some $(p^*, q^*) \in (0, 1)^2$ such that $p^* + q^* = 1$. Men's average payoff is $p^* U_{aa} + (1 - p^*) U_{bb}$. Under WOSM, the steady state is

$(p^*, q^*) = (1, 0)$, and men's payoff is U_{aa}. When $U_{aa} > U_{bb}$ (resp., $U_{aa} < U_{bb}$), men are strictly better off (worse off) under MOSM than under WOSM.

Under MOSM, women's average is $p^* V_{aa} + (1-p^*) V_{bb}$. Under WOSM, the steady state is $(p^*, q^*) = (1, 0)$, so women's payoff is V_{aa}. When $V_{aa} > V_{bb}$ (resp., $V_{aa} < V_{bb}$), women are strictly better (resp., worse off) under MOSM than under WOSM. □

A.4 Proof of Lemma 1

Proof of Lemma 1. Because women are homophilic, at any stable matching, there is a maximum feasible mass of couples of homophilic men and women who have the same cultural type. Hence, if $p^t h_m \geqslant q^t$, every type-a woman is matched with a type-a man; and if $(1 - p^t) h_m \geqslant 1 - q^t$ ($\Leftrightarrow q^t \geqslant 1 - h_m + p^t h_m$), every type-$b$ woman is matched with a type-b man. Otherwise, some type-a and some type-b women are matched with men of the opposite type. Hence, we must successively address the following three cases with a (i) small, (ii) intermediate, and (iii) large mass of type-a women: (i) $q^t \leqslant \pi^t_{a_1} = p^t h_m$; (ii) $q^t \in (\pi^t_{a_1}, 1 - \pi^t_{b_1}) = (p^t h_m, p^t h_m + 1 - h_m)$; and (iii) $q^t \geqslant 1 - \pi^t_{b_1} = p^t h_m + 1 - h_m$.

In Case (i), q^t is smaller than $\pi^t_{a_1}$: The mass of type-a women is smaller than the mass of homophilic type-a men. Under men-optimal stable matching, we must have mass q^t of aa couples. As all men of type a are matched with women of type a, we must also have mass $p^t - q^t$ of ab couples and mass $1 - p^t$ of bb couples. This case corresponds to $(p, q) \in \Omega_1$.

In Case (ii), $\pi^t_{a_1} = p^t h_m < q^t$ and $\pi^t_{b_1} = (1 - p^t) h_m < 1 - q^t$. Because women are homophilic, at any stable matching, we must have at least mass $p^t h_m$ of aa couples and at least mass $(1 - q^t) h_m$ of bb couples. The rest of the market consists of mass $(1 - h_m) p^t$ of type-a men who prefer type-b women, mass $(1 - h_m)(1 - p^t)$ of type-b men who prefer type-a women, mass $q^t - p^t h_m$ of type-a women who prefer type-a men, and mass $1 - q^t - (1 - p^t) h_m$ of type-b women who prefer type-a men. Depending on whether some type-a heterophilic men marry type-a women, we have to consider two different subcases.

First, suppose $(1 - h_m)(1 - p^t) > q^t - p^t h_m$, which implies $q^t < 1 - p^t(1 - h_m) - h_m(1 - p^t)$. In this case, there are more remaining women of type b than remaining men of type a and more remaining men of type b than remaining women of type a. Hence, we must have mass $q^t - p^t h_m$ of ba couples, mass $(1 - h_m) q^t$ of ab couples, and mass $1 - h_m - (q^t - p^t h_m) - (1 - h_m) p^t$ of additional bb couples. Hence, we have in total mass $1 - q^t - p^t(1 - h_m)$ of bb couples. This subcase corresponds to $(p, q) \in \Omega_2$.

Second, suppose $(1 - h_m) p^t > 1 - q^t - (1 - p^t) h_m$, which implies $q^t > 1 - p^t(1 - h_m) - h_m 1 - p^t)$. In this case, there are more remaining women of type a than remaining men of type b and more remaining men of type a than remaining women of type b. Hence, we must have mass $1 - q^t - (1 - p^t) h_m$ of ab couples, mass $(1 - h_m)(1 - p^t)$ of ba couples and mass $1 - h_m - [1 - q^t - (1 - p^t) h_m] - (1 - h_m)(1 - p^t)$ additional aa couples. Hence, at the end, we have in total mass $q^t - (1 - p^t)(1 - h_m)$ of aa couples. This subcase corresponds to $(p, q) \in \Omega_3$.

In Case (iii), $\pi^t_{b_1}$ is larger than $1 - q^t$: There are more type-b homophilic men than type-b women. Since women are homophilic, at any stable matching, we must have mass $1 - q^t$ of bb couples. As all of type-b women are matched with type-b men, we must also have mass $q^t - p^t$ of ba couples and mass p^t of aa couples. This case corresponds to $(p, q) \in \Omega_4$.

The three cases (four subcases in total) provide a complete characterization of MOSM when proposers mix homophilic and heterophilic men and receivers are homophilic women. □

A.5 Proof of Proposition 3

Construction of the phase diagram in Figure 5b. Let us focus on regions Ω_1 and Ω_2, because symmetric reasoning would apply for regions Ω_3 and Ω_4. We also assume that $h_m < 1/2$. The analysis for $h_m \geqslant 1/2$ is similar. In region Ω_1, we have $q^t \leqslant p^t h_m < p^t$ such that $q^{t+1} > q^t$ and $p^{t+1} < p^t$. Hence, (p^t, q^t) ultimately leaves region Ω_1 and enters region Ω_2. In region Ω_2, we have

$$\begin{aligned} & q^t < \pi_{a_1} + \pi_{a_2} = p^t h_m + (1-p^t)(1-h_m) \\ \Leftrightarrow \quad & h_m + q^t + p^t - 2h_m p^t < 1 \\ \Leftrightarrow \quad & p^t h_m + \left[q^t + p^t - 2h_m p^t\right] p^t < p^t \\ \Leftrightarrow \quad & p^{t+1} < p^t. \end{aligned}$$

Hence, p^t decreases overtime. Note that, when (p^t, q^t) is located on the boundary between region Ω_2 and region Ω_3, $p^{t+1} = p^t$. Hence this boundary constitutes a stationnary locus for p^t that will be denoted pp. This locus is unstable since we have shown that, for any (p^t, q^t) below the pp locus ($\Leftrightarrow (p^t, q^t) \in \Omega_1 \cup \Omega_2$), p^t moves away from it.

Still in region Ω_2, we have $q^{t+1} > q^t$ if and only if

$$\begin{aligned} & p^t h_m + \left[q^t + p^t - 2p^t h_m\right] q^t > q^t \\ \Leftrightarrow \quad & p^t > \frac{q^t(1-q^t)}{h_m + (1-2h_m)q^t} =: k_m(q^t), \end{aligned}$$

where $k_m(q^t)$ is the equation of the qq curve. It is easy to check that $k_m(0) = 0$ and $k_m(1/2) = 1/2$. Moreover, we have

$$k_m'(q^t) = \frac{h_m - 2h_m q^t - (1-2h_m)q^t q^t}{[h_m + (1-2h_m)q^t]^2},$$

which is positive if and only if

$$q^t < \frac{\sqrt{h_m - h_m^2} - h_m}{1 - 2h_m} \in (0, 1/2).$$

Hence, $k_m(q^t)$ is concave and reaches a maximum on the interval $(0, 1/2)$. Finally, note that $k_m'(0) = 1/h_m$. This implies that the slope of the qq locus at $(0, 0)$ is the same as the slope of the boundary between Ω_1 and Ω_2, which equals h_m. Also we have $k_m'(1/2) = 2h_m - 1$. This implies that the slope of the qq locus at $(1/2, 1/2)$, which equals $\frac{1}{2h_m - 1}$, is steeper than that of the boundary between region Ω_2 and Ω_3, which equals $2h_m - 1$, because $h_m < 1/2$. Hence, as depicted in Figure A.1, for $q^t \in [0, 1/2]$, the qq locus belongs to region Ω_2 and is stable in the sense that, for all $(p^t, q^t) \in \Omega_2$, q^t evolves toward the qq locus.

As indicated by the phase diagram in Figure A.1, the joint dynamics of p^t and q^t features three steady states $(1/2, 1/2)$, which is the crossing point between the pp locus and the qq locus, $(0, 0)$ and $(1, 1)$.

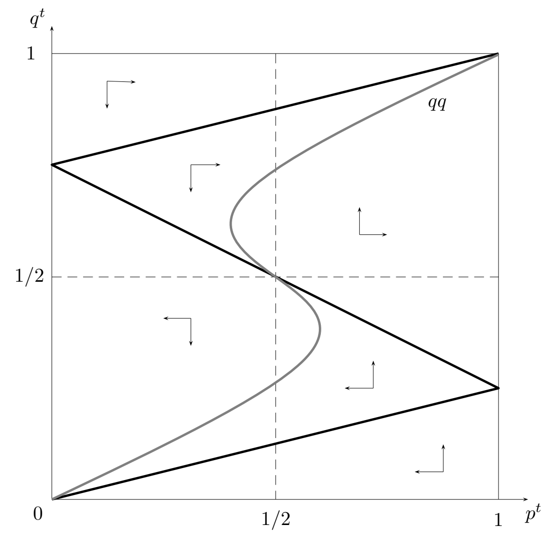


Figure A.1: Phase diagram.

Proof of Proposition 3. We first show that $(0,0)$ is asymptotically stable. Consider the region T_ε in which $p^t + q^t \leqslant 1 - \varepsilon$, for some arbitrarily small $\varepsilon > 0$. T_ε is a compact set since it is closed and bounded. For any $(p^t, q^t) \in T_\varepsilon \cap \Omega_1$, we have

$$p^{t+1} + q^{t+1} = 2q^t + (p^t - q^t)(p^t + q^t) < 2q^t + (p^t - q^t) = p^t + q^t.$$

For any $(p^t, q^t) \in T_\varepsilon \cap \Omega_2$, we have

$$\begin{aligned} p^{t+1} + q^{t+1} &= 2p^t h_m + [q^t + p^t - 2p^t h_m](p^t + q^t) \\ &< 2p^t h_m + q^t + p^t - 2p^t h_m = p^t + q^t \end{aligned}$$

For any $(p^t, q^t) \in T_\varepsilon \cap (\Omega_3 \cup \Omega_4)$, we can similarly show that $p^{t+1} + q^{t+1} < p^t + q^t$.

This implies that for any $(p^t, q^t) \in T_\varepsilon$, $p^t + q^t > p^{t+1} + q^{t+1}$. Hence, $p^t + q^t$ converges to 0 as time approaches infinity by the monotone convergence theorem, implying that (p^t, q^t) converges to 0 as time approaches infinity as well. Hence, $(0,0)$ is attracting. Also, for any $\delta > 0$, for any (p^0, q^0) satisfies that $\sqrt{(p^0)^2 + (q^0)^2} < \frac{\sqrt{2}}{2}\delta$, we have $\sqrt{(p^t)^2 + (q^t)^2} \leqslant p^t + q^t < p^0 + q^0 < \delta$, given that $p^t + q^t$ is monotonically decreasing in t. Hence $(0,0)$ is stable. Therefore, it is asymptotically stable. Since ε is arbitrarily small, we can say that for any (p^0, q^0) such that $p^0 + q^0 < 1$, (p^t, q^t) converges to $(0,0)$ as time approaches infinity.

By applying the same logic, we can show that $(1,1)$ is asymptotically stable and for any (p^0, q^0) such that $p^0 + q^0 > 1$, (p^t, q^t) converges to $(1,1)$ as time approaches infinity.

Finally, we show that $(1/2, 1/2)$ is a saddle point. First, we check the Jacobian matrix of the dynamics in region Ω_2 evaluated at $(1/2, 1/2)$, which is

$$\begin{bmatrix} h_m + q + (1-2h_m)2p & p \\ h_m + (1-2h_m)q & 2q + (1-2h_m)p^t \end{bmatrix}_{(p=\frac{1}{2},q=\frac{1}{2})} = \begin{bmatrix} \frac{3}{2} - h_m & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} - h_m \end{bmatrix}$$

Then we check the Jacobian matrix of the dynamics in region Ω_3 evaluated at $(\frac{1}{2}, \frac{1}{2})$, which is also

$$\begin{bmatrix} 3 - q - 3h_m + (2h_m - 1)2p & 1-p \\ (1-h_m) + (2h_m - 1)q & 3 - 2h_m + (2h_m - 1)p - 2q \end{bmatrix}_{(p=\frac{1}{2},q=\frac{1}{2})} = \begin{bmatrix} \frac{3}{2} - h_m & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} - h_m \end{bmatrix}$$

This implies that the dynamics is a continuously differentiable mapping at $(\frac{1}{2}, \frac{1}{2})$. The eigenvalues are given by $1 - h_m < 1$ and $2 - h_m > 1$. Hence, $(\frac{1}{2}, \frac{1}{2})$ is a saddle point.

Note that when $(p^t, q^t) \in \Omega_1$ and $q^t = 1 - p^t$, we have:

$$\begin{aligned} q^{t+1} &= q^t + \left[p^t - q^t\right](1-p^t) \\ &= p^t + q^t - \left\{q^t + \left[p^t - q^t\right] p^t\right\} = 1 - p^{t+1} \end{aligned}$$

where the last equality comes form the fact that we have assumed $p^t + q^t = 1$. When $(p^t, q^t) \in \Omega_2$ and $q^t = 1 - p^t$, we have:

$$\begin{aligned} q^{t+1} &= p^t h_m + [q^t + p^t - 2p^t h_m](1-p^t) \\ &= p^t + q^t - \left\{p^t h_m + [q^t + p^t - 2p^t h_m]p^t\right\} = 1 - p^{t+1}. \end{aligned}$$

Similarly, we can show that when $(p^t, q^t) \in \Omega_3 \cup \Omega_4$ and $q^t = 1 - p^t$, we have $q^{t+1} = 1 - p^{t+1}$. Note that we have shown that when $p^t + q^t < 1$, we have $p^{t+1} + q^{t+1} < 1$; when $p^t + q^t > 1$, we have $p^{t+1} + q^{t+1} > 1$.

