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Borrowing Stigma and Lender of Last Resort Policies

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Abstract

How should the lender of last resort provide liquidity to banks during periods of financial distress? During the 2008–2010 crisis, banks avoided borrowing from the Fed's long-standing discount window but actively participated in its special monetary program, the Term Auction Facility, although both programs had the same borrowing requirements. Using an adverse selection model with endogenous borrowing decisions, we explain why the two programs suffer from different stigma costs and how the introduction of TAF incentivized banks' borrowing. We discuss the empirical relevance of the model's predictions.

> [Banks] deliberately did not ask for the liquidity they needed for fear of damaging their reputation—the 'stigma' problem… I do not think we were conscious of this before the crisis started and I do not think central banks have a convincing answer to it… This is, I think, still a challenge in how to manage the process of central bank provision of liquidity support. This is one of the big intellectual issues that has not been fully resolved. (Governor Mervyn King, Bank of England (2016))

> For various reasons, including the competitive format of the auctions, [Term Auction Facility] has not suffered the stigma of conventional discount window lending and has proved effective for injecting liquidity into the financial system… Another possible reason that [Term Auction Facility] has not suffered from stigma is that auctions are not settled for several days, which signals to the market that auction participants do not face an immediate shortage of funds. (Ben Bernanke, testimony to U.S. House of Representatives (2010))

I. Introduction

Financial crises are typically accompanied by liquidity shortages in the banking sector, in which case the central bank should act as the lender of last resort (LOLR) (Bagehot (1873)). How should LOLR lend to depository institutions and

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provide liquidity during such episodes? The answer is not obvious. The discount window (DW) has been the primary lending facility used by the Fed, but it was severely underutilized when the interbank market froze at the beginning of the financial crisis in late 2007 (Armantier, Ghysels, Sarkar, and Shrader (2015)). A main reason for the underutilization is believed to be the stigma associated with DW borrowing: Tapping DW conveys a negative signal about borrowers' financial condition to their counterparties, competitors, regulators, and the public.[1]

In response to the credit crunch and banks' reluctance to borrow from DW, the Fed created a temporary program, the Term Auction Facility (TAF), in Dec. 2007. TAF held an auction every other week and provided a preannounced amount of loans with *identical* loan maturity, collateral margins, and eligibility criteria to those of DW.

Surprisingly, TAF provided much more liquidity than DW: Graph A of Figure 1 shows that the outstanding balance in TAF far exceeded that in DW during 2007–2010; the outstanding balance in DW was sometimes less than one-fifth of that in TAF between 2007 and 2010. Even more surprisingly, banks sometimes paid a higher interest rate to obtain liquidity through TAF auction: Graph B shows that the *stop-out rate* (the rate that clears the auction) was higher than the concurrent *discount rate* (the rate readily available in DW) in 21 of the 60 auctions, especially from Mar. to Sept. 2008, the peak of the financial crisis.[2]

This episode suggests the importance of the design of emergency lending programs to cope with liquidity shortages effectively. In particular, it raises a series of questions about LOLR policies. Why could TAF overcome the stigma and generate more borrowing than DW? Should not the same stigma also prevent banks from participating in TAF? How did banks decide to borrow from DW and/or TAF? Was there any systematic difference between the banks that borrowed from the two facilities? How could the program be further improved? There is no consensus on the answers to the questions (Armantier and Sporn (2013), Bernanke (2015)).

This study provides a theory of LOLR in the presence of borrowing stigma. We introduce a model in which banks have private information about their financial condition. Weaker banks have more urgent liquidity needs and enjoy higher

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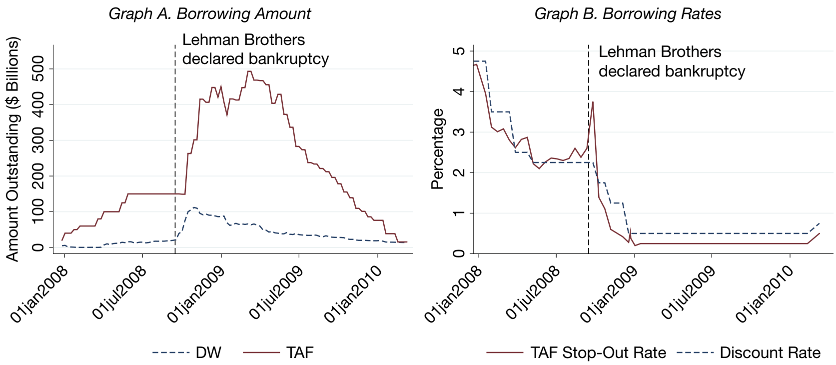
[1]Banks have regularly paid more for loans from the interbank market than for loans they could readily get from DW (Peristiani (1998), Furfine (2001), (2003), (2005)). Although the Fed does not publicly disclose which institutions have received loans from DW, the Board of Governors publishes weekly the total amount of DW lending by each of the 12 Federal Reserve districts. Therefore, a surge in total DW borrowing could send the market scrambling to identify loan recipients. Because of the interconnectedness of the interbank lending market, it is not impossible for other banks to infer which institutions went to DW. Market participants and social media could also infer from other activities. See footnote 7 for some anecdotal evidence.

[2]The stop-out rate ranged from 1.5 percentage points above (on Sept. 25, 2008) to 0.83 percentage points below (on Dec. 4, 2008) the concurrent discount rate. The stop-out rate was above the concurrent discount rate for almost all auctions between Mar. 2008 (when Bear Stearns filed for bankruptcy) and Sept. 2008 (when Lehman Brothers filed for bankruptcy).

FIGURE 1

Borrowing Amounts and Rates in DW and TAF from 2008 to 2010

Figure 1 plots the borrowing amount and borrowing interest rates of the DW and the TAF between Dec. 2007 and Apr. 2010. In both graphs, the red solid line describes TAF borrowing, whereas the blue dashed line shows DW borrowing. Source: Federal Reserve Board.



borrowing benefits. Two lending facilities are available. An auction allocates a set amount of liquidity, and DW is always available—before, during, and after the auction. Importantly, TAF delays its release of funds. Borrowing from each facility incurs a stigma cost, which is endogenously determined by the financial condition of participating banks.

In equilibrium, banks self-select into different programs. The weakest banks borrow immediately from DW because they have the highest demand for liquidity, and it will be very costly for them to wait. Stronger banks, in contrast, are lured to participate in the auction because the potential of borrowing cheap renders the auction more attractive than DW. Their liquidity needs are not as imperative, and they value the lower expected price in the auction more than weaker banks do. Some banks that participate in TAF may bid higher than the discount rate because they would like to avoid DW stigma brought by being pooled with the weakest banks. As a result, the clearing price in the auction may exceed the discount rate. Of the banks that have lost in TAF, relatively weaker ones might still borrow from DW.

We demonstrate that TAF, used in accordance with DW, could increase liquidity provision through three channels. First, by setting a low reserve price in the auction, TAF attracted moderately weak banks (that would have borrowed from DW without TAF) to participate and take their chances on borrowing cheaply. Second, participating banks can submit bids to internalize any stigma cost associated with TAF, so TAF also attracted moderately strong banks (that would not have borrowed at all without TAF) to participate. Finally, due to the selection by stronger banks into the auction, the auction stigma is endogenously lower than DW stigma, which further encourages stronger banks to participate in TAF. Hence, the combination of TAF and DW expands the set of banks who try to, and may obtain, liquidity, thus increasing the overall supply of short-term credit to the economy.

Our model generates some empirically testable implications. First, financially weaker banks borrowed relatively more from DW than TAF, compared with their stronger peers. Given so, DW carries a higher stigma cost than TAF. This result also explains why banks might want to bid more in TAF than the concurrent discount rates. Moreover, DW alone may not effectively provide liquidity during the crisis. Indeed, when banks face higher liquidity risks, they might borrow less from DW. In addition, introducing TAF could further increase the stigma of DW relative to the situation when there is only DW.

Literature

The paper contributes to the literature on LOLR policies, starting from Bagehot (1873). Freixas, Giannini, Hoggarth, and Soussa (1999) offer an earlier review of this literature. Theoretically, our paper discusses how to design LOLR facilities to mitigate the participation stigma. Philippon and Skreta (2012) and Tirole (2012) use a mechanism design approach to study government intervention in markets plagued by adverse selection. In the dynamic context, Fuchs and Skrzypacz (2015) show trading restrictions and subsidies could be optimal. Our paper contributes to this literature by allowing for multiple and dynamic policy intervention programs, which have the potential to separate heterogeneous participants. We show how one program could have a higher stigma cost than the other, although both have *identical* requirements. More relevantly, our paper contributes to the theoretical understanding of LOLR (Rochet and Vives (2004)) and the associated stigma (Ennis and Weinberg (2013), Lowery (2014), and Ennis (2019)). La'O (2014) also explains how TAF may alleviate DW stigma from the perspective of predatory trading. The explanation focuses on the signaling perspective of TAF borrowing. We offer a complementary explanation of how delayed funding settlement creates separation, which according to Bernanke (2015) is crucial to the design of TAF. Moreover, La'O (2014) predicts that in equilibrium, banks always pay a premium for TAF loans over the discount rate, which is at odds with the empirical observation. Che, Choe, and Rhee (2024) show that a stigma could have a salutary effect: Refusing bailouts could be a useful signal that firms send to their market participants. Gorton and Ordoñez (2020) also study central bank liquidity provision and show that stigma is desirable to implement opacity. Our paper rationalizes the borrowing behavior in the last financial crisis and improves the understanding of appropriate interventions during a financial crisis.

The rest of the paper is organized as follows: Section II describes LOLR facilities during the financial crisis. Section III sets up the model. Section IV characterizes the equilibrium of the model and discusses liquidity provision under different settings. Section V discusses the empirical relevance of the model. Section VI concludes, and the Appendix contains omitted proofs, while the Supplementary Material contains our empirical analysis.

II. Background

Stress in the interbank lending market began to loom in the summer of 2007 (Figure 1 of Angelini, Nobili, and Picillo (2011)). Two of Bear Stearns' mortgage-heavy hedge funds reported large losses in June. On July 31, they declared bankruptcy. On Aug. 9, BNP Paribas, France's largest bank, barred investors from withdrawing money from investments backed by U.S. subprime mortgages, citing evaporated liquidity as the main reason (Paribas (2007)). Subsequently, many other banks and financial institutions experienced liquidity dry-ups in wholesale funding in the form of asset-backed commercial paper or repurchase agreements (see Kacperczyk and Schnabl (2010), Gorton and Metrick (2012)).

With the growing scarcity of short-term funding, banks were supposed to borrow from LOLR.[3] In the United States, the role of LOLR has largely been fulfilled by DW, which allows eligible institutions—mostly commercial banks—to borrow money from the Fed on a short-term basis to meet temporary shortages of liquidity caused by internal and external disruptions. DW loans were extended to sound institutions with good collateral. Since its founding in 1913, the Fed has never lost a penny on a DW loan.[4] However, banks were reluctant to use DW, due to the widely held perception that a stigma was associated with borrowing from the Fed. As advised by Bagehot (1873), a penalty—1 percentage point above the target federal funds rate—was charged on DW loans to encourage banks to look first to private markets for funding. However, this penalty generated a side effect for banks: Banks would look weak if it became known that they had borrowed from the Fed.

Individual banks' DW borrowing was kept confidential.[5] However, banks were nervous that investors, particularly money market participants, could guess when they had come to the window by observing banks' behavior and carefully analyzing the Fed's balance sheet figures.[6]

The Fed subsequently made a few changes to DW policies. In particular, on Aug. 16, 2007, it halved the interest rate penalty on DW loans. The maturity of loans was also extended from overnight to up to 30 days with an implicit promise of further renewal. Moreover, the Fed tried to persuade some leading banks to borrow at the window, thereby suggesting that borrowing did not equal weakness. On Aug. 17, Timothy Geithner and Donald Kohn hosted a conference call with the Clearing House Association, claiming that the Fed would consider borrowing at DW "a sign of strength." Following the call, on Aug. 22, Citi announced that it was

[3]The Federal Home Loan Bank (FHLB) system also helped reduce financial stress at the onset of the crisis. However, Ashcraft, Bech, and Frame (2010) show that FHLB system was not enough to ease liquidity stress by the end of 2007. Also, many institutions such as foreign banks and primary dealers were ineligible for FHLB membership. For example, Dexia Group, the bank that borrowed the most from DW, was not a member of FHLB. A list of FHLB member banks is maintained by the Federal Housing Finance Agency (2023).

[4]https://www.federalreserve.gov/faqs/banking_12841.htm. Initially, DW was a teller window staffed by a lending officer, hence the name.

[5]The Dodd–Frank Act required the disclosure of details of DW loans after July 2010 on a 2-year lag from the date on which the loan was made.