In addition, for $p^t = 1 - q^t > \frac{1}{2}$ and $p^t \in \Omega_1$, we have

$$p^{t+1} = q^t + (p^t - q^t)p^t = 1 - p^t + (2p^t - 1)p^t,$$

which is less than p^t but larger than $\frac{1}{2}$.[A.1] Similarly, for $p^t = 1 - q^t > \frac{1}{2}$, and $p^t \in \Omega_2$, we have

$$p^{t+1} = p^t h_m + (q^t + p^t - 2p^t h_m)p^t = (1 + h_m - 2p^t h_m)p^t,$$

which is less than p^t but larger than $\frac{1}{2}$.[A.2] In sum, for $p^t = 1 - q^t > \frac{1}{2}$, we have $p^t > p^{t+1} > \frac{1}{2}$. By the same logic, we can show that for $p^t = 1 - q^t < \frac{1}{2}$ (in region Ω_3 and region Ω_4), we have $p^t < p^{t+1} < \frac{1}{2}$. Then by the monotone convergence theorem, for any $p^0 = 1 - q^0$, as time goes to infinity, p^t converges to $\frac{1}{2}$, which automatically implies that q^t converges to $\frac{1}{2}$ as well.

Hence, the unique saddle path that converges toward $(1/2, 1/2)$ and that splits the state space between

[A.1] $1 - p^t + (2p^t - 1)p^t < p^t$ is equivalent to $(1 - 2p^t)(1 - p^t) < 0$, which is is true for $p^t > \frac{1}{2}$. $1 - p^t + (2p^t - 1)p^t > \frac{1}{2}$ is equivalent to $2(p^t - \frac{1}{2})^2 > 0$, which is true for $p^t > \frac{1}{2}$.

[A.2] $(1 + h_m - 2p^t h_m)p^t < p^t$ is equivalent to $1 + h_m - 2p^t h_m < 1$, which is is true for $p^t > \frac{1}{2}$. $(1 + h_m - 2p^t h_m)p^t > \frac{1}{2}$ is equivalent to $-\frac{1}{2} + (1 + h_m)p^t - 2h_m p^t p^t > 0$, which is true for $p^t > \frac{1}{2}$ and $h_m < \frac{1}{2}$.

the basin of attraction of $(0,0)$ ($p^t + q^t < 1$) and the basin of attraction of $(1,1)$ ($p^t + q^t > 1$) exactly corresponds to the straight line $q^t = 1 - p^t$. □

A.6 Stable matching with mixtures of homophilic and heterophilic proposers and receivers (Section 3.2.2)

Consider the stable matching with mass p of type-a men and mass q of type-b women. Homophilic men and women of the same type, M_{θ_1} and W_{θ_1}, want to be matched together. Hence, at any stable matching the mass of $a_1 a_1$ couples is $\min\{p^t h_m, q^t h_w\}$, i.e., $p^t h_m$ if $q > h_m p/h_w =: h_1(p)$, and $q^t h_w$ otherwise. The mass of $b_1 b_1$ couples is $\min\{(1-p^t)h_m, (1-q^t)h_w\}$, i.e., $(1-p^t)h_m$ if $q^t < 1 - h_m(1-p^t)/h_w =: h_2(p^t)$, and $(1-q^t)h_w$ otherwise.

Moreover, for $\theta \neq \theta'$, heterophilic men and women of the opposite types, M_{θ_2} and $W_{\theta_2'}$, want to be matched together. Hence, at any stable matching the mass of ab couples must be at least

$$\begin{cases} p^t(1-h_m) & \text{if} \quad q^t < 1 - \left(\frac{1-h_m}{1-h_w}\right) p^t =: h_3(p^t), \\ (1-q^t)(1-h_w) & \text{otherwise.} \end{cases}$$

and the mass of bb couples must be at least

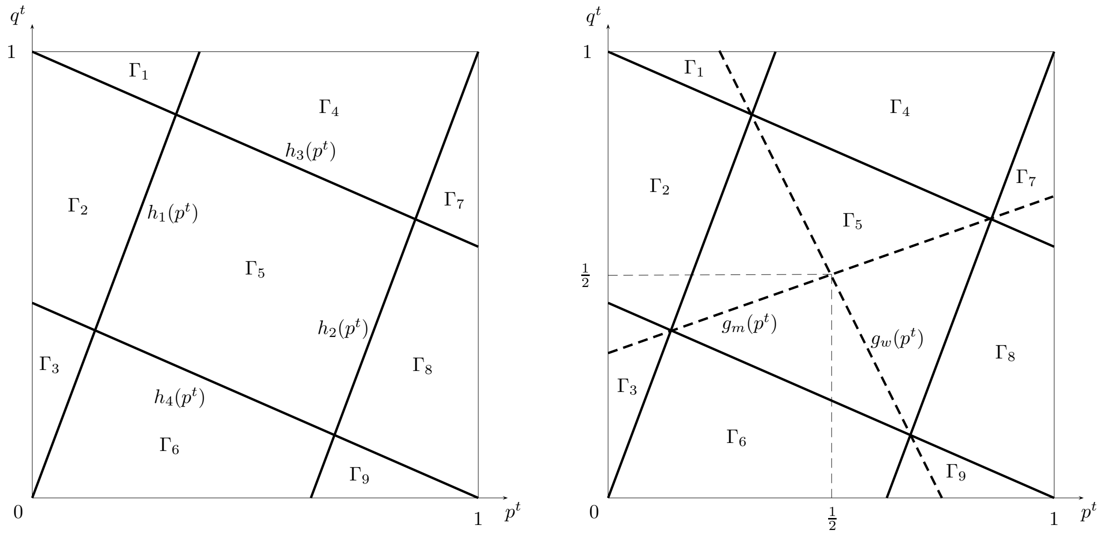
$$\begin{cases} (1-p^t)(1-h_m) & \text{if} \quad q^t > \left(\frac{1-h_m}{1-h_w}\right)(1-p^t) =: h_4(p^t), \\ q^t(1-h_w) & \text{otherwise.} \end{cases}$$

As Figure A.2a depicts, the unit square can be partitioned in nine disjoint sets according to the position of (p^t, q^t) with respect to the four functions $h_1(p^t)$, $h_2(p^t)$, $h_3(p^t)$ and $h_4(p^t)$.

Below, we describe the stable matching in each region. Except region Γ_5, there is a unique stable matching, summarized in the table below.

Region	μ_{aa}	μ_{ba}	μ_{bb}	μ_{ab}
Γ_1	$p-(1-q)(1-h_w)$	$1-p-(1-q)h_w$	$(1-q)h_w$	$(1-q)(1-h_w)$
Γ_2	ph_m	$q-ph_m$	$1-q-p(1-h_m)$	$p(1-h_m)$
Γ_3	ph_m	$q-ph_m$	$1-q-p(1-h_m)$	$p(1-h_m)$
Γ_4	$p-(1-q)(1-h_w)$	$1-p-(1-q)h_w$	$(1-q)h_w$	$(1-q)(1-h_w)$
Γ_6	qh_w	$q(1-h_w)$	$1-p-q(1-h_w)$	$p-qh_w$
Γ_7	$q-(1-p)(1-h_m)$	$(1-p)(1-h_m)$	$(1-p)h_m$	$1-q-(1-p)h_m$
Γ_8	$q-(1-p)(1-h_m)$	$(1-p)(1-h_m)$	$(1-p)h_m$	$1-q-(1-p)h_m$
Γ_9	qh_w	$q(1-h_w)$	$1-p-q(1-h_w)$	$p-qh_w$

If $(p,q) \in \Gamma_5$, MOSM and WOSM do not coincide. Define $g_w(p) := (1-h_w-p)/(1-2h_w)$ and $g_m(p) := 1-h_m-p(1-2h_m)$. The stable matching is described as follows.



(a) Partitioned regions

(b) Additional partition of region Γ_5

Figure A.2: Partitioning regions for characterization of stable matching

	μ_{aa}	μ_{ba}	μ_{bb}	μ_{ab}
WOSM				
$q < g_w(p)$	qh_w	$q(1-h_w)$	$1-p-q(1-h_w)$	$p-qh_w$
$q \geqslant g_w(p)$	$p-(1-q)(1-h_w)$	$1-p-(1-q)h_w$	$(1-q)h_w$	$(1-q)(1-h_w)$
MOSM				
$q < g_m(p)$	$q-(1-p)(1-h_m)$	$(1-p)(1-h_m)$	$(1-p)h_m$	$1-q-(1-p)h_m$
$q \geqslant g_m(p)$	ph_m	$q-ph_m$	$1-q-p(1-h_m)$	$p(1-h_m)$

As depicted in Figure A.2b, $g_w(p^t)$ passes through the crossing point between $h_1(p^t)$ and $h_3(p^t)$ and the crossing point between $h_2(p^t)$ and $h_4(p^t)$ while $g_m(p^t)$ passes through the crossing point between $h_1(p^t)$ and $h_4(p^t)$ and the crossing point between $h_2(p^t)$ and $h_3(p^t)$. Moreover, $g_w(p^t)$ and $g_m(p^t)$ intersect at $(1/2, 1/2)$.

A.7 Proof of Proposition 4

Construction of the phase diagrams in Figure 6. Cultural evolution is driven by the following two dimensional dynamical system:

$$\begin{aligned} p^{t+1} &= \mu^t_{aa} + (\mu^t_{ab} + \mu^t_{ba})p^t \\ q^{t+1} &= \mu^t_{aa} + (\mu^t_{ab} + \mu^t_{ba})q^t \end{aligned}$$

In the following, for each region partitioning the unit square in Figure A.2, we will replace the proportions μ_{aa}, μ_{ab} and μ_{ba} by those corresponding to the stable matching (see Section A.6). Before that, let us

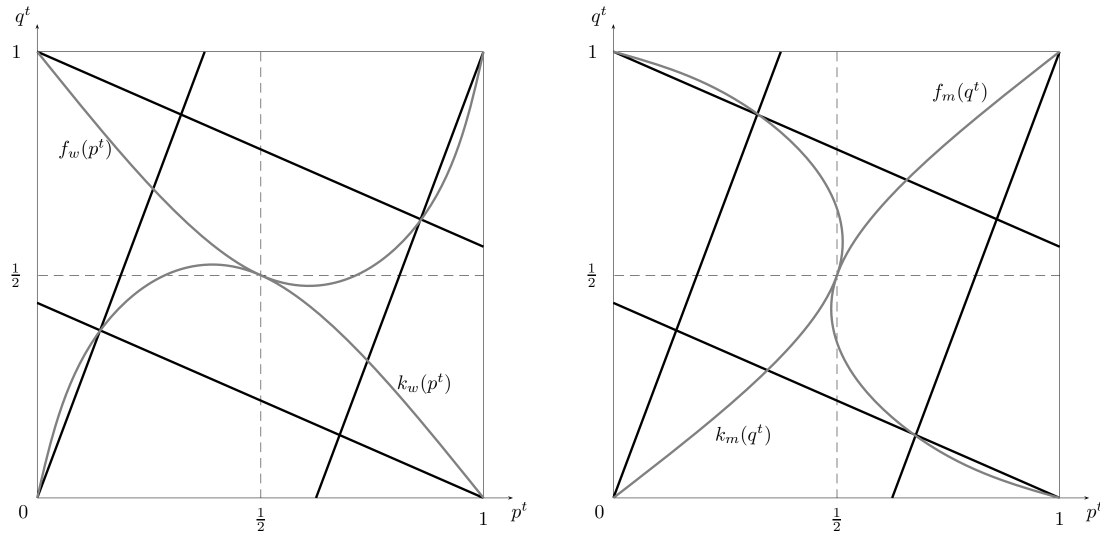


Figure A.3: Cultural evolution in the general model with mixed proposers and mixed receivers

define the two following functions that will be usefull in our analysis:

$$\begin{aligned} f_i(x) &:= 1 - \frac{x(1-x)}{1-h_i-x(1-2h_i)} \\ k_i(x) &:= \frac{x(1-x)}{h_i+x(1-2h_i)} \end{aligned}$$

with $f_i(0) = f_i(1) = 1$, $k_i(0) = k_i(1) = 0$, $f_i(1/2) = k_i(1/2) = 1/2$, $f_i''(.) > 0$ and $k_i''(.) < 0$ for all $i \in \{m, f\}$. Moreover, we can verify that $f_w(p^t)$ passes through the crossing point between $h_2(p^t)$ and $h_3(p^t)$; $k_w(p^t)$ passes through the crossing point between $h_1(p^t)$ and $h_4(p^t)$; $f_m(q^t)$ passes through the crossing point between $h_2(p^t)$ and $h_4(p^t)$; and $k_m(q^t)$ passes through the crossing point between $h_1(p^t)$ and $h_3(p^t)$. Those functions are depicted in Figure A.3a.

If $(p^t, q^t) \in \Gamma_1$**:**

$$\begin{aligned} p^{t+1} &= p^t - (1-q^t)(1-h_w) + \left[1 - p^t + (1-q^t)(1-2h_w)\right] p^t \\ q^{t+1} &= p^t - (1-q^t)(1-h_w) + \left[1 - p^t + (1-q^t)(1-2h_w)\right] q^t \end{aligned}$$

Hence, $p^{t+1} > p^t$ iff $q^t > f_w(p^t)$. As shown in Figure A.3a, for all $(p^t, q^t) \in \Gamma_1$, q^t is higher than $f_w(p^t)$ such that $p^{t+1} > p^t$. If $q^t = g_w(p^t)$, $q^{t+1} = q^t$ s.t. $q^t = g_w(p^t)$ is the equation of the stationnary locus of q^t (qq curve) in Γ_1 ($g_w(p^t)$ has been defined in Section A.6).

If $(p^t, q^t) \in \Gamma_9$**:**

$$\begin{aligned} p^{t+1} &= q^t h_w + \left[p^t + q^t(1-2h_w)\right] p^t \\ q^{t+1} &= q^t h_w + \left[p^t + q^t(1-2h_w)\right] q^t \end{aligned}$$

Hence, $p^{t+1} > p^t$ iff $q^t > k_w(p^t)$. As shown in Figure A.3a, for all $(p^t, q^t) \in \Gamma_9$, q^t is lower than $k_w(p^t)$ such that $p^{t+1} < p^t$. Moreover, as in region Γ_1, $q^t = g_w(p^t)$ corresponds to the qq locus (when $q^t = g_w(p^t)$, $q^{t+1} = q^t$) which is unstable ($q^{t+1} > q^t$ iff $q^t > g_w(p^t)$).