[6]The stigma associated with borrowing from the government was also significant in the UK. In Aug. 2007, Barclays twice tapped the emergency lending facility offered by the Bank of England. The news came out on Thursday, Aug. 30, when the Bank of England said it had supplied almost 1.6 billion pounds as a LOLR without naming the borrower(s). Journalists and the market scrambled to find out. Barclays declined to confirm that it had used the central bank's standing borrowing facility, but later, it cited a technical breakdown in the clearing system as the reason for the large pile of cash. In its statement, Barclays said, "Had there not been a technical breakdown, this situation would not have occurred." Shin (2009) described the bank run on Northern Rock, the UK's fifth-largest mortgage lender. On Sept. 13, 2007, the BBC broke the news that Northern Rock had sought the Bank of England's support. The next morning, the Bank of England announced that it would provide emergency liquidity support. It was only *after* the announcement (i.e., after the central bank had announced its intervention to support the bank) that retail depositors started queuing outside the branch offices.

borrowing $500 million for 30 days. JPMorgan Chase, Bank of America, and Wachovia subsequently announced that they had borrowed the same amount, increasing the total amount borrowed at DW by $2 billion. However, the 4 big banks—with the borrowing stigma in mind—made it clear in their announcements that they did not need the money. Thirty days later, DW borrowing fell back to $207 million.[7] On Dec. 11, 2007, the Fed lowered its discount rate to 4.75%, but the attempt was unsuccessful in injecting liquidity to the financial system. The weekly average balance of DW's primary credit program, $3009 million in the week of Dec. 13, 2007 (see, e.g., Federal Reserve (2007), was tiny compared with the amount of outstanding borrowing during the rest of the crisis (see Graph A of Figure 1).

To further relieve stress in the short-term lending market, the Fed established TAF in Dec. 2007. The rule of the auction was as follows: Every other Monday, banks phoned their local Fed regional banks to submit bids specifying their interest rate (and loan amount) and post collaterals. On the next day, the Fed secretly informed the winners and publicly announced the stop-out rate (as well as the number of banks receiving loans), determined by the highest losing bid (or the minimum reserve price if the auction was undersubscribed). On Thursday of the same week, the Fed released the loans to the banks. Throughout the whole auction process, banks were free to borrow from DW. The following Monday, each regional Fed published total lending from last week; banks may be inferred from these summaries or other channels. The first auction, held on Dec. 17, released $20 billion in the form of 28-day loans. The participation requirement was the same as for DW. The Fed received over $61 billion in bids and released the full $20 billion to 93 institutions. In Feb. 2008, Dick Fuld, CEO of Lehman Brothers, urged the Fed to include Wall Street investment banks in auctions, which would require invoking Section 13(3) to allow the Fed to have authority to lend to nonbank institutions, but the Fed refused. From Mar. to Sept. 2008, the stop-out rate in TAF consistently exceeded the concurrent discount rate. The final auction was held on Mar. 8, 2010, as the auctions had been consistently undersubscribed since 2009.

As shown in Figure 1, TAF was clearly more successful than DW in providing liquidity, and banks were also willing to pay a higher interest rate in TAF than the concurrent discount rate in DW. As Bernanke (2015) acknowledged, before implementing TAF, policymakers were also concerned that the stigma that had kept banks away from DW would also be attached to the auctions. The program was implemented as "give it a try and see what happens," but turned out to be quite successful.

III. The Model

We introduce a static model.

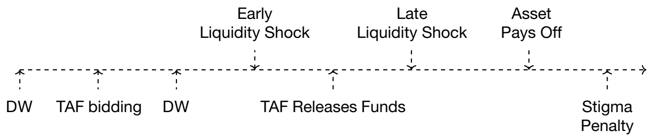
The number of banks, n, is finite. Each bank is endowed with one unit of an illiquid asset, which pays off at the end. Given the potential of liquidity shocks (explained below), each bank can preemptively borrow liquidity from one of the two facilities sponsored by the government: DW and TAF. DW is available before

[7]Records released later show that JPMorgan and Wachovia returned most of the money the next day, whereas Bank of America and Citi, already showing signs of problems, kept the money for a month (Bernanke (2015)).

FIGURE 2

Timeline of the Model

Figure 2 describes the timeline of the model. Each bank can preemptively borrow liquidity from one of the two facilities: DW and TAF. DW is available before and after TAF bidding, and the borrowing bank can immediately obtain funding from it. There is a gap between TAF bidding funds release. Each bank faces the potential of a liquidity shock before the asset pays off. The liquidity shock could be before or after TAF releases funds. Borrower banks may incur a stigma penalty if detected of borrowing.



and after TAF bidding date, and the borrowing bank can immediately obtain funding from it. By contrast, TAF releases funds with a delay: There is a gap between TAF bidding date and TAF funding release. Before the asset pays off, each bank faces the potential of a liquidity shock. The liquidity shock could be early (i.e., before TAF releases funds) or late (i.e., after TAF releases funds). When the liquidity shock hits, the bank fails if it has not obtained liquidity yet. In this case, the asset is liquidated with zero payoff. Finally, borrowing banks may incur a penalty if detected borrowing.

Below, let us provide more details.

A. Preferences, Technology, and Shocks

All banks are risk-neutral and do not discount future cash flows. Each bank has one unit of long-term, illiquid assets that will mature at the end of the game. The asset generates cash flows of R upon maturity, but nothing if the bank fails and the asset gets liquidated early. Each bank may be hit with a liquidity shock similar to Holmström and Tirole (1998). Throughout the paper, we normalize the size of the liquidity shock to one unit. Let $1-\theta_i \in [0,1]$ be the probability that the liquidity shock affects bank i, where θ_i follows the independently and identically distributed cumulative distribution function (cdf) F with associated probability density function (pdf) f on the support $[0,1]$. Assume that F is log-concave. This assumption is not restrictive, as many standard distributions satisfy it; it is imposed to guarantee equilibrium uniqueness.[8] We assume that θ_i is private information only known by the bank. For the rest of the paper, we drop subscript i whenever no confusion arises. Type θ is also referred to as a bank's financial strength. We sometimes refer to a type θ bank as bank θ.[9]

[8]Distributions on a bounded support with a log-concave pdf, which implies a log-concave cdf, include i) uniform distribution on any convex set and beta if both shape parameters are no less than 1 and ii) truncated distributions of the following distributions on unbounded support: normal, exponential, uniform over any convex set, logistic, extreme value, Laplace, chi, Dirichlet if all parameters are no less than 1, gamma if the shape parameter is no less than 1, Weibull if the shape parameter is no less than 1, and chi-square if the number of degrees of freedom is no less than 2 (Bagnoli and Bergstrom (2005), Theorem 9). iii) For any distribution F, we can redefine banks' type as $F(\theta)$ so that banks' types are distributed according to uniform $[0,1]$, which is a log-concave distribution.

[9]In reality, one can proxy a bank's strength θ by its reserve of liquid assets net the level of its demandable liabilities that can be quickly withdrawn. Following such an interpretation, financially

Conditional on a liquidity shock hitting, the bank immediately fails and receives a zero payoff if it does not have one unit of liquidity in stock to defray it. Therefore, if the bank never borrows any liquidity, its expected payoff is θR. The liquidity shock can be early or late. In particular, let $1-\delta$ be the probability of the shock being early and δ be the probability of the shock being late. Receiving a loan with an interest rate of r before the early liquidity shock will help the bank defray the liquidity shock with certainty so that the bank's payoff becomes $R-r$. Therefore, bank θ's expected payoff from borrowing a rate r loan is $(1-\theta)R-r$ if it receives the loan before the early liquidity shock, and $\delta(1-\theta)R-r$ if it receives the loan between the early and the late liquidity shock.[10]

We describe the two lending facilities in the next subsection.

B. Lending Facilities

Any bank can borrow from either DW or TAF.[11]

1. Discount Window

DW is a facility that offers loans at a fixed interest rate r_D, commonly referred to as the discount rate and exogenously set by the Fed. As a bank can always borrow from DW with certainty, the net borrowing benefit is $(R-r_D)-\theta R=(1-\theta)R-r_D$.

2. Term Auction Facility

TAF allocates preannounced m units of liquidity through an auction. In the auction, banks that decide to participate submit simultaneously their sealed bids, which are required to be higher than the preannounced minimum bid r_A. After receiving all of the bids, the auctioneer ranks them from highest to lowest. The auction takes a uniform price format: All winners pay the same interest rate, which is referred to as the stop-out rate s, and losers do not pay anything. If there are fewer bids than the units of liquidity provided, each bidder receives a loan and pays r_A. If there are more bidders than the total liquidity, each of the m highest bidders receives one unit of liquidity by paying the highest *losing* bid. Formally, suppose there are l bidders in total. If $l \le m$, each bidding bank receives a loan by paying $s=r_A$. If $l>m$, each of the m highest bidding banks receives one unit of liquidity by paying the $m+1^{\text{st}}$ highest bid. The remaining $l-m$ banks do not pay anything and, of course, do not receive any liquidity.

weaker banks are more likely to run into liquidity shortages and, therefore, have a higher demand for liquidity. Another interpretation is that financially weaker banks have more toxic assets on their balance sheet, and the liquidation value of these assets is necessarily low. These banks are also more likely to run into liquidity shortages in the crisis as well. If a bank only invested in safe (and liquid) assets and had no risky projects, it would not be considered weak.

[10]According to Bernanke (2015), one main reason to implement TAF was that it would take time to conduct an auction and determine the winning bids so that borrowers would receive funds with a delay and thus signal that they were not desperate for cash.

[11]Note that for simplicity, we do not allow banks to borrow from the interbank market. Previous research has documented that during the 2007–2008 financial crisis, the interbank market was stressed but not completely frozen (Afonso, Kovner, and Schoar (2011)). In addition, our results are unchanged if the interbank rate gets very high, which was the case during most time of the crisis (see, e.g., Figures 1 and 2 of Thornton (2009)).

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We have modeled TAF as an extended second-price auction: All winning parties pay the highest losing bid. In practice, TAF is closer to an extended first-price auction: All winning banks pay the lowest winning bid. The two auctions generate the same revenue for the auction and the same expected payoffs for the bidders, by the revenue equivalence theorem (Myerson (1981)), and consequently make the same borrowing decisions. We present the analysis with the extended second-price auction because it is notationally simpler, as it is a weakly dominant strategy for each bank to bid the maximum interest rate it is willing to pay (Vickrey (1961)).[12]

In reality, winners receive their TAF funds 3 days after the auction. Recall that there is a probability, $1-\delta$, that an early liquidity shock hits each bank before it receives the funds. Note that if the early liquidity shock has occurred, but a TAF winning bank is still waiting for funds to be settled, it cannot borrow from DW. In reality, both DW and TAF loans are collateralized. Thus, if a bank has already pledged its collaterals to TAF, it could no longer borrow from DW had a liquidity shock hit. This assumption is consistent with the narratives in Bernanke (2015), which emphasizes that winning in TAF signals that the bank will likely survive at least during the 3-day settlement period.[13] Hence, the expected net borrowing benefit of a winner who pays stop-out rate s is $\delta(1-\theta)R-s$, where $\delta(1-\theta)R$ is the discounted expected investment return when the bank does not face a liquidity shock before TAF fund arrives and s is the borrowing cost of TAF fund, regardless of whether it solves liquidity issues. Losers, upon learning the result of the auction, may borrow from DW if needed.

C. Borrowing Stigma Costs

Banks are assumed to incur a facility-dependent stigma cost. We have argued that a key reason that banks were reluctant to borrow from LOLR is stigma cost. Detected borrowing may signal financial weakness to counterparties, investors, and regulators. Although θ is private information, the public can infer based on whether the bank has borrowed or which facility the bank has used if it has borrowed. We

[12]In contrast, in the first-price auction, banks shade their bids, which depend on the liquidity supply m and the number of potentially participating banks n.

[13]On page 157, Bernanke (2015) wrote, "because it takes time to conduct an auction and determine the winning bids, borrowers would receive their funds with a delay, making clear that they were not desperate for cash." Moreover, Carlson and Rose (2017) wrote, "TAF had several features designed to minimize stigma. TAF featured delayed settlement, with funds generally being delivered 2 days after the auction, so use of the facility would not signal that the bank had an immediate funding need. The rate at which institutions could borrow at TAF was determined by auction so that it was market-determined." Courtois and Ennis (2010) wrote in an economic brief, "A 3-day settlement period between the close of the auction and disbursement of funds may have reduced the appearance of a desperate need for cash and thus financial distress."However, we would like to stress that the same endogenous separation in bank borrowing from DW and TAF can be generated even without the delayed settlement. In Supplementary Material B, we present such a model, in which TAF is only held once every other week, whereas DW is always immediately available. The only qualitative difference between the two models is whether bids in TAF are monotonic in the bank's type. We decided to focus on the current model because the remarks by policymakers and bank regulators have highlighted the particular feature of the 3-day delay in settlement.

assume that upon detection, the public can perfectly tell whether the borrowing has been achieved through DW or TAF.