If $(p^t, q^t) \in \Gamma_2$**:**

$$\begin{aligned} p^{t+1} &= p^t h_m + \left[q^t + p^t(1-2h_m)\right] p^t \\ q^{t+1} &= p^t h_m + \left[q^t + p^t(1-2h_m)\right] q^t \end{aligned}$$

Hence, $p^{t+1} > p^t$ iff $q^t > g_m(p^t)$. Since, for all $(p^t, q^t) \in \Gamma_2$ we have $q^t > g_m(p^t)$ (see Figure A.2b), we must have $p^{t+1} > p^t$. Moreover, $q^{t+1} > q^t$ iff $p^t > k_m(q^t)$. As shown in Figure A.3a, for all $(p^t, q^t) \in \Gamma_2$, $p^t < k_m(q^t)$ such that $q^{t+1} < q^t$.

If $(p^t, q^t) \in \Gamma_8$**:**

$$\begin{aligned} p^{t+1} &= q^t - (1-p^t)(1-h_m) + \left[1 - q^t + (1-p^t)(1-2h_m)\right] p^t \\ q^{t+1} &= q^t - (1-p^t)(1-h_m) + \left[1 - q^t + (1-p^t)(1-2h_m)\right] q^t \end{aligned}$$

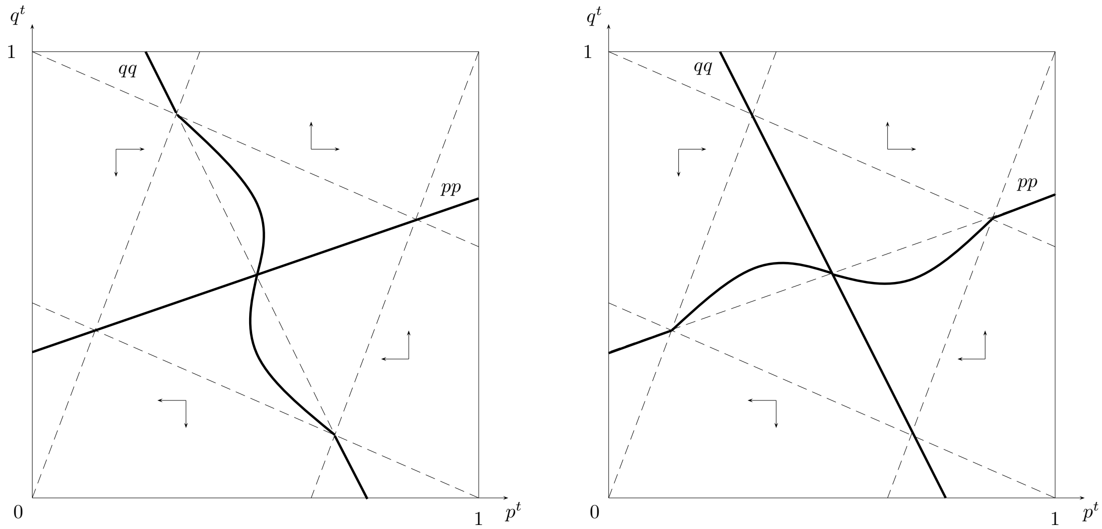
Hence, $p^{t+1} > p^t$ iff $q^t > g_m(p^t)$. Since, for all $(p^t, q^t) \in \Gamma_8$ we have $q^t < g_m(p^t)$ (see Figure A.2b), we must have $p^{t+1} < p^t$. Moreover, $q^{t+1} > q^t$ iff $p^t > f_m(q^t)$. As shown in Figure A.3a, for all $(p^t, q^t) \in \Gamma_8$, $p^t > f_m(q^t)$, we must have $q^{t+1} > q^t$.

If $(p^t, q^t) \in \Gamma_3$**:** The dynamics is the same as in the case $(p^t, q^t) \in \Gamma_2$. Since for all $(p^t, q^t) \in \Gamma_3$, $p^t < k_m(q^t)$, we must have $q^{t+1} < q^t$. Moreover, the line with equation $q^t = g_m(p^t)$ passes through Γ_3. Hence, this line corresponds to the pp locus in that region and this locus is stable. Indeed, the pp locus is upward slopping and $p^{t+1} > p^t$ iff $q^t > g_m(p^t)$.

If $(p^t, q^t) \in \Gamma_7$**:** The dynamics is the same as in the case $(p^t, q^t) \in \Gamma_8$. Since for all $(p^t, q^t) \in \Gamma_7$, $p^t > f_m(q^t)$, we must have $q^{t+1} > q^t$. Moreover, as in region Γ_3, the line with equation $q^t = g_m(p^t)$ corresponds to the pp locus which is stable ($p^{t+1} > p^t$ iff $q^t > g_m(p^t)$).

If $(p^t, q^t) \in \Gamma_4$**:** The dynamics is the same than in the case $(p^t, q^t) \in \Gamma_1$. Since for all $(p^t, q^t) \in \Gamma_4$, $q^t > f_w(p^t)$, we must have $p^{t+1} > p^t$, and since for all $(p^t, q^t) \in \Gamma_4$, $q^t > g_w(p^t)$, we must have $q^{t+1} > q^t$.

If $(p^t, q^t) \in \Gamma_6$**:** The dynamics is the same than in the case $(p^t, q^t) \in \Gamma_9$. Since for all $(p^t, q^t) \in \Gamma_6$, $q^t < k_w(p^t)$, we must have $p^{t+1} < p^t$, and since for all $(p^t, q^t) \in \Gamma_6$, $q^t < g_w(p^t)$, we must have $q^{t+1} < q^t$.



(a) MOSM (b) WOSM

Figure A.4: The cultural dynamics

If $(p^t, q^t) \in \Gamma_5$**:**

- Consider WOSM. If $q^t < g_w(p^t)$, the dynamics is the same as in the case $(p^t, q^t) \in \Gamma_9$ such that $q^{t+1} < q^t$ and $p^{t+1} > p^t$ iff $q^t > k_w(p^t)$. If $q^t > g_w(p^t)$, the dynamics is the same as in the case $(p^t, q^t) \in \Gamma_1$ such that $q^{t+1} > q^t$ and $p^{t+1} > p^t$ iff $q^t > f_w(p^t)$. To sum-up, the line of equation $q^t = g_w(p^t)$ corresponds to the locus of stationarity of q^t and is unstable; while the curve of equation $q^t = k_w(p^t)$ if $q^t \leqslant g_w(p^t)$ and $q^t = f_w(p^t)$ for $q^t > g_w(p^t)$ corresponds to the stationnary locus of p^t and is stable.

- Consider MOSM. If $q^t < g_m(p^t)$, the dynamics is the same as in the case $(p^t, q^t) \in \Gamma_8$ such that $p^{t+1} < p^t$ and $q^{t+1} > q^t$ iff $p^t > f_m(q^t)$. If $q^t > g_w(p^t)$, the dynamics is the same as in the case $(p^t, q^t) \in \Gamma_2$ such that $p^{t+1} > p^t$ and $q^{t+1} > q^t$ iff $p^t > k_m(q^t)$. To sum-up, the line of equation $q^t = g_m(p^t)$ corresponds to the stationnary locus of p^t and is stable, while the curve of equation $p^t = k_m(q^t)$ if $q^t \geqslant g_m(p^t)$ and $p^t = f_m(q^t)$ if $q^t < g_m(p^t)$ corresponds to the stationnary locus of q^t and is unstable.

The phase diagrams in Figure A.4 summarize these results. The qq and pp curves correspond to the stationary locus of q^t and p^t respectively. As shown on this diagram, under both MOSM and WOSM, there are three steady states: $(0, 0)$, $(1, 1)$ and $(1/2, 1/2)$, which is the crossing point between the pp locus and the qq locus.

Proof of Proposition 4. Similar to what we have done in the proof of Proposition 3, we can use the fact that when $p^t + q^t < 1$, $p^{t+1} + q^{t+1} < p^t + q^t$ in regions Γ_2, Γ_3, Γ_5, Γ_6, Γ_8 and Γ_9 to show that $(0, 0)$ is asymptotically stable and for any (p^0, q^0) such that $p^0 + q^0 < 1$, (p^t, q^t) converges to $(0, 0)$ as time approaches infinity. Similarly, by using the fact that when $p^t + q^t > 1$, $p^{t+1} + q^{t+1} > p^t + q^t$ in regions $\Gamma_1, \Gamma_2, \Gamma_4, \Gamma_5, \Gamma_7$ and Γ_9, we can also show that $(1, 1)$ is asymptotically stable and for any (p^0, q^0) such that $p^0 + q^0 > 1$, (p^t, q^t) converges to $(1, 1)$ as time approaches infinity.

Finally, we can show that $(\frac{1}{2}, \frac{1}{2})$ is a saddle point by check the Jacobian matrix of the dynamics in region Γ_5 evaluated at $(\frac{1}{2}, \frac{1}{2})$. It turns out that under either WOSM or MOSM, the Jacobian matrix evaluated at $(\frac{1}{2}, \frac{1}{2})$ is identical to the one we have found in the proof of Proposition 3, which implies that the dynamics is a continuously differentiable mapping at $(\frac{1}{2}, \frac{1}{2})$. Also, the eigenvalues are still given by $1 - h_m < 1$ and $2 - h_m > 1$, implying that $(\frac{1}{2}, \frac{1}{2})$ is a saddle point.

Note that the line of equation $q^t = 1 - p^t$ passes through regions Γ_2, Γ_5 and Γ_8. When $(p^t, q^t) \in \Gamma_2$ and $q^t = 1 - p^t$, we have:

$$\begin{aligned} q^{t+1} &= p^t h_m + [q^t + p^t - 2p^t h_m](1 - p^t) \\ &= p^t + q^t - \left\{p^t h_m + [q^t + p^t - 2p^t h_m]p^t\right\} = 1 - p^{t+1} \end{aligned}$$

When $(p^t, q^t) \in \Gamma_8$ and $q^t = 1 - p^t$, we have:

$$\begin{aligned} q^{t+1} &= (1 - p^t)h_m + \left[1 - q^t + (1 - p^t)(1 - 2h_m)\right](1 - p^t) \\ &= 1 - \left\{(1 - p^t)h_m + \left[1 - q^t + (1 - p^t)(1 - 2h_m)\right] p^t\right\} \\ &= 1 - \left\{q^t - (1 - p^t)(1 - h_m) + \left[1 - q^t + (1 - p^t)(1 - 2h_m)\right] p^t\right\} \\ &= 1 - p^{t+1} \end{aligned}$$

When $(p^t, q^t) \in \Gamma_5$, under MOSM, the dynamics is the same as in the case $(p^t, q^t) \in \Gamma_2$ or $(p^t, q^t) \in \Gamma_8$. Hence, when $q^t = 1 - p^t$ we must have $q^{t+1} = 1 - p^{t+1}$. Under WOSM, the dynamics is the same as in the case $(p^t, q^t) \in \Gamma_1$ or $(p^t, q^t) \in \Gamma_9$. When the dynamics is the same as in the case $(p^t, q^t) \in \Gamma_1$ and $q^t = 1 - p^t$ we have:

$$\begin{aligned} q^{t+1} &= p^t h_w + \left[1 - p^t + (1 - q^t)(1 - 2h_w)\right](1 - p^t) \\ &= 1 - \left\{p^t h_w + \left[1 - p^t + (1 - q^t)(1 - 2h_w)\right] p^t\right\} \\ &= 1 - \left\{p^t - (1 - q^t)(1 - h_w) + \left[1 - p^t + (1 - q^t)(1 - 2h_w)\right] p^t\right\} \\ &= 1 - p^{t+1} \end{aligned}$$

When the dynamics is the same as in the case $(p^t, q^t) \in \Gamma_9$ and $q^t = 1 - p^t$, we have:

$$\begin{aligned} q^{t+1} &= q^t h_w + \left[p^t + q^t(1 - 2h_w)\right](1 - p^t) \\ &= p^t + q^t - \left\{q^t h_w + \left[p^t + q^t(1 - 2h_w)\right] p^t\right\} \\ &= 1 - p^{t+1} \end{aligned}$$

Similar to what we have done in the proof of Proposition 3, we can use the monotone convergence theorem to show that, for any $p^0 = 1 - q^0$, as time goes to infinity, p^t converges to $\frac{1}{2}$, which automatically implies that q^t converges to $\frac{1}{2}$ as well.

Hence, the unique saddle path that converges to $(1/2, 1/2)$ and splits the state space between the basin of attraction of $(0,0)$ and the basin of attraction of $(1,1)$ exactly corresponds to the line $q^t = 1 - p^t$. $\square$

B Omitted details with imitative logit transmission in heterogamies

B.1 Microfounding imitative logit transmission in heterogamies

Consider a boy from a mixed family (symmetric reasoning applies for a girl). He randomly meets a role model in the men's population. Suppose the role model is type θ. Through observing the role model, the boy receives a noisy signal about the expected utility associated with trait θ, $U^t_\theta + \varepsilon_\theta$, and he compares with a noisy signal about the expected utility associated with the alternative trait $\theta' \neq \theta$, $U^t_{\theta'} + \varepsilon_{\theta'}$.[B.3] If the signal associated with trait θ is higher than the signal associated with trait θ', the boy imitates the role model. It happens with probability

$$\xi_\theta = \Pr\left(\varepsilon_{\theta'} - \varepsilon_\theta \leqslant U^t_\theta - U^t_{\theta'}\right).$$

Assume that ε_i and $\varepsilon_{i'}$ are distributed independently according to a Gumbel distribution with 0 mean and a scale parameter δ, the difference $\varepsilon_{i'} - \varepsilon_i$ is distributed according to a logistic distribution with 0 mean and a scale parameter δ, such that:

$$\xi_\theta = \frac{1}{1 + \exp\left[\frac{U^t_{\theta'} - U^t_\theta}{\delta}\right]} = \frac{\exp[U^t_\theta/\delta]}{\exp[U^t_\theta/\delta] + \exp[U^t_{\theta'}/\delta]}. \tag{B.1}$$

If the boy does not imitate the role model, he draws a new role model at random and repeats the procedure, only stopping when imitation occurs. The probability that the boy adopts type a is

$$\begin{aligned}
P^t &= p^t\xi_a + \left[p^t\xi_b + (1-p^t)\xi_a\right] \times \left[p^t\xi_a + \left[p^t\xi_b + (1-p^t)\xi_a\right] \times [\ldots]\right] \\
&= p^t\xi_a \sum_{j=0}^{+\infty} \left[p^t\xi_b + (1-p^t)\xi_a\right]^j \\
&= \frac{p^t\xi_a}{1 - [p^t\xi_b + (1-p^t)\xi_a]} = \frac{p^t\xi_a}{p^t\xi_a + (1-p^t)\xi_b},
\end{aligned}$$

where the last equality comes from the fact that $\xi_a = 1 - \xi_b$. Then, replacing ξ_a and ξ_b by their expressions given by equation (B.1), we get that

$$P^t = \frac{p^t \exp(U^t_a/\delta)}{p^t \exp(U^t_a/\delta) + (1-p^t)\exp(U^t_b/\delta)}.$$

[B.3] Note that, we implicitly assume that this expected utility is evaluated under myopic expectations. This is consistent with the fact that the child observes and imitates a role model from the adult generation. Hence, this assumption is in line with the original imitative logit model proposed by Weibull (1995) and Björnerstedt and Weibull (1996) and is also used by Besley (2017) and Besley and Persson (2019).