We capture the notion of stigma cost in a parsimonious way. We assume that after all of the borrowings are complete, banks that have successfully borrowed may be detected independently. Denote the probability of a bank's being detected borrowing from a particular facility to be p. Let G_D and G_A be the type distributions of the banks borrowed from DW and TAF, respectively. Let the stigma cost depend on the expected financial condition of the bank. For simplicity, we assume linear dependence. That is, for any detected borrowing decision $\omega \in \{D, A\}$,

$$k_\omega \equiv k(G_\omega) = K - \kappa \int_0^1 \theta dG_\omega(\theta).$$

If the dependence is nonlinear, our model will in general have multiple equilibria, but the qualitative features remain unchanged. For the same reason, we assume that the degree of stigma is low relative to the borrowing benefits: $\kappa \leq \min\left\{\frac{\delta R}{p}, \frac{(1-\delta)R}{p}\right\}$. For the rest of the paper, we normalize the stigma cost of a bank believed to have an unconditional average condition to be 0, $k_\varnothing \equiv 0$.[14]

Note that financially weaker banks (i.e., those with lower θ) will receive a higher stigma cost upon detection. This cost can be understood as the bank's deteriorated reputation, a reduced chance to find counterparties, the cost of a heightened chance of runs and increasing withdrawals by creditors, fines imposed by regulatory authorities, and an increase in future regulatory scrutiny and compliance costs.

D. Definition of Equilibrium

In summary, the setting is summarized by the return R, type distribution F of banks, discount rate r_D in DW, number m of units of liquidity auctioned, minimum bid r_A in TAF, and the penalty function $k: G \mapsto \mathbb{R}_+$ attached to different belief distributions of bank's type.

Without loss of generality, we restrict each bank's strategy to be type-symmetric. Each bank θ's strategy can be succinctly described by $\sigma(\theta) = (\sigma_{D1}(\theta), \sigma_A(\theta), \beta(\theta), \sigma_{D2}(\theta))$, where $\sigma_\omega(\theta)$ is the probability of borrowing from $\omega \in \{D1, A, D2\}$, and $\beta(\theta)$ is its bid if it participates in the auction. $D1$ and $D2$ refer to borrowing from DW before and after TAF, respectively. Given strategies σ, beliefs about the financial situation can be inferred following Bayes' rule; in this case, we say that aggregate strategies σ generate a posterior belief system $G = (G_A, G_D)$.

Definition 1. Borrowing and bidding strategies σ^* and belief system G^* form an equilibrium if (i) each type θ bank's strategy $\sigma^*(\theta)$ maximizes its expected payoff given belief system G^* and (ii) the belief system G^* is consistent with banks' aggregate strategies σ^*.

[14]This implies $K \equiv \kappa \int_0^1 \theta dF(\theta)$.

Clearly, the best (i.e., type 1) bank has no intention of borrowing because it would pay a price, incur a stigma cost, and receive no benefit from borrowing. We assume that the borrowing benefit of the worst (i.e., type 0) bank is so high that it has a strict incentive to borrow even given the most pessimistic belief about banks that borrow: $R - r_D - k(\underset{\bar{}}{G}) > 0$, where $\underset{\bar{}}{G}(\theta) = 1$ for all $\theta > 0$.

IV. Theoretical Analysis

We present the solution of the benchmark design (only DW) and the solution of the actual design (DW and TAF with a delayed release of funds). Then, we discuss four alternative designs (only TAF, DW, and TAF with immediate release of funds, two DWs with different releases of funds, and two DWs with different interest rates). Finally, as it remains unclear how the public detects banks' borrowing decisions, we discuss our results under alternative detection technologies.

A. Only DW

We start by examining the equilibrium when the government only sets up DW. The optimal borrowing decision can be characterized by one threshold: Weaker banks borrow from DW, and stronger banks do not borrow at all.

Note (again) that the best bank never borrows because it knows that a liquidity shock could never affect it, and therefore, it never needs the liquidity; instead, borrowing incurs an interest cost and a stigma cost. The larger the probability a liquidity shock affects the bank, the more incentive the bank has to borrow. Under the assumption $r_D < R - k(\underset{\bar{}}{G})$, the worst bank is incentivized to borrow from DW.

Furthermore, there is a unique equilibrium, which is guaranteed by the assumption of a log-concave cdf F.

Theorem 1 (equilibrium with only DW). Suppose only DW is available (i.e., $m = 0$). There exists a unique equilibrium characterized by a threshold $\theta^{DW} > 0$: Banks $\theta \in [0, \theta^{DW}]$ borrow from DW, and banks $\theta \in (\theta^{DW}, 1]$ do not borrow. The equilibrium DW stigma is

$$k^{DW}(\theta^{DW}) = K - \kappa \int_0^{\theta^{DW}} \theta dF(\theta) / F(\theta^{DW}),$$

where the threshold θ^{DW} satisfies

(DW) $$(1 - \theta^{DW})R - r_D - pk^{DW}(\theta^{DW}) = 0.$$

DW provides liquidity to all banks worse than θ^{DW}, but banks better than θ^{DW} do not borrow because the real economic benefits of borrowing to save the unrealized assets are dwarfed by the interest cost and the stigma cost. The change in the returns, interest rate, and stigma costs will affect liquidity provision as follows:

Proposition 1 (liquidity provision with only DW). The expected total liquidity to be provided with only DW, L^{DW}, is $nF(\theta^{DW})$. It increases as (i) the return R increases,

(ii) the discount rate r_D decreases, (iii) the probability of detection p decreases, and (iv) the stigma severity κ decreases.

How total liquidity depends on the change in the distribution of banks' types is interesting, though it may decrease when banks face higher liquidity risks overall.

Proposition 2 (market condition and liquidity provision with only DW). Total liquidity with only DW, L^{DW}, changes ambiguously when the type distribution F shifts in a first-order stochastic dominance (FOSD) way.

To understand this result, note that there are two effects. First, when the distribution of banks becomes worse, holding the stigma cost unchanged, more banks would choose to borrow from DW, increasing total liquidity provision. However, there is a second countervailing force. When banks worse than θ^{DW} face even higher liquidity risks than before, banks that borrow from DW are perceived to be of even lower quality than before. As a result, the stigma cost rises, and bank θ^{DW}, which was indifferent between borrowing from DW and not, is no longer interested in borrowing. In other words, *the worsened conditions of infra-marginal borrowing banks adversely affect the borrowing decision of the marginal borrowing bank.* Due to the stigma cost, DW may not effectively provide liquidity when the worst banks become worse.

This result implies that banks might borrow less from DW when they face higher liquidity risks because the heightened stigma cost may dominate the increased liquidity demand. The fact that banks were initially reluctant to borrow from DW before introducing TAF suggests that the worst banks in the economy faced higher liquidity risks.

B. DW and TAF

We now solve for the equilibrium when both DW and TAF with delayed release of funds are available.

Lemma 1. Only banks $\theta \leq \theta_D$ would borrow from DW if they have lost in the auction, where $\theta_D = 1 - (r_D + pk_D)/R$ and k_D is the equilibrium stigma cost from DW borrowing.

Lemma 2. Banks $\theta \in (\theta_1, \theta_A]$ participate in the auction, where

$$\theta_1 = 1 - \frac{r_D - r_A + pk_D - pk_A}{(1-\delta)R}, \quad \theta_A = 1 - \frac{r_A + pk_A}{\delta R}.$$

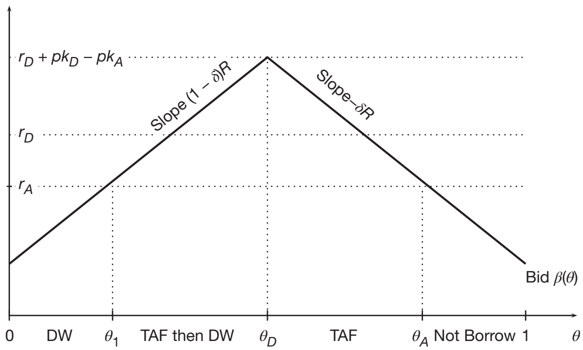
and bid

$$\beta(\theta) = \begin{cases} r_D + pk_D - pk_A - (1-\delta)R(1-\theta) & \textit{if } \theta < \theta_D \\ \delta R(1-\theta) - pk_A & \textit{if } \theta \geq \theta_D \end{cases}.$$

FIGURE 3

Facility Choice and TAF Bids in the DW-and-TAF Design

Figure 3 illustrates the optimal bid and optimal facility choice made by different banks. The solid line plots the willingness to pay as a function of the bank's type. The horizontal axis marks the optimal facility choice made by each bank.



Note that bids are increasing in θ when $\theta<\theta_D$ and decreasing in θ when $\theta \geq \theta_D$. Intuitively, banks $\theta<\theta_D$ will always borrow; they will still tap DW after losing in TAF. However, if they win in the auction, chances are that the early liquidity shock could hit them before the funds get settled. In this case, the bank will have to fail. Therefore, delayed settlement is more costly for worse banks that are more likely to be hit by the early liquidity shock. As a result, they bid less. In fact, the bids increase at the rate of $(1-\delta)R$ for banks worse than θ_D. On the other hand, banks $\theta>\theta_D$ will choose not to borrow at all after losing in TAF, so they are borrowing only to hedge the late liquidity shock. Among them, worse banks will bid more as they are more likely to be hit by the late liquidity shock. In fact, the bids decrease at the rate of δR for banks better than θ_D. Therefore, bank θ_D has the highest willingness to pay, and banks further away from θ_D have a lower willingness to pay. The auction winners will be the banks that are the closest to θ_D. For any bank, as long as its willingness to pay is above r_A, it will participate in the auction by submitting a bid higher than r_A. Figure 3 shows the willingness to pay (i.e., bid) in TAF and the optimal facility choice of different banks.

The difference in the stigma cost between the two borrowing facilities could lead to banks bidding more than the discount rate r_D. In particular, bank θ_D is willing to bid up to $r_D+pk_D-pk_A$ to avoid the stigma cost. As we will show later, $k_D>k_A$ in equilibrium, so that bank θ_D always bids more than r_D. If the realized bank distribution is concentrated around θ_D, the stop-out rate in TAF will be above the discount rate r_D. The relation between θ_1 and θ_D in the lemmas depends on the equilibrium stigma costs and will be determined in equilibrium, as characterized by Corollary 2 below.

Lemma 3 (equilibrium with both DW and TAF: high chance of early liquidity shock). Suppose DW and TAF are both available, and there is a sufficiently high chance of an early liquidity shock: $m>0$, $r_D<R-k(\underline{G})$, and $\delta \leq$

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$\left[r_A+k\left(\theta^{DW}\right)\right]/\left[r_D+pk^{DW}\left(\theta^{DW}\right)\right]$. In the unique equilibrium, banks $\theta\in\left[0,\theta^{DW}\right]$ borrow from DW, and banks $\theta\in\left(\theta^{DW},1\right]$ do not borrow.

Note that the condition on δ is less likely to satisfy as r_A gets higher. We can interpret a low δ as a longer delay in releasing the funds from TAF. Therefore, delaying the release of the funds from TAF for too long (and/or setting the minimum bid too high) will render the program ineffective.

Theorem 2 (equilibrium with both DW and TAF: low chance of early liquidity shock). Suppose DW and TAF are both available, and there is a sufficiently low chance of an early liquidity shock: $m>0$, $r_D<R-k(\underline{G})$, and $\delta>\left[r_A+k\left(\theta^{DW}\right)\right]/\left[r_D+pk^{DW}\left(\theta^{DW}\right)\right]$. Suppose $\delta R\geq p\kappa$ and $(1-\delta)R\geq p\kappa$. In the unique equilibrium, there exist three thresholds θ_1, θ_D, and θ_A such that (i) banks $\theta\in[0,\theta_1]$ are indifferent between borrowing from DW before the auction and borrowing from DW after the auction; (ii) banks $\theta\in(\theta_1,\theta_D]$ bid in the auction and borrow from DW if they lose in the auction; (iii) banks $\theta\in(\theta_D,\theta_A]$ bid in the auction and do not borrow if they lose in the auction; and (iv) banks $\theta\in(\theta_A,1]$ neither borrow from DW nor participate in the auction.[15]

Theorem 2 immediately implies.

Corollary 1. In equilibrium, DW stigma k_D^* is larger than auction stigma k_A^*.

Three forces separate banks that borrow in DW and those that borrow in TAF. First, the possibility of early liquidation due to the delayed release of funds in TAF forces the worst banks to borrow from DW and deters them from participating in TAF. Second, excluding the worst banks from the auction ensures that the average quality of banks that borrow from TAF is not too low, which implies that the stigma associated with TAF is not too high, thus further attracting more banks to borrow from TAF. Finally, the competitive nature of the auction attracts banks that would not have borrowed with only DW by offering them a chance to borrow cheaper than the discount rate. TAF serves as an alternative to DW for banks close to and worse than θ^{DW}. They try borrowing in the auction first before borrowing in DW. TAF serves as a complement for DW in terms of total lending. Banks that are close to and better than θ^{DW} switch to borrowing in the auction from not borrowing. This result implies that the presence of TAF could increase the stigma of DW, consistent with some arguments made by policymakers (Carlson and Rose (2017)).

Our next result offers a definitive comparison of the marginal borrower θ^{DW} when only DW is offered and θ_D and θ_1 when TAF is offered in addition to DW.

Corollary 2. In comparison, $\theta_1<\theta_D<\theta^{DW}$.

[15]The indifference result (i) can be easily broken. For example, if the early liquidity shock has a probability $\varepsilon>0$ of occurring between the first DW and TAF bids, then banks between 0 and θ_1 will strictly prefer DW before TAF. Our baseline model can be thought of as the limiting case whereby $\varepsilon\to0$.