B.2 Proof of Proposition 5

Proof of Proposition 5. We show that (p, q), where $p \neq q$, cannot be a steady state. First, suppose $p^t > q^t$. Cultural evolution can be written as

$$
\begin{aligned}
p^{t+1} - p^t &= (p^t - q^t)(P^t - 1); \\
q^{t+1} - q^t &= (p^t - q^t)Q^t.
\end{aligned}
$$

Unless $P^t = 1$, $p^{t+1} - p^t < 0$, and unless $Q^t = 0$, $q^{t+1} - q^t > 0$. P^t could be 1 and Q^t could be 0 only if the intergenerational transmission is Darwinian. Second, suppose $p^t < q^t$. The argument proceeds in a similar manner. Cultural evolution can be written as

$$
\begin{aligned}
p^{t+1} - p^t &= (q^t - p^t)P^t; \\
q^{t+1} - q^t &= (q^t - p^t)(Q^t - 1).
\end{aligned}
$$

Unless $P^t = 0$, $p^{t+1} - p^t > 0$, and unless $Q^t = 1$, $q^{t+1} - q^t < 0$. P^t could be 0 and Q^t could be 1 only if the intergenerational transmission is Darwinian. In sum, only (r, r), where $r \in (0, 1)$, is a steady state.

The set $\{(r, r) : r \in (0, 1)\}$ forms a stable set, because $\lim_{t\to\infty} |p^t - q^t| = 0$. When $p^t > q^t$ and also when $p^t < q^t$,

$$
|p^{t+1} - q^{t+1}| = |(p^t - q^t)(P^t - Q^t)|.
$$

Since $P^t, Q^t \in (0, 1)$ under imitative logit transmission and when $p^t q^t \neq 0$, $|P^t - Q^t| < 1$, so $|p^{t+1} - q^{t+1}| < |p^t - q^t|$ when $p^t \neq q^t$. □

B.3 Proof of Proposition 6

Proof of Proposition 6. (Construction of the phase diagrams is embedded in this proof.) First consider the case in which $\delta = 0$. Our assumptions imply that $\min\{U_{aa}, U_{bb}\} > U_{ab} = U_{ba}$ and $V_{ab} = V_{ba} > \max\{V_{aa}, V_{bb}\}$. Hence, according to equation (3), when $p^t + q^t > 1$, $P^t = 1$ and $Q^t = 0$ and when $p^t + q^t < 1$, $P^t = 0$ and $Q^t = 1$. Hence, when $p^0 + q^0 > 1$, according to Equations (14) and (15), $p(1) = 1$ and $q(1) = p^0 + q^0 - 1$. Hence, $p(1) + q(1) > 1$ such that $P(1) = 1$ and $Q(1) = 0$. Which implies that $p(2) = 1 = p(1)$ and $q(2) = p(1) + q(1) - 1 = q(1)$. Hence, p^t and q^t have reached a steady state. Similarly, when $p^0 + q^0 < 1$, according to Equations (16) and (17), $p(1) = 0$ and $q(1) = p^0 + q^0$. Hence, $p(1) + q(1) = p^0 + q^0 < 1$ such that $P(1) = 0$ and $Q(1) = 1$. Which implies that $p(2) = 0 = p(1)$ and $q(2) = p(1) + q(1) = q(1)$.

Let us now consider the configuration where $\delta \neq 0$. Note that when $p^t + q^t = 1$, $p^{t+1} = P^t = p^t$ and $q^{t+1} = Q^t = q^t$ such that $p = q$ are steady states. We will now successively address the configurations $p^t + q^t > 1$ and $p^t + q^t < 1$.

When $p^t + q^t > 1$, we know from equation (14) that $p^{t+1} > p^t$ if and only if $P^t > \frac{1-q^t}{2-q^t-p^t}$. The latter inequality can be rewritten as

$$\frac{p^t}{1-q^t} > \exp\left[\frac{U_b^t - U_a^t}{\delta}\right] = \exp\left[-\left(\frac{U_{aa}-U_{ab}}{\delta}\right)\frac{p^t+q^t-1}{p^t}\right].$$

Note that the LHS of the above inequality is increasing in q^t and is equal to 1 when $q^t = 1-p^t$ while the RHS is decreasing in q^t and is equal to 1 when $q^t = 1-p^t$. Hence, for all $p^t+q^t > 1$, the above inequality is satisfied such that $p^{t+1} > p^t$. Now, according to (15),

$$q^{t+1}\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} q^t \quad\Leftrightarrow\quad Q^t\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}\frac{1-p^t}{2-q^t-p^t}.$$

Let us focus on the condition $Q^t > \frac{1-p^t}{2-q^t-p^t}$ which might be rewritten as

$$\frac{q^t}{1-p^t} > \exp\left[\frac{V_b^t - V_a^t}{\delta}\right] = \exp\left[\left(\frac{V_{ab}-V_{aa}}{\delta}\right)\frac{p^t+q^t-1}{q^t}\right]. \tag{B.2}$$

Let us define $\lambda := \frac{V_{ab}-V_{aa}}{\delta} > 0$ and $z^t := -\lambda\left(\frac{1-p^t}{q^t}\right)$. Then, the above inequality rewrites as:

$$\begin{aligned}\frac{-\lambda}{z^t} > \exp\left[\lambda + z^t\right] \quad&\Leftrightarrow\quad -\lambda < z^t\frac{\exp\left[z^t\right]}{\exp(-\lambda)}\\ &\Leftrightarrow\quad -\lambda\exp(-\lambda) < z^t\exp\left[z^t\right] =: g(z^t)\end{aligned}$$

The function $g(z^t)$ is decreasing and then increasing reaching a minimum in $z^t = -1$, at that point $g(z^t) = -1/e$. Moreover, note that, since $p^t + q^t > 1$, z^t varies from $-\lambda$ and 0 with $g(-\lambda) = -\lambda\exp(-\lambda)$ and $g(0) = 0$. Hence, as illustrated in Figure B.5a and B.5b, when $\lambda < 1$, $g(z^t) > -\lambda\exp(-\lambda)$; and when $\lambda > 1$, $g(z^t) > -\lambda\exp(-\lambda)$ iff $z^t > \omega(\lambda)$ with $\omega(\lambda) := W_0\left(-\lambda\exp(-\lambda)\right)$ where $W_0(.)$ is known as the Lambert W function. Then, using the definition of z^t, we get that the inequality (B.2) is satisfied iff $\lambda < 1$ or $\lambda > 1$ and

$$q^t > \frac{-\lambda}{\omega(\lambda)}\left(1-p^t\right)$$

with $\frac{-\lambda}{\omega(\lambda)} > 1$ since $\omega(\lambda) \in (-1, 0)$ while $-\lambda < -1$.

When $p^t + q^t > 1$, we know from (16) that

$$p^{t+1}\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} p^t \quad\Leftrightarrow\quad P^t\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}\frac{1-q^t}{2-q^t-p^t}$$

Let us focus on the condition

When $p^t + q^t < 1$, we know from equation (16) that $p^{t+1} < p^t$ iff $P^t < \frac{p^t}{p^t+q^t}$ which might be rewritten

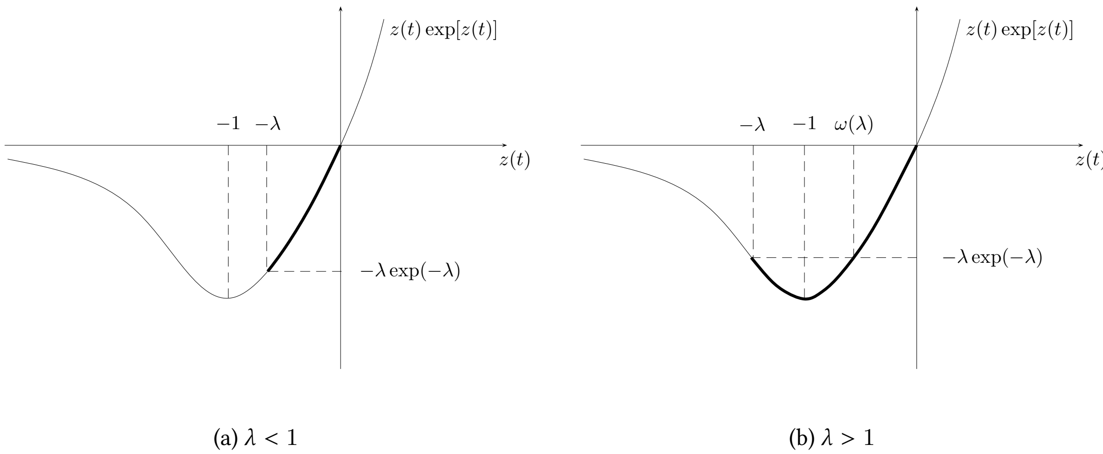


Figure B.5

as

$$\frac{q^t}{1-p^t} < \exp\left[\frac{U_b^t - U_a^t}{\delta}\right] = \exp\left[\left(\frac{U_{bb} - U_{ab}}{\delta}\right)\frac{1-p^t-q^t}{1-p^t}\right].$$

Note that the LHS of the above inequality is increasing in q^t and is equal to 1 when $q^t = 1 - p^t$ while the RHS is decreasing in q^t and is equal to 1 when $q^t = 1 - p^t$. Hence, for all $p^t + q^t < 1$, the above inequality is satisfied such that $p^{t+1} < p^t$. Then, according to equation (17),

$$q^{t+1}\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} q^t \quad \Leftrightarrow \quad Q^t \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \frac{q^t}{p^t + q^t}.$$

Let us focus on the condition $Q^t < \frac{q^t}{p^t+q^t}$ which might be rewritten as

$$\frac{p^t}{1-q^t} < \exp\left[\frac{V_b^t - V_a^t}{\delta}\right] = \exp\left[-\left(\frac{V_{ba} - V_{bb}}{\delta}\right)\frac{1-p^t-q^t}{1-q^t}\right]. \tag{B.3}$$

Let us define $\rho := \frac{V_{ba}-V_{bb}}{\delta} > 0$ and $k^t := -\rho\left(\frac{p^t}{1-q^t}\right)$. Then, the above inequality rewrites as:

$$\begin{aligned} \frac{k^t}{-\rho} < \exp\left[-\rho - k^t\right] \quad &\Leftrightarrow \quad k^t > -\rho\frac{\exp(-\rho)}{\exp\left[k^t\right]} \\ &\Leftrightarrow \quad -\rho\exp(-\rho) < k^t \exp\left[k^t\right] \end{aligned}$$

From the analysis developed in the previous case and using the definition of z^t, we deduce that the in-

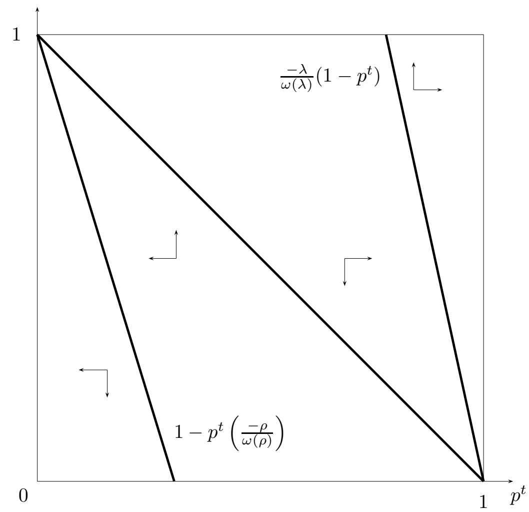


Figure B.6: Phase diagram for $V_{ab} - V_{aa} > \delta$ and $V_{ba} - V_{bb} > \delta$.

equality (B.3) is satisfied iff $\rho < 1$ or $\rho > 1$ and

$$q^t < 1 - p^t \left(\frac{-\rho}{\omega(\rho)} \right)$$

with $\frac{-\rho}{\omega(\rho)} > 1$.

We can derive from this analysis the following phase diagram in Figure B.6, illustrating how p^t and q^t evolve over time.

Now we proceed to check the stability of the steady states. When $p^t + q^t > 1$, we have shown that $p^{t+1} > p^t$ and $q^{t+1} > q^t$ iff $\lambda < 1$ or $\lambda > 1$ and $q^t > \frac{-\lambda}{\omega(\lambda)}\left(1 - p^t\right)$. If $\lambda < 1$, then we can using the same argument as in the proof of Proposition 2 to show that the dynamics is a contraction mapping in T_ε, where T_ε is the region that $p^t + q^t \geqslant 1 - \varepsilon$, for some arbitrarily small $\varepsilon > 0$. By the contraction mapping theorem, (p^t, q^t) converges to $(1, 1)$ as time approaches infinity. Since ε is arbitrarily small, we can say that for any (p^0, q^0) such that $p^0 + q^0 > 1$, $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t = 1$. If $\lambda > 1$, consider the region C_ε, in which $q^t > \frac{-\lambda}{\omega(\lambda)}\left(1 - p^t\right) + \varepsilon$ (the northeast region in Figure B.6) for some arbitrarily small $\varepsilon > 0$. In this region, we have $p^{t+1} > p^t$ and $q^{t+1} > q^t$, then the dynamics is a contraction mapping in C_ε, and by the contraction mapping theorem, (p^t, q^t) converges to $(1, 1)$ as time approaches infinity. Since ε is arbitrarily small, we can say that for any (p^0, q^0) such that $q^0 > \frac{-\lambda}{\omega(\lambda)}\left(1 - p^0\right)$, $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t = 1$. The above arguments automatically give that $(1, 1)$ is asymptotically stable.

By using the same logic, we can prove that when $\rho < 1$, for any (p^0, q^0) that satisfies $p^0 + q^0 < 1$, $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t = 0$. When $\rho < 1$, for any (p^0, q^0) that satisfies $q^0 < 1 - p^0 \left(\frac{-\rho}{\omega(\rho)}\right)$, $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t = 0$. Also, $(0, 0)$ is asymptotically stable.