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When some banks bid in the auction in equilibrium (i.e., the setting described in Theorem 2), introducing TAF will attract some marginal borrowers of the original DW to try the auction first before settling on DW. Furthermore, the expected marginal borrowers of DW in the presence of TAF (i.e., θ_D) will be worse than the marginal borrowers of DW without TAF (i.e., θ^{DW}), because the higher stigma cost associated with DW in the presence of TAF discourages the marginal borrowers in DW-only setting.

Liquidity Provision. For total liquidity, consider the expected marginal borrower. The expected marginal borrower is better than θ^{DW}, because they borrow from the auction, and the distribution of the types of banks participating in the auction in DW-and-TAF setting first-order stochastically dominates the distribution of the types of banks borrowing from DW.

Proposition 3 (liquidity provision with both DW and TAF). The combination of TAF and DW provides more total liquidity in expectation than does DW alone: $L^* > L^{DW}$. The liquidity provided by DW decreases when TAF is introduced.

Even though the combination of TAF and DW provides more liquidity in expectation, it is still possible that the combination of the two facilities can lead to less liquidity provision in realization. In particular, if many realized banks' types are slightly below θ_D, then they will bid in TAF, hoping to take advantage of the low reserve price. The losing banks, which would have borrowed from DW if TAF were unavailable, would choose not to borrow at all.

Remark. An important decision the Fed makes is on m. In the model, it is the number of winners in the auction. It is also the amount of liquidity released by TAF (or the quantity limit on each bank so that more banks receive the funding and get pooled together in TAF). On the one hand, an increase in m will bring more healthy banks into TAF and pool them with less healthy banks to create a lower stigma. More participation may also reduce the chance of banks being detected. However, on the other hand, this will also bring in less healthy banks who are now more willing to wait for the lower stigma. In equilibrium though, the first effect must dominate the second effect; a proof of contradiction can show this claim: If the stigma cost of TAF actually increases when m increases, then there should be more banks that borrow from DW directly, which in turn lowers the stigma cost of TAF. Hence, an increase in m lowers TAF stigma and hence increases participation in TAF and liquidity provision.

C. Alternative Designs

Instead of the combination of a periodic TAF and the always available DW, could the Fed have improved liquidity provision? We explore a few alternative designs in this subsection.

1. Only TAF

Next, we examine the equilibrium when the government only sets up the auction. The equilibrium can also be characterized by one threshold: Weaker banks bid their willingness to pay in the auction, and stronger banks do not participate in the auction or borrow at all.

Proposition 4 (equilibrium with only TAF). Suppose only TAF is available. Assume $\delta R - pk(0) > r_A$. There exists a unique equilibrium characterized by a threshold θ^{TAF}: (i) Banks $\theta \in [0, \theta^{TAF}]$ bid $\beta^{TAF}(\theta) = \delta(1-\theta)R - pk_A$ in TAF, and (ii) banks $\theta \in (\theta^{TAF}, 1]$ do not bid. Equilibrium auction stigma is

$$k^{TAF}(\theta^{TAF}) = K - \kappa \int_0^{\theta^{TAF}} \int_0^{\theta_s} \frac{\theta dF(\theta)}{F(\theta_s)} h(\theta_s) d\theta_s - \kappa \int_{\theta^{TAF}}^{1} \int_0^{\theta^{TAF}} \frac{\theta dF(\theta)}{F(\theta^{TAF})} h(\theta_s) d\theta_s,$$

where $h\theta_s = \binom{n}{m} F^{m-1}(\theta_s) f(\theta_s)(1 - F(\theta_s))^{n-m}$ is the pdf of the m^{th} weakest bank, and the threshold θ^{TAF} satisfies

$$\text{(TAF)} \qquad \delta R(1 - \theta^{TAF}) - r_A - pk_A^{TAF}(\theta^{TAF}) = 0.$$

The two double integrals correspond to the case where the realization of the mth weakest bank falls below and above θ^{TAF}, respectively. Our result shows that TAF alone is not necessarily more effective than DW in providing liquidity. If the facilities are used alone, it is unclear which one will provide more liquidity. Therefore, the combination of DW and TAF is needed to increase liquidity provision compared with DW-only design.

2. DW and Immediate TAF

Suppose TAF immediately releases funds to winners, and DW is always available. This is essentially a special case of the DW-and-TAF design previously, with a probability $1 - \delta = 0$ of encountering a liquidity shock between winning the auction and receiving the loan. In this case, TAF becomes a free option. DW no longer possesses an immediacy advantage, so all of the weakest banks bid in the auction first. All of the banks that would borrow from DW after losing in the auction (banks $\theta \le \theta'_D$) bid the same rate $r_D + pk_D - pk_A$, and all of the banks that would not borrow from DW after losing in the auction – banks $\theta > \theta'_D$ – bid lower rates. In summary, as Figure 4 illustrates, banks $\theta \in [0, \theta'_A]$ participate in the auction. Winners receive loans from TAF, and losers with sufficiently weak financial conditions —banks $\theta \le \theta'_D$—borrow from DW afterward.

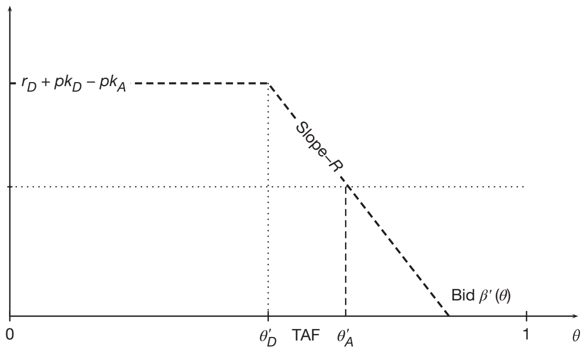
Proposition 5 (equilibrium with DW and immediate TAF). Suppose TAF releases funds immediately and DW is always available. In the unique equilibrium, there exist two thresholds θ'_D and θ'_A such that banks $\theta \in [0, \theta'_D]$ bid in TAF and borrow from DW if they lose in TAF, and banks $\theta \in (\theta'_D, \theta'_A)$ bid in TAF and do not borrow if they lose in TAF.

This design could provide less liquidity for two reasons than the original design. First, the weakest banks —banks $\theta \le \theta_1$— no longer immediately borrow from DW but participate in the auction, so they take away liquidity from stronger banks that would not have borrowed from DW if they lost in the auction (i.e., banks $\theta \in [\theta'_D, \theta'_A]$). Second, the increased participation of the weakest banks in TAF

FIGURE 4

Facility Choice and TAF Bids in the DW-and-Immediate-TAF Design

Figure 4 illustrates the optimal bid and optimal facility choice made by different banks in the alternative design where TAF immediately releases funds to winners. The dashed line plots each bank's willingness to pay.



increases its stigma, discouraging stronger banks from bidding in TAF and further increasing its stigma cost.