Finally, for any (p, q) such that $p+q = 1$, and $p \neq 1$ and $q \neq 1$, consider an initial state $(p^0, q^0) = (p+\varepsilon, q)$ for some $\varepsilon > 0$, then by the above analysis, $p^{t+1} > p^t$, for any $t > 0$. Hence, it is not stable. For

$(p, q) = (1, 0)$, consider an initial state $(p^0, q^0) = (1, q + \varepsilon)$ for some $\varepsilon > 0$, then by the above analysis, $q^{t+1} > q^t$, for any $t > 0$. Hence, it is not stable. Similar logic applies to show that $(p, q) = (0, 1)$ is not stable. □

B.4 Proof of Proposition 7

Proof of Proposition 7. We assume that $h_m < \frac{1}{2}$. The analysis for $h_m \geqslant \frac{1}{2}$ is similar. The proof will be in two steps. We will successively consider the cases where (p^t, q^t) initially belongs to regions Ω_1, Ω_2, Ω_3 and Ω_4 and analyze how the cultural distribution evolves from this initial condition. Then, we will discuss the stability of the steady states $(0, 0)$, $(1, 1)$ and $(1/2, 1/2)$.

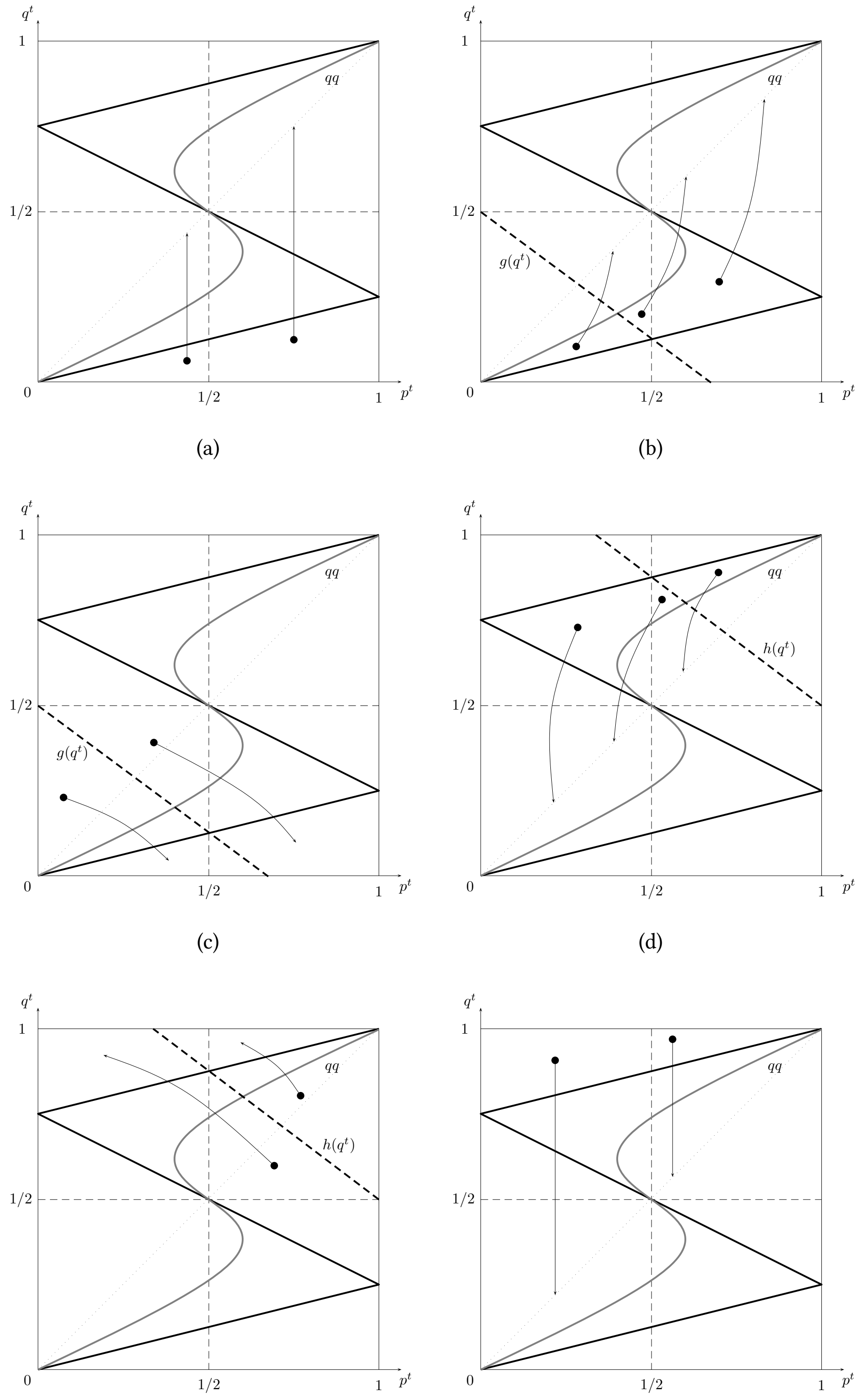
If $(p^t, q^t) \in \Omega_1$**:**

$$\begin{aligned}
U_a^t &= \frac{q^t U_{a_1 a} + (p^t h_m - q^t) U_{a_1 b} + p^t (1 - h_m) U_{a_2 b}}{p^t},\\
U_b^t &= h_m U_{b_1 b} + (1 - h_m) U_{b_2 b},\\
V_a^t &= V_{aa},\\
V_b^t &= \frac{(p^t - q^t) V_{ba} + (1 - p^t) V_{bb}}{1 - q^t}.
\end{aligned}$$

Hence, $Q^t = 1$, while $P^t = 1$ if and only if $q^t > p^t(2h_m - 1)$ which is verified since $h_m < 1/2$. This implies that $q^{t+1} = p^t$ and $p^{t+1} = p^t$. Hence, (p^t, q^t) is upward shifted (q^t jumps while p^t remains constant) and joins the first diagonal. As a consequence, if $p^t < 1/2$, (p^{t+1}, q^{t+1}) will be in Ω_2 (under the qq curve); otherwise, (p^{t+1}, q^{t+1}) will be in Ω_3 above the qq curve. See Figure B.7a below.

If $(p^t, q^t) \in \Omega_2$**:**

$$\begin{aligned}
U_a^t &= h_m U_{a_1 a} + (1 - h_m) U_{a_2 b},\\
U_b^t &= \left[(1 - p^t) h_m U_{b_1 b} + (q^t - p^t h_m) U_{b_2 a} + ((1 - p^t)(1 - h_m) - q^t + p^t h_m) U_{b_2 b}\right] \Big/ [1 - p^t],\\
V_a^t &= \frac{p^t h_m V_{aa} + (q^t - p^t h_m) V_{ab}}{q^t},\\
V_b^t &= \frac{p^t (1 - h_m) V_{ba} + (1 - q^t - p^t (1 - h_m)) V_{bb}}{1 - q^t}.
\end{aligned}$$



(a) (b) (c) (d) (e) (f)

Figure B.7: Equilibrium dynamics

Hence, $P^t = 1$ and $Q^t = 0$ (resp., $Q^t = 1$) if $V_a^t < V_b^t$ (resp., $V_a^t > V_b^t$) with $V_b^t > V_a^t$ iff

$$\begin{aligned}
& q^t\left[p^t(1-h_m)V_{ba} + (1-q^t-p^t(1-h_m))V_{bb}\right] > (1-q^t)\left[p^t h_m V_{aa} + (q^t - p^t h_m)V_{ab}\right] \\
\Leftrightarrow \quad & q^t\left[p^t(1-h_m)V_{ab} + (1-q^t-p^t(1-h_m))V_{aa}\right] > (1-q^t)\left[p^t h_m V_{aa} + (q^t - p^t h_m)V_{ab}\right] \\
\Leftrightarrow \quad & V_{ab}\left\{p^t\left[q^t(1-2h_m)+h_m\right] - q^t(1-q^t)\right\} > V_{aa}\left\{p^t\left[q^t(1-2h_m)+h_m\right] - q^t(1-q^t)\right\} \\
\Leftrightarrow \quad & (V_{aa}-V_{ab})\left\{p^t\left[q^t(1-2h_m)+h_m\right] - q^t(1-q^t)\right\} < 0 \\
\Leftrightarrow \quad & p^t < \frac{q^t(1-q^t)}{q^t(1-2h_m)+h_m} = k_m(q^t)
\end{aligned}$$

with $k_m(q^t)$ already defined in Appendix A.7.

Hence, if (p^t, q^t) is under the qq curve (defined in Section 3.2.1), we have $Q^t = P^t = 1$ and $p^{t+1} = q^{t+1} = p^t(1-h_m) + q^t$. Here $(0,0)$ is clearly a steady state but for all $(q^t, p^t) \neq (0,0)$ we have $q^{t+1} > q^t$ and $p^{t+1} = p^t + q^t - h_m p^t > p^t$ since $q^t > p^t h_m$ in Ω_2. Moreover, $p^{t+1} = q^{t+1}$ such that (p^{t+1}, q^{t+1}) will lie on the first diagonal. Finally, $p^{t+1} = q^{t+1} > 1/2$ iff

$$p^t > \frac{1}{1-h_m}\left(\frac{1}{2} - q^t\right) =: g(q^t)$$

with $g(1/2) = 0$ and $g(0) < 1$ since $h_m < 1/2$. Note also that $g(q^t) = 1/2$ for $q^t = h_m/2$ such that $g(q^t)$ crosses the boundary between Ω_1 and Ω_2 in $q^t = h_m/2$. Then, if $p^t > g(q^t)$, (p^{t+1}, q^{t+1}) will be in Ω_3 (above the qq curve); otherwise (q^{t+1}, p^{t+1}) will be in Ω_2 (under the qq curve). See Figure B.7b.

Now if $(p^t, q^t) \in \Omega_2$ but above the qq curve, we have $P^t = 1$ and $Q^t = 0$ and

$$\begin{aligned}
p^{t+1} &= q^t + p^t(1-h_m) \\
q^{t+1} &= p^t h_m
\end{aligned}$$

Again $(0,0)$ is a steady state but for all $(q^t, p^t) \neq (0,0)$ we have $p^{t+1} > p^t$ and $q^{t+1} = p^t h_m < q^t$. Moreover, $q^{t+1} = p^t h_m < p^{t+1} h_m$ such that (p^{t+1}, q^{t+1}) will belong to Ω_1. Finally, it is easy to verify that $p^{t+1} < 1/2$ iff $p^t < g(q^t)$. See Figure B.7c.

If $(p^t, q^t) \in \Omega_3$:

$$\begin{aligned}
U_a^t &= \Big[p^t h_m U_{a_1 a} + (1 - q^t - (1-p^t)h_m)U_{a_2 b} \\
&\qquad + ((1-h_m)p^t - 1 + q^t + (1-p^t)h_m)U_{a_2 a}\Big]\Big/ p^t, \\
U_b^t &= h_m U_{b_1 b} + (1-h_m)U_{b_2 a}, \\
V_a^t &= \frac{(q^t - (1-h_m)(1-p^t))V_{aa} + (1-h_m)(1-p^t)V_{ab}}{q^t}, \\
V_b^t &= \frac{(1-p^t)h_m V_{bb} + (1-q^t-(1-p^t)h_m)V_{ba}}{1-q^t}.
\end{aligned}$$

Then, $U_b^t > U_a^t$ iff

$$(U_{a_1a} - U_{a_2a})\left[1 - q^t - (1-p^t)h_m\right] > 0$$

which is always verified since $U_{a_1a} > U_{a_2a}$ and, $q^t < 1 - (1-p^t)h_m$ in Ω_3. Thus, $P^t = 0$. Moreover, we have $V_b^t > V_a^t$ iff

$$\begin{aligned}
& q^t\left[(1-p^t)h_m V_{bb} + (1 - q^t - (1-p^t)h_m)V_{ba}\right] \\
> \quad & (1-q^t)\left[(q^t - (1-h_m)(1-p^t))V_{aa} + (1-h_m)(1-p^t)V_{ab}\right] \\
\Leftrightarrow \quad & (V_{aa} - V_{ab})\left\{[1 - h_m - q^t(1-2h_m)](1-p^t) - q^t(1-q^t)\right\} > 0 \\
\Leftrightarrow \quad & p^t < 1 - \frac{q^t(1-q^t)}{1 - h_m - q^t(1-2h_m)} = f_m(q^t)
\end{aligned}$$

with $f_m(q^t)$ already defined in Appendix A.7. Thus, $Q^t = 0$ if (p^t, q^t) is above the qq curve while $Q^t = 1$ if (p^t, q^t) is under the qq curve.

Hence, if (p^t, q^t) is above the qq curve, we have $Q^t = P^t = 0$ and $p^{t+1} = q^{t+1} = q^t - (1-p^t)(1-h_m)$. Thus, $(1,1)$ is a steady state and for all $(q^t, p^t) \neq (1,1)$ we have $q^{t+1} < q^t$ and $p^{t+1} = p^t + q^t - 1 + (1-p^t)h_m < p^t$ since, in Ω_3 $q^t < 1 - (1-p^t)h_m$. Moreover, $p^{t+1} = q^{t+1}$ such that (p^{t+1}, q^{t+1}) will lie on the first diagonal. Finally, $p^{t+1} = q^{t+1} > 1/2$ iff

$$p^t > 1 + \frac{1}{1-h_m}\left(\frac{1}{2} - q^t\right) =: h(q^t)$$

with $h(1/2) = 1$ and $h(1) > 0$ since $h_m < 1/2$. Note also that, $h(q^t) = 1/2$ for $q^t = 1 - h_m/2$ such that $h(q^t)$ crosses the boundary between Ω_3 and Ω_4 in $q^t = 1 - h_m/2$. Then, if $p^t > h(q^t)$, (p^{t+1}, q^{t+1}) will be in Ω_3 (above the qq curve); otherwise (p^{t+1}, q^{t+1}) will be in Ω_2 (under the qq curve). See Figure B.7d.

Now if $(p^t, q^t) \in \Omega_3$ but under the qq curve, we have $P^t = 0$ and $Q^t = 1$ and

$$\begin{aligned}
p^{t+1} &= q^t - (1-p^t)(1-h_m) \\
q^{t+1} &= 1 - (1-p^t)h_m
\end{aligned}$$

Again $(1,1)$ is a steady state and for all $(q^t, p^t) \neq (1,1)$, since $(p^t, q^t) \in \Omega_3$, we must have $q^{t+1} > q^t$ and $p^{t+1} = p^t + q^t - (1-h_m) - p^t h_m < p^t$. Moreover, $q^{t+1} = 1 - h_m + p^t h_m > 1 - h_m + p^{t+1} h_w$ such that (p^{t+1}, q^{t+1}) will belong to Ω_4. Finally, it is easy to verify that $p^{t+1} < 1/2$ iff $p^t < h(q^t)$. See Figure B.7e.

If $(p^t, q^t) \in \Omega_4$:

$$\begin{aligned}
U_a^t &= h_m U_{a_1a} + (1-h_m)U_{a_2a}; \\
U_b^t &= \frac{(1-q^t)U_{b_1b} + ((1-p^t)h_m - (1-q^t))U_{b_1a} + (1-p^t)(1-h_m)U_{b_2a}}{1-p^t}; \\
V_a^t &= \frac{p^t V_{aa} + (q^t - p^t)V_{ab}}{q^t}; \\
V_b^t &= V_{bb}.
\end{aligned}$$

In that configuration, $U_b^t > U_a^t$ if and only if

$$(U_{a_1 a} - U_{a_2 a})\left[1 - q^t + (1-p^t)(1-2h_m)\right] > 0,$$

which is always verified since $U_{a_1 a} > U_{a_2 a}$ and $h_m < 1/2$. Hence, $P^t = 0$. We also have $V_b^t > V_a^t$ such that $Q^t = 0$. Hence, in that configuration we have $p^{t+1} = q^{t+1} = p^t < q^t$. Hence, (p^t, q^t) is downward shifted (q^t jumps while p^t remains constant) and joins the first diagonal. As a consequence, if $p^t < 1/2$, (p^{t+1}, q^{t+1}) will be in Ω_2 (above the qq curve); otherwise, (p^{t+1}, q^{t+1}) will be in Ω_3 under the qq curve. See Figure B.7f.

Instability of the steady states. It is easy to verify that when $(p^t, q^t) = (0,0)$, $(p^{t+1}, q^{t+1}) = (0,0)$ and when $(p^t, q^t) = (1,1)$, $(p^{t+1}, q^{t+1}) = (1,1)$ such that $(0,0)$ and $(1,1)$ are steady states. Now, as already shown, when (p^t, q^t) is close to $(0,0)$ we must have $p^{t+1} > p^t$, if $(p^t, q^t) \in \Omega_2$ and is above the qq locus; or $q^{t+1} > q^t$ if $(p^t, q^t) \in \Omega_2$ and is under the qq locus or if $(p^t, q^t) \in \Omega_1$. Hence, $(0,0)$ is unstable. Similar arguments can be used to show that $(1;1)$ is unstable.

Let us now consider $(1/2, 1/2)$. It corresponds to the crossing point between the qq locus, on which $U_a^t = U_b^t$, and the boundary between Ω_2 and Ω_3, on which $V_a^t = V_b^t$. Hence, when $(p^t, q^t) = (1/2, 1/2)$, $P^t = Q^t = 1/2$ such that $p^{t+1} = q^{t+1} = 1/2$. Hence, $(1/2, 1/2)$ is a steady state. To show that it is unstable let us consider a (p^t, q^t) close to $(1/2, 1/2)$ but belonging to Ω_2 and being located above the qq locus. We have already shown that, in such a configuration, (p^{t+1}, q^{t+1}) jumps to Ω_1 which is far away from $(1/2, 1/2)$. □

B.5 Proofs of Propositions 8 and 9

Proof of Proposition 8. A simple inspection of the dynamical system (18) leads us to conclude that, for all $(p^t, q^t) \in \Omega_1$, $p^{t+1} < p^t$ and $q^{t+1} > q^t$; and for all $(p^t, q^t) \in \Omega_4$, $p^{t+1} > p^t$ and $q^{t+1} < q^t$.

Let us now consider the case $(p^t, q^t) \in \Omega_2$. Considering equation (18) as well as the expression of P^t we get that

$$p^{t+1} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} p^t \quad \Leftrightarrow \quad P^t \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \frac{(1-h_m)p^t}{q^t + (1-2h_m)p^t}.$$

Let us focus on the condition $P^t > \frac{(1-h_m)p^t}{q^t + (1-2h_m)p^t}$. From Lemma 1 we can deduce that:

$$\begin{aligned} U_b^t - U_a^t &= \left(\frac{q^t - (1-h_m) + p^t(1-2h_m)}{1-p^t}\right)\Delta U \\ V_b^t - V_a^t &= \left(\frac{q^t(1-q^t) - p^t[h_m + q^t(1-2h_m)]}{q^t(1-q^t)}\right)\Delta V \end{aligned}$$

with $\Delta U := U_{a_2 b} - U_{b_2 b} > 0$ and $\Delta V := V_{aa} - V_{ab} > 0$. Putting the expression of $U_b^t - U_a^t$ into the expression of P^t, we get that the previous inequality might be rewritten as:

$$\frac{q^t - h_m p^t}{(1-h_m)(1-p^t)} > \exp\left[\left(\frac{q^t - (1-h_m) + p^t(1-2h_m)}{1-p^t}\right)\frac{\Delta U}{\delta}\right]. \tag{B.4}$$

Let us define $\lambda := \frac{\Delta U}{\delta}(1-h_m)$ and $z^t := -\lambda\left(\frac{q^t - h_m p^t}{(1-h_m)(1-p^t)}\right)$. Then, the above inequality rewrites as:

$$\frac{-z^t}{\lambda} > \exp\left[-\lambda - z^t\right] \quad \Leftrightarrow \quad -\lambda\exp(-\lambda) > z^t\exp\left[z^t\right] =: g(z^t) \tag{B.5}$$

Then, we deduce from the analysis of function $g(z)$ developed in Appendix B.3 that:

- If $\lambda \leqslant 1$: $p^{t+1} < p^t$.
- If $\lambda > 1$:

$$p^{t+1}\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} p^t \quad \Leftrightarrow \quad z^t\begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \omega(\lambda)$$

  with $\omega(\lambda) := W_0(-\lambda\exp(-\lambda)) < 0$ where $W_0(.)$ is known as the Lambert W function. Finally, using the expression of z^t we get that the condition $z^t < \omega(\lambda)$ might be rewritten as:

$$q^t > p^t\left[h_m + \frac{\omega(\lambda)}{\lambda}(1-h_m)\right] - \frac{\omega(\lambda)}{\lambda}(1-h_m)$$

Now, considering equation (18) we deduce that:

$$q^{t+1}\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} q^t \quad \Leftrightarrow \quad Q^t\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \frac{q^t - p^t h_m}{q^t + p^t(1-2h_m)h_m}.$$

Let us focus on the condition $Q^t > \frac{q^t - p^t h_m}{q^t + p^t(1-2h_m)h_m}$. Putting the expression of $V_b^t - V_a^t$ into the expression of Q^t, we get that the previous inequality might be rewritten as:

$$\frac{q^t}{1-q^t}\left(\frac{p^t(1-h_m)}{q^t - p^t h_m}\right) > \exp\left[\left(1 - \frac{p^t[h_m + q^t(1-2h_m)]}{q^t(1-q^t)}\right)\frac{\Delta V}{\delta}\right].$$

The LHS of the above equation is increasing in p^t while the RHS is decreasing in p^t. Moreover, both the LHS and the RHS equals 1 when $p^t = \frac{q^t(1-q^t)}{h_m + q^t(1-2h_m)} = k_m(q^t)$. Hence we have

$$q^{t+1}\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} q^t \quad \Leftrightarrow \quad p^t\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} k_m(q^t).$$

Let us finally consider the case in which $(p^t, q^t) \in \Omega_3$. Using equation (18) and the expression of P^t

we get that

$$p^{t+1}\left\{\begin{array}{c}>\\=\\<\end{array}\right\}p^t \quad\Leftrightarrow\quad P^t\left\{\begin{array}{c}>\\=\\<\end{array}\right\}\frac{p^t-q^t+(1-p^t)(1-h_m)}{2-q^t-p^t-2(1-p^t)h_m}.$$

Let us focus on the condition $P^t > \frac{p^t-q^t+(1-p^t)(1-h_m)}{2-q^t-p^t-2(1-p^t)h_m}$. From Lemma 1 we can deduce that:

$$\begin{aligned}
U_b^t - U_a^t &= \left(\frac{q^t-(1-h_m)+p^t(1-2h_m)}{p^t}\right)\Delta U \\
V_b^t - V_a^t &= \left(\frac{(1-p^t)[1-h_m-q^t(1-2h_m)]-q^t(1-q^t)}{q^t(1-q^t)}\right)\Delta V
\end{aligned}$$

Putting the expression of $U_b^t - U_a^t$ into the expression of P^t, we get that the previous inequality might be rewritten as:

$$\frac{p^t(1-h_m)}{1-h_m+h_m p^t-q^t} > \exp\left[\left(\frac{q^t-(1-h_m)+p^t(1-2h_m)}{p^t}\right)\frac{\Delta U}{\delta}\right].$$

As previously, we define $\lambda := \frac{\Delta U}{\delta}(1-h_m)$ and we also define $k^t := -\lambda\left(\frac{1-h_m+h_m p^t-q^t}{p^t(1-h_m)}\right)$. Then, the above inequality rewrites as:

$$\frac{-\lambda}{k^t} > \exp\left[\lambda+k^t\right] \quad\Leftrightarrow\quad -\lambda\exp(-\lambda) > k^t\exp\left[k^t\right] =: g(k^t)$$

The, we have:

- If $\lambda \leqslant 1$: $p^{t+1} < p^t$.
- If $\lambda > 1$:

$$p^{t+1}\left\{\begin{array}{c}>\\=\\<\end{array}\right\}p^t \quad\Leftrightarrow\quad k^t\left\{\begin{array}{c}<\\=\\>\end{array}\right\}\omega(\lambda)$$

  with $\omega(\lambda)$ already defined. Finally, using the expression of z^t we get that the condition $k^t < \omega(\lambda)$ might be rewritten as:

$$q^t < (1-h_m)+p^t\left[h_m+\frac{\omega(\lambda)}{\lambda}(1-h_m)\right]$$

Now, considering (18) we deduce that:

$$q^{t+1}\left\{\begin{array}{c}>\\=\\<\end{array}\right\}q^t \quad\Leftrightarrow\quad Q^t\left\{\begin{array}{c}>\\=\\<\end{array}\right\}\frac{(1-h_m)(1-p^t)}{2-q^t-p^t-2(1-p^t)h_m}.$$

Let us focus on the condition $Q^t > \frac{(1-h_m)(1-p^t}{2-q^t-p^t-2(1-p^t)h_m}$. Putting the expression of $V_b^t - V_a^t$ into the expression

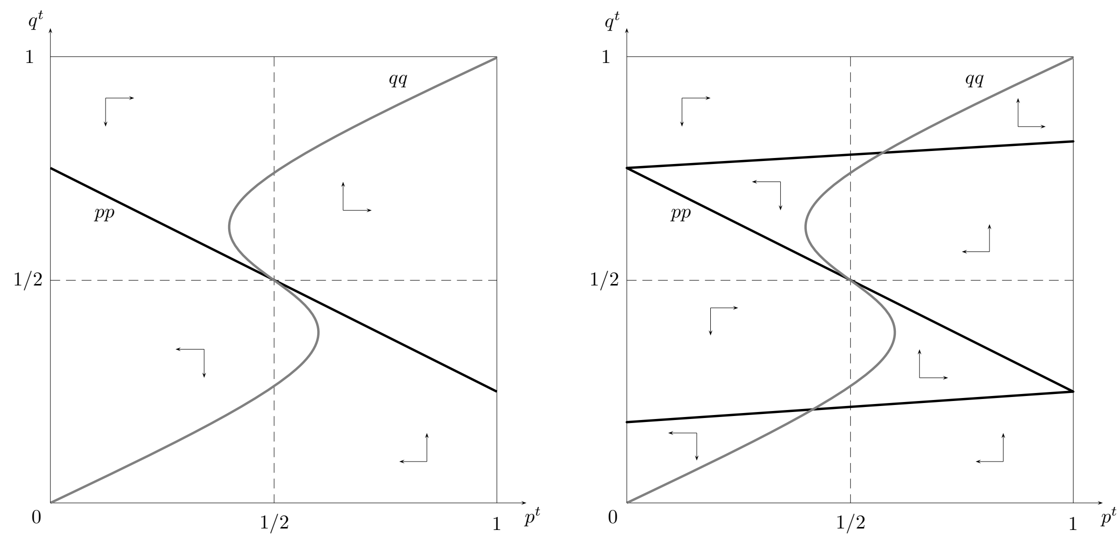


Figure B.8: Phase diagram

of Q^t, we get that the previous inequality might be rewritten as:

$$\frac{q^t}{1-q^t}\left(\frac{1-q^t-(1-p^t)h_m}{(1-p^t)h_m}\right) > \exp\left[\left(\frac{(1-p^t)[1-h_m-q^t(1-2h_m)]}{q^t(1-q^t)}-1\right)\frac{\Delta U}{\delta}\right].$$

The LHS of the above equation is increasing in p^t while the RHS is decreasing in p^t. Moreover, both the LHS and the RHS equals 1 when $p^t = 1 - \frac{q^t(1-q^t)}{1-h_m-q^t(1-2h_m)} = f_m(q^t)$. Hence we have

$$q^{t+1}\left\{\begin{array}{c} > \\ = \\ < \end{array}\right\} q^t \quad \Leftrightarrow \quad p^t \left\{\begin{array}{c} > \\ = \\ < \end{array}\right\} f_m(q^t).$$

From the above analysis, we get that the stationary locus of q^t corresponds to the qq curve already defined in Appendix A.5. While the stationary locus of p^t (pp locus) is given by the boundary between Ω_2 and Ω_3 and the two straight lines of equations:

$$\begin{aligned} q^t &= p^t\left[h_m + \frac{\omega(\lambda)}{\lambda}(1-h_m)\right] - \frac{\omega(\lambda)}{\lambda}(1-h_m) \\ q^t &= (1-h_m) + p^t\left[h_m + \frac{\omega(\lambda)}{\lambda}(1-h_m)\right] \end{aligned}$$

The corresponding phase diagram is illustrated in Figure B.8 (the motion arrows are also deduced from the above analysis).

Now we proceed to check the stability of the steady states. We first show that $(0,0)$ is locally asymptotically stable. Consider the region T_ε in which $p^t + q^t \leqslant \varepsilon$ for some $\varepsilon > 0$. Let us also define

$$\tilde{\Omega}_2 := \left\{(p^t, q^t) \in \Omega_2 : p^t > k_m(q^t)\right\},$$

which corresponds to the values of (p^t, q^t) that are in Ω_2 and are above the qq locus. As can be seen from Figure B.8 that for $(p^t, q^t) \in \Omega_2 \setminus \tilde{\Omega}_2$, when q^t is low enough we have $p^{t+1} < p^t$ and $q^{t+1} < q^t$ such that $p^{t+1} + q^{t+1} < p^t + q^t$. For any $(p^t, q^t) \in \Omega_1 \cup \Omega_2$, we have $p^{t+1} + q^{t+1} < p^t + q^t$ iff $P^t + Q^t < 1$. This condition holds true if both P^t and Q^t are lower than $1/2$. We find that the condition $P^t < 1/2$ might be reexpressed as

$$\frac{p^t}{1-p^t} < \exp\left[\left(2h_m - 1 - \frac{q^t}{p^t}\right)\frac{\Delta U}{\delta}\right] \quad \text{if} \quad (p^t, q^t) \in \Omega_1 \tag{B.6}$$

$$\frac{p^t}{1-p^t} < \exp\left[\left(\frac{q^t - (1-h_m) + p^t(1-2h_m)}{1-p^t}\right)\frac{\Delta U}{\delta}\right] \quad \text{if} \quad (p^t, q^t) \in \tilde{\Omega}_2 \tag{B.7}$$

The RHS of (B.6) is decreasing in q^t and, since $(p^t, q^t) \in \Omega_1$ we must have $q^t < p^t h_m$; while the RHS of (B.7) is increasing in q^t and, since $(p^t, q^t) \in \Omega_2$ we must have $q^t \geqslant p^t h_m$. Hence, replacing q^t by $p^t h_m$, we get that the RHS of both equations are bounded below by $\exp(-\lambda)$ (remember that $\lambda = \frac{\Delta U}{\delta}(1-h_m)$). Hence, a sufficient condition for both inequalities to hold is $\frac{p^t}{1-p^t} < \exp(-\lambda)$ which is obvioulsy verified for sufficiently low p^t.

Let us now consider the condition $Q^t < 1/2$. It can be rewritten as

$$\frac{q^t}{1-q^t} < \exp\left[\left(\frac{q^t - p^t}{1-q^t}\right)\frac{\Delta V}{\delta}\right] \quad \text{if} \quad (p^t, q^t) \in \Omega_1 \tag{B.8}$$

$$\frac{q^t}{1-q^t} < \exp\left[\left(1 - \frac{p^t[h_m + q^t(1-2h_m)]}{q^t(1-q^t)}\right)\frac{\Delta V}{\delta}\right] \quad \text{if} \quad (p^t, q^t) \in \tilde{\Omega}_2 \tag{B.9}$$

The RHS of both inequalities are decreasing in p^t. Hence, the RHS of (B.8) is minimized when $p^t = q^t/h_m$. Hence, a sufficient condition for inequality (B.8) to be satisfied is

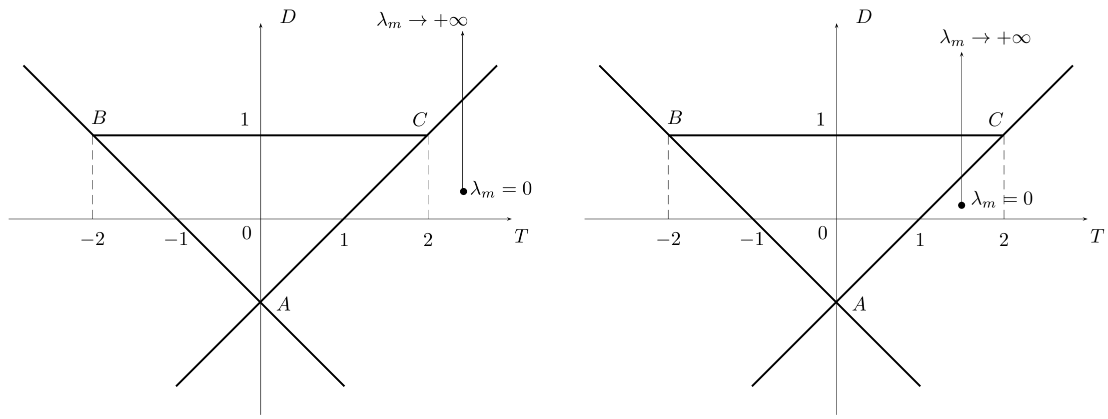
$$\frac{q^t}{1-q^t} < \exp\left[\frac{-\lambda}{h_m}\left(\frac{q^t}{1-q^t}\right)\right],$$

which is obviously satisfied for sufficiently low q^t. Indeed the LHS is increasing in q^t and equals to 0 when $q^t = 0$ while the RHS is decreasing in q^t and equals to 1 when $q^t = 0$. Finally, the RHS of (B.9) is minimized when $p^t = k_m(q^t)$. Hence, inequality (B.9) is satisfied if $\frac{q^t}{1-q^t} < 1$ which is true for sufficiently low q^t.

We have proven that, there exists a positive value of ε such that for all $(p^t, q^t) \in T_\varepsilon$, $p^{t+1} + q^{t+1} < p^t + q^t$. Then, we can show that $(0,0)$ is locally asymptotically stable. Similar arguments might be mobilized to show that $(1,1)$ is locally asymptotically stable.

The stability of $(1/2, 1/2)$ is analyzed below, in the proof of Proposition 9. □

Proof of Proposition 9. In order to analyze the stability of $(\frac{1}{2}, \frac{1}{2})$ we check the Jacobian matrix of the



(a) $h_m \leqslant 1/2$

(b) $h_m > 1/2$

Figure B.9: Local stability property of $(1/2, 1/2)$

dynamics evaluated at $(\frac{1}{2}, \frac{1}{2})$, which is (in both regions Ω_2 and Ω_3):

$$\begin{bmatrix} \frac{(3-2h_m)(1+h_m\lambda_m)-\lambda_m}{2} & \frac{1-(1-h_m)\lambda_m}{2} \\ \frac{1+(1-h_m)\lambda_w}{2} & \frac{(3-2h_m)(1-h_m\lambda_w)+\lambda_w}{2} \end{bmatrix}$$

with $\lambda_m := \Delta U/\delta$ and $\lambda_w := \Delta V/\delta$. To assess the behavior of the dynamic system in the neighborhood of $(1/2, 1/2)$ we can rely on the comparison between the Determinant (D) and the Trace (T) of the Jacobian matrix. Simple algebra lead us to conclude that:

$$\begin{aligned} D &= \frac{(1-h_m)\left\{4-2h_m[1-(1-h_m)^2\lambda_m^2]+\Delta\lambda\left[1-2h_m[2-h_m-(1-h_m)^2\lambda_m]\right]\right\}}{2} \\ T &= \frac{6-4h_m+\Delta\lambda(1-h_m)(1-2h_m)}{2} \end{aligned}$$

with $\Delta\lambda := \lambda_w - \lambda_m$. As proposed by Grandmont et al. (1998), we analyze the stability properties of $(1/2, 1/2)$ by studying the variations of T and D in the (T, D) plane as one of the parameters, namely λ_m, varies continuously on its admissible range $[0, +\infty)$. In the (T, D) plane, the steady state is a sink if $|T| < D+1$ and $D < 1$ (inside the triangle ABC in Figure B.9); it is a saddle point if (T, D) lies either on the left side of the line (AB) or on the right side of the line (AC) ($|T| > |D+1|$). It is a source anywhere else.

As a preliminary result, let us show that, D_0 (the value of D when $\lambda_m = 0$) is higher than $-T-1$ when $\Delta V \leqslant \Delta U + \frac{4\delta}{3h_m-1}$ ($\Leftrightarrow \Delta\lambda \leqslant \frac{4}{3h_m-1}$). After some algebra, the condition $D_0 \geqslant -T-1$ rewrites as $\Delta\lambda(1-h_m)[h_m(3-h_m)-1] \leqslant (3-h_m)(2-h_m)$ which is always true if $h_m(3-h_m) < 1$; otherwise, it holds if

$$\Delta\lambda \leqslant \frac{(3-h_m)(2-h_m)}{(1-h_m)[h_m(3-h_m)-1]}.$$

Observing that the RHS of the above inequality is higher than $\frac{4}{3h_m-1}$ for all values of h_m such that $h_m(3-h_m) \geqslant 1$, we conclude that $D_0 \geqslant -T-1$. Hence, when $\lambda_m = 0$, the point (T, D) is always above the (AB)

line as depicted in Figure B.9.

Let us now consider the case $h_m < 1/2$, illustrated in Figure B.9a. Note that, in that configuration, T is increasing in $\Delta\lambda \geqslant 0$. Hence, $T \geqslant 3 - 2h_m \geqslant 2$. Note also that D is strictly increasing in λ_m while T is independent of λ_m and $D = T - 1$ iff $\lambda_m = 1/(1-h_m)$. Hence, for $\lambda_m = 0$, (T, D) lies on the right of the (AC) line. Then as λ_m increases, (T, D) is upward shifted and crosses the (A, C) line when $\lambda_m = 1/(1-h_m)$ ($\Leftrightarrow$ $\delta = (1-h_m)\Delta U$). Since $T \geqslant 2$, when (T, D) crosses the (AC) line, the local stability property of $(1/2, 1/2)$ changes from saddle to source.

Let us finally consider the case $h_m > 1/2$, illustrated in Figure B.9b. In that case, T is decreasing $\Delta\lambda$ such that $T < 2$. Moreover, $T \geqslant 1$ iff $\lambda_m < \frac{4}{2h_m-1}$ which is true since we have assumed $\lambda_m < \frac{4}{3h_m-1}$. Again, D is increasing in λ_m and is equal to $T-1$ when $\lambda_m = 1/(1-h_m)$. Hence, for $\lambda_m = 0$, (D, T) lies on the right of the (AC) line. Then as λ_m increases, (D, T) is upward shifted and crosses the (A, C) line when $\lambda_m = \frac{1}{1-h_m}$ ($\Leftrightarrow$ $\delta = (1-h_m)\Delta U$). Since $T \in [1, 2)$, when (T, D) crosses the (AC) line, it enters the (ABC) triangle such that the local stability property of $(1/2, 1/2)$ changes from saddle to sink. Then, as λ_m further increases, it crosses the line (BC) line, then the local stability property of $(1/2, 1/2)$ changes from sink to source. □

C Omitted details with imperfect vertical transmission in homogamies

C.1 Microfounding cultural substitutability in homogamies

Let us consider a couple in which both spouses hold the trait i. They have the possibility to transmit this trait to her daughter (symmetric reasonning would apply for their son) with a probability τ_i^t. This probability corresponds to a socialization effort which cost is given by a strictly increasing and convex function $c\left(\tau_i^t\right)$ with $c(0) = 0$ and $\partial c(0)/\partial \tau_i^t = 0$. We also assume a form of *cultural intolerance* (Bisin and Verdier, 2001): Parents prefer their child not to deviate from their own culture. To make things as simple as possible, we assume that parents derive a utility $v \in (0, 1)$ from having a child belonging to the group i while this utility is normalized to 0 if their child belongs to the group $j \neq i$. Then, being given the cultural transmission process describes in Section 5, parents choose their socialization effort τ_i^t in order to maximize:

$$\tau_i^t v + (1-\tau_i^t) q^t v - c\left(\tau_i^t\right).$$

Hence, the optimal socialization effort (that also corresponds to the probability of direct vertical transmission) must be such that:

$$(1-q^t)v = c'\left(\tau_i^t\right).$$

By the properties of $c(.)$ it is clear that the corresponding value of τ_i^t is decreasing in q^t and equal 0 when $q^t = 1$.

Note that, if we assume the following quadratic form for the cost function, $c\left(\tau_i^t\right) = \frac{\left(\tau_i^t\right)^2}{2}$, the optimal socialization effort is linear in q^t: $\tau_t^i = (1-q^t)v$. In that case, the condition stated in Proposition 12,

$$h_m > \frac{d(1/2)}{d(1/2) - d'(1/2)/2}$$

is simply rewritten as $h_m > 1/2$.

C.2 Proof of Proposition 10

Proof of Proposition 10. We look for steady states p^* and q^*. We first show that there is no steady state such that $p^* \neq q^*$. Suppose there is. Without loss of generality, by symmetry, suppose $p^* > q^*$; by cultural substitutability, $d(p^*) < d(q^*)$ and $d(1-p^*) > d(1-q^*)$. The cultural evolution equations become

$$\begin{aligned} 0 &= q^* d(p^*)(1-p^*) - (1-p^*)d(1-p^*)p^*; \\ 0 &= q^* d(q^*)(1-q^*) - (1-p^*)d(1-q^*)q^*. \end{aligned}$$

Equating the two equations yields

$$q^*[d(p^*)(1-p^*) - d(q^*)(1-q^*)] = (1-p^*)[d(1-p^*)p^* - d(1-q^*)q^*].$$

By $p^* > q^*$, because $d(p^*) < d(q^*)$, the terms in the square brackets on the left-hand side of the equation are negative, and because $d(1-p^*) > d(1-q^*)$, the terms in the square brackets on the right-hand side of the equation are positive. Hence, the equation cannot hold, and we cannot have a steady state such that $p^* > q^*$ (or $p^* < q^*$ by symmetry).

We now consider the gender-symmetric steady states such that $p^* = q^* := r$. The steady state must satisfy

$$r(1-r)[d(r) - d(1-r)] = 0.$$

The equation holds when $r = 0$, $r = 1$, or $r = 1/2$. First, we show that $(0, 0)$ and $(1, 1)$ are unstable. Take any $p^t = q^t := r^0$. The system of equations becomes

$$\begin{aligned} p^{t+1} - p^t &= r^0(1-r^0)[d(r^0) - d(1-r^0)]; \\ q^{t+1} - q^t &= r^0(1-r^0)[d(r^0) - d(1-r^0)]. \end{aligned}$$

Both are positive if $r^0 < 1/2$ and negative if $r^0 > 1/2$. Hence, $(0, 0)$ and $(1, 1)$ cannot be stable. Next, we prove that $(1/2, 1/2)$ is stable. Maintain the assumption $p^t \geqslant q^t$. When $p^t > 1/2$,

$$p^{t+1} - p^t = (1-p^t)[q^t d(p^t) - p^t d(1-p^t)] < 0,$$

where the inequality is derived from $d(p^t) < d(1-p^t)$ and $q^t \leqslant p^t$. Similarly, when $q^t < 1/2$,

$$q^{t+1} - q^t = q^t[d(q^t)(1-q^t) - (1-p^t)d(1-q^t)] > 0.$$

When $p^t < 1/2$, $q^t d(p^t) - p^t d(1-p^t) > 0$ when $p^t = q^t$, so by continuity of $d(\cdot)$, $p^{t+1} - p^t$ is positive for (p^t, q^t) sufficiently close to $(1/2, 1/2)$. Similarly, when $q^t > 1/2$, $q^{t+1} - q^t$ is negative for (p^t, q^t) sufficiently close to $(1/2, 1/2)$, because $d(q^t)(1-q^t) - (1-p^t)d(1-q^t) < 0$ when $p^t = q^t$.

We can analogously derive results on the sign of $p^{t+1} - p^t$ and $q^{t+1} - q^t$ for the case $p^t \leqslant q^t$.

The Lyapunov function that helps prove global stability is

$$\mathcal{V}(p,q) = \begin{cases} (q-1/2)^2 & \text{if } p \geqslant q \text{ and } p+q<1 \\ (p-1/2)^2 & \text{if } p \geqslant q \text{ and } p+q \geqslant 1 \\ (q-1/2)^2 & \text{if } p < q \text{ and } p+q \geqslant 1 \\ (p-1/2)^2 & \text{if } p < q \text{ and } p+q<1 \end{cases}$$

The function satisfies: (i) $\mathcal{V}(1/2,1/2)=0$, (ii) $\mathcal{V}(p^{t+1},q^{t+1}) < \mathcal{V}(p^t,q^t)$ for all $(p^t,q^t) \neq (1/2,1/2)$, and (iii) $\mathcal{V}(p^t,q^t) > 0$ for all (p^t,q^t), and (iv) $\|(p,q)\| \to \infty, \mathcal{V}(p,q) \to \infty$. In addition, the dynamic system is Lipschitz continuous, because $d(\cdot)$ is Lipschitz by assumption ($d(\cdot)$ differentiable and bounded on the closed interval $[0,1]$ implies a bounded first derivative, which implies Lipshitz continuity). By Theorem 1.4 of Bof et al. (2018), the existence of such a Lyapunov function implies global asymptotic stability. □

C.3 Proof of Proposition 11

Proof of Proposition 11. When $p^t+q^t<1$, the cultural evolution is characterized by

$$p^{t+1}-p^t = -\mu_{bb}^t d(1-p^t)p^t = -(1-p^t-q^t)d(1-p^t)p^t < 0; \tag{C.10}$$
$$q^{t+1}-q^t = -\mu_{bb}^t d(1-q^t)q^t = -(1-p^t-q^t)d(1-q^t)q^t < 0. \tag{C.11}$$

By Equations (C.10) and (C.11), the system tends toward $(0,0)$. Using the same argument of the contraction mapping theorem as in Proposition 2, we can say that for any (p^0,q^0) such that $p^0+q^0<1$, $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t = 0$. When $p^t+q^t>1$, the cultural evolution is characterized by

$$p^{t+1}-p^t = \mu_{aa}^t d(p^t)(1-p^t) = (p^t+q^t-1)d(p^t)(1-p^t) > 0; \tag{C.12}$$
$$q^{t+1}-q^t = \mu_{aa}^t d(p^t)(1-q^t) = (p^t+q^t-1)d(p^t)(1-q^t) > 0. \tag{C.13}$$

By Equations (C.12) and (C.13), the system tends toward $(1,1)$. Using the same argument of the contraction mapping theorem as in Proposition 2, we can say that for any (p^0,q^0) such that $p^0+q^0>1$, $\lim_{t\to\infty} p^t = \lim_{t\to\infty} q^t = 1$. And when $p^t+q^t=1$, the cultural evolution is characterized by

$$\begin{aligned} p^{t+1}-p^t &= 0; \\ q^{t+1}-q^t &= 0. \end{aligned}$$

In summary, the asymptotically stable steady states are $(0,0)$ and $(1,1)$, and any (p^*,q^*) such that $p^*+q^*=1$ is an unstable steady state. □

C.4 Proof of Proposition 12

Proof of Proposition 12. First, suppose receivers are homophilic. The different phases of cultural evolution are depicted by Figure 5a. When $(p^t,q^t) \in \overline{\Omega}_1 = \{(p,q) : q \leqslant ph\}$, the cultural evolution is characterized by

$$\begin{aligned} p^{t+1}-p^t &= q^t d(p^t)(1-p^t)-(1-p^t)d(1-p^t)p^t; &\text{(C.14)}\\ q^{t+1}-q^t &= q^t d(q^t)(1-q^t)-(1-q^t)d(1-q^t)q^t = q^t(1-q^t)[d(q^t)-d(1-q^t)]. &\text{(C.15)}\end{aligned}$$

equation (C.15) indicates that only $q=0$, $q=1$, or $q=1/2$ can be part of a steady state in $\overline{\Omega}_1$. Because $h<1$, $q=1$ cannot be in $\overline{\Omega}_1$. When $q=1/2$, the evolution of p^t becomes

$$p^{t+1}-p^t=(1-p^t)[d(p^t)/2-d(1-p^t)p^t],$$

which equates 0 only when $p^t=1/2$ or $p^t=1$. However, neither $(1/2,1/2)$ nor $(1,1/2)$ is in $\overline{\Omega}_1$. When $q=0$, the evolution of p^t becomes

$$p^{t+1}-p^t=-(1-p^t)d(1-p^t)p^t,$$

which equates 0 only when $p^t=0$ or $p^t=1$. Hence, in $\overline{\Omega}_1$, only $(0,0)$ or $(1,0)$ can be a steady state. First, consider the neighborhood of $(1,0)$ and take the element $(p^t,q^t)=(1-\varepsilon,\varepsilon)$. The evolution of p^t becomes

$$p^{t+1}-p^t=\varepsilon d(1-\varepsilon)\varepsilon-\varepsilon d(\varepsilon)(1-\varepsilon)=\varepsilon[\varepsilon d(1-\varepsilon)-(1-\varepsilon)d(\varepsilon)],$$

which is negative for any ε sufficiently small. Hence, $(1,0)$ is not stable. Consider $(0,0)$ and take $(p^t,q^t)\in\overline{\Omega}_1$. The evolution of q^t is

$$q^{t+1}-q^t=q^t(1-q^t)[d(q^t)-d(1-q^t)],$$

which is positive for any $q^t\in(0,1/2)$. Hence, no evolution with initial state in $\overline{\Omega}_1$ converges to $(0,0)$.

When $(p^t,q^t)\in\overline{\Omega}_2=\{(p,q):ph_m\leqslant q\leqslant ph_m+(1-p)(1-h_m)\}$, the cultural evolution is

$$\begin{aligned} p^{t+1}-p^t &= p^t h_m d(p^t)(1-p^t)-[1-q^t-p^t(1-h_m)]d(1-p^t)p^t;\\ q^{t+1}-q^t &= p^t h_m d(q^t)(1-q^t)-[1-q^t-p^t(1-h_m)]d(1-q^t)q^t.\end{aligned}$$

A steady state (p^*,q^*) satisfies

$$\begin{aligned} 0 &= p^* h_m d(p^*)(1-p^*)-[1-q^*-p^*(1-h_m)]d(1-p^*)p^*;\\ 0 &= p^* h_m d(q^*)(1-q^*)-[1-q^*-p^*(1-h_m)]d(1-q^*)q^*.\end{aligned}$$

Equating the two equations above yields

$$p^*h_m[d(p^*)(1-p^*)-d(q^*)(1-q^*)]+[1-q^*-p^*(1-h_m)][d(1-q^*)q^*-d(1-p^*)p^*]=0.$$

We claim that $p^*=q^*$. Suppose by contradiction that $p^*\neq q^*$, and without loss of generality, suppose $p^*>q^*$. The left-hand side is strictly negative, because $d(p^*)(1-p^*)<d(q^*)(1-q^*)$ and $d(1-q^*)q^*<d(1-p^*)p^*$. Hence, a contradiction for the equation arises.

Now we can consider steady states $(p^*, q^*) = (r, r)$ such that r satisfies

$$rh_m d(r)(1-r) - [1-r-r(1-h_m)]d(1-r)r = 0 \Leftrightarrow r[h(1-r)d(r) - (1-2r+rh_m)d(1-r)] = 0.$$

Two of the solutions of the equation are $r = 0$ and $r = 1/2$. We have shown that $(0, 0)$ is not stable. We now investigate the stability of $(1/2, 1/2)$. The Jacobian matrix near $(1/2, 1/2)$ is continuous and evaluated at $(1/2, 1/2)$ is

$$\mathcal{J}|_{p=1/2,q=1/2} = \begin{bmatrix} \frac{1}{2}h_m d'(\frac{1}{2}) + (\frac{1}{2} - h_m)d(\frac{1}{2}) & \frac{1}{2}d(\frac{1}{2}) \\ \frac{1}{2}d(\frac{1}{2}) & \frac{1}{2}h_m d'(\frac{1}{2}) + (\frac{1}{2} - h_m)d(\frac{1}{2}) \end{bmatrix}. \tag{C.16}$$

The two eignenvalues are $-2d(1/2)+d'(1/2)h_m$ and $2d(1/2)-2d(1/2)h_m+d'(1/2)h_m$. Because $d' < 0$, the first eigenvalue is negative. If the second eigenvalue is negative, then $(1/2, 1/2)$ is asymptotically stable; otherwise, it is a saddle point. The second eigenvalue is negative if

$$h_m > \frac{d(1/2)}{d(1/2) - d'(1/2)/2},$$

which is the specified condition for $(1/2, 1/2)$ to be stable.

Suppose there is a mixture of homophilic and heterophilic receivers ($0 < h_w < 1$) in addition to a mixture of homophilic and heterophilic proposers. Consider the stable matching characterized in Figure A.2 in Section A.6. When $(p^t, q^t) \in \Omega_5 \cap \{q^t \geqslant g_m(p^t)\}$, the evolution is exactly the same as equation (C.16). When $(p^t, q^t) \in \Omega_5 \cap \{q^t < g_m(p^t)\}$, the evolution is

$$\begin{aligned} p^{t+1} - p^t &= [q^t - (1-p^t)(1-h_m)]d(p^t)(1-p^t) - (1-p^t)h_m d(1-p^t)p^t; \\ q^{t+1} - q^t &= [q^t - (1-p^t)(1-h_m)]d(q^t)(1-q^t) - (1-p^t)h_m d(1-q^t)q^t. \end{aligned}$$

The Jacobian matrix evaluated at $(1/2, 1/2)$ is the same as equation (C.16). Hence, the condition for the stability of $(1/2, 1/2)$ is the same for when all receivers are homophilic and when there is a strict mixture of homophilic and heterophilic receivers.

Suppose all receivers are heterophilic (and there is a strict mixture of homophilic and heterophilic proposers). Suppose $q^t < (1-p^t)(1-h_m)$. The evolution becomes

$$\begin{aligned} p^{t+1} - p^t &= -(1-p^t-q^t)d(1-p^t)p^t < 0; \\ q^{t+1} - q^t &= -(1-p^t-q^t)d(1-q^t)q^t < 0. \end{aligned}$$

Hence, $(0, 0)$ is an asymptotically stable steady state by the contraction mapping theorem. Similarly, when receivers are all heterophilic, $(1, 1)$ is also an asymptotically stable steady state. □

D Omitted proofs with implications

D.1 Proof of Lemma 2

Proof of Lemma 2. The two following intermediary results are useful for proving Lemma 2.

Lemma 3. *At any stable matching, if there exist a positive mass of unmatched type-a men, all type-b women must be matched with a type-a man; and if there exists a positive mass of unmatched type-b men, all type-a women must be matched with a type-b man.*

Proof. Let us prove the first point, the proof of the second point would follow exactly the same logic. By contradiction, assume that there exists a positive mass of unmatched type-a men and a positive mass of type-b woman who are either unmatched or matched with a type-b man. Then these women will form blocking pairs with the unmatched type-a men. Hence, the matching cannot be stable. □

Lemma 4. *At any stable matching, either all type-b women are matched with a type-a man; or all type a-women are matched with a type b-man.*

Proof. There are more men than women such that, at any matching, some men (either of type-a, of type-b or of both types) will remain unmatched. The result of the Lemma is directly derived from this simple fact associated with Lemma 3. □

We can now successively address the three configurations listed in Lemma 2. First, if $q < \phi_1(p)$, there are more type-b women than type-a men such that not all type-b women can be matched with a type-a man. Hence, by Lemma 4, all type-a women must be matched with a type-b man implying that $\mu_{ba} = q$ and $\mu_{aa} = 0$. Since type-b women prefer to be matched with type-a rather than type-b men while type-a men prefer to be matched with type-b women rather than remain unmatched, all type-a men must be matched with type-b women. The remaining type-b women will be matched with type-b men. Hence, $\mu_{ab} = (1+\lambda)p$ and $\mu_{bb} = 1 - q - (1+\lambda)p$.

Second, if $q > \phi_2(p)$, not all type-a women can be matched with a type-b man. Hence, by Lemma 4, all type-b women must be matched with a type-a man such that $\mu_{ab} = 1 - q$ and $\mu_{aa} = 0$. Since type-a women prefer to be matched with type-b rather than type-a men while type-b men prefer to be matched with type-a women rather than remain unmatched, all type-b men must be matched with type-a women. The remaining type-a women will be matched with type-a men. Hence, $\mu_{ba} = (1+\lambda)(1-p)$ and $\mu_{aa} = q - (1+\lambda)(1-p)$.

Third and finally, if $q \in [\phi_1(p), \phi_2(p)]$, if all type-$b$ women are matched with a type-a man, the remaining type-a women want to be matched with type-b rather than type-a men and type-b men prefer to be matched with type-a women rather than remain unmatched. Hence, all type-a women are matched with a type-b man. A symmetric reasoning applies when all type-a women are matched with a type-b man. Hence, in both cases, $\mu_{ab} = 1 - q$, $\mu_{ba} = q$ and $\mu_{aa} = \mu_{bb} = 0$. □