3. DW or Immediate TAF

Suppose DW and TAF are simultaneously offered, and banks can *only* choose to borrow from one facility. Then, in equilibrium, there continues to be a separation between TAF and DW borrowing.

Proposition 6 (equilibrium with simultaneous DW and immediate TAF). Suppose DW and TAF are simultaneously offered. In the unique equilibrium, there exist two thresholds θ''_D and θ''_A such that banks $\theta \in [0, \theta''_D]$ bid borrow from DW, and banks $\theta \in (\theta''_D, \theta''_A)$ bid in TAF and do not borrow if they lose in TAF.

This hypothetical situation highlights the importance of the competitive nature of the auction in the separation of banks, in addition to the channel of delayed release of funds. Intuitively, TAF introduces uncertainty regarding whether a bidding bank can borrow at a low rate, lower than its willingness to pay, at the cost of potentially failing to borrow and hedge the early liquidity shock. This cost is lower for stronger banks because their borrowing benefits are lower. Therefore, they are more inclined to participate in the auction and take advantage of the opportunity to borrow when rates are sufficiently low. In this case, borrowing can divide borrowers into two groups by the so-called single-crossing condition. It is worthwhile to point out that our result on separation does not depend on the assumption that delaying cost is higher for weaker banks. To see this, note that a bank's overall payoff has three components that vary with θ. First, a stronger bank has lower borrowing benefits. Second, in equilibrium, a stronger bank submits a lower bid and is less likely to win in the auction. However, third, conditional on winning in the auction, it pays less in expectation. When a bank bids optimally, it is indifferent between raising the bid to increase the winning probability and paying more conditional on winning. Therefore, the last two effects cancel out. As a result, the overall effect is the decreasing benefits of borrowing times the probability of winning in the auction, which is increasing the bank's financial weakness.

4. DWs with Immediate and Delayed Release of Funds

If the delay in releasing funds is important, why does not the Fed simply set up a separate DW D' that releases funds later? The main problem with this separate DW is that banks are separated into the two facilities only for certain combinations of discount rate r_D and discount factor δ. Let us explore this possibility and see how this design does not inject liquidity as desired. Suppose DW D' charges the interest rate r'_D.

Proposition 7 (equilibrium with two differentially timed DWs). Suppose there are two DWs: D releases funds immediately and D' releases funds with a delay. Suppose $\delta R \geq p\kappa$ and $(1-\delta) \geq p\kappa$. In the unique equilibrium, there exist two thresholds θ_1 and θ_2 such that banks $\theta \in [0, \theta_1]$ borrow from D, and if $\theta_2 \geq \theta_1$, banks $\theta \in [\theta_1, \theta_2]$ borrow from D'.

To guarantee the separation of banks into two facilities, the conditional probability of the early liquidity shock $1-\delta$ can be neither too large nor too small.[16] Otherwise, all banks borrow from the early DW (when the chance of an early liquidity shock is high) or borrow from the late DW (when the chance of an early liquidity shock is low). The possible inability to separate banks into two facilities may render the design less useful, as the main purpose of such a design is to separate banks to inject liquidity into stronger banks with a delay. The DW-and-TAF design circumvents this potential problem by setting a relatively low minimum required bid to attract banks to participate in the auction and to allow individual bids so that those willing to pay the most emerge as winners and separate themselves from other banks.

5. Cheap and Expensive DWs

Setting up two DWs with different interest rates does not provide more liquidity. It provides less liquidity than simply setting up the cheaper DW.

Proposition 8 (equilibrium with two differentially priced DWs). Suppose there are two DWs: D charges interest rate r_D and D' charges interest rate $r_{D'} > r_D$. In equilibrium, banks are indifferent between the two DWs. The design offers less liquidity than setting up only the cheaper DW D.

In equilibrium, it must be that all banks are indifferent between the two DWs; otherwise, they would borrow from the one with strictly lower total costs, including borrowing and the stigma cost. Bank θ gets $(1-\theta)R - r_D - pk_D$ from D and gets $(1-\theta)R - r_{D'} - pk_{D'}$ from D'. All banks are indifferent between the two facilities if $r_D + pk_D^* = r_{D'} + pk_{D'}^*$. Therefore, the average bank borrowing from D is worse than the average bank borrowing from D', and consequently, the average bank of all borrowing banks is better than the average bank borrowing from D.

[16]The specific expression is $\frac{r_D + pk_D^*}{R}\left[1 - \frac{r_{D'} + pk_{D'}^*}{r_D + pk_D^*}\right] < 1 - \delta < 1 - \frac{r_{D'} + pk_{D'}^*}{r_D + pk_D^*}$.

D. Alternative Detection Technologies

In this subsection, we discuss how alternative assumptions on detection technology could affect our equilibrium results.

1. Pooled DW and TAF Detection

Suppose borrowing from DW faces the same stigma cost and the same probability of detection as borrowing from TAF. In other words, the public can only tell whether a bank has borrowed from the Fed but not whether the borrowing was from DW or TAF. The equilibrium borrowing behavior is qualitatively the same as characterized in Section IV.B: Weaker banks immediately borrow from DW, and stronger banks first bid in the auction. However, no bank would be willing to bid more than the discount rate because the auction would not have a lower stigma cost than DW, as the borrowing cannot be distinguished. This predicted borrowing behavior—bids being capped at the concurrent discount rate—is against the observed pattern that in more than a third of the auctions, each winning bank was paying more than the discount rate, and in more than two-thirds of the auctions, some banks were bidding more than the discount rate.

2. Separate Early and Late DW Detection

Suppose non-auction-week DW and auction-week DW borrowing can be separately detected, as the Fed publishes its balance sheets weekly. Such finer detection technology could further deter banks from borrowing immediately from the early (i.e., non-auction-week) DW, as the stigma cost of early DW increases. It would encourage more banks to bid in the auction, as it substitutes for the early DW. It would also encourage more banks to borrow from the late DW, because the weakest banks that borrow in the early DW are not associated with the late DW stigma anymore. A consequence of a lower late DW stigma cost is lower bids submitted by banks in TAF; nonetheless, the late DW stigma cost is still higher than TAF stigma cost, so some banks still bid higher than the concurrent discount rate.

Separate TAF Participation and Borrowing Stigma. Suppose participating in but not borrowing from TAF also incurs a stigma cost. This additional stigma cost would decrease the participation in the auction (as some stronger banks choose not to try in the auction) and consequently may reduce aggregate borrowing, as the auction may end up undersubscribed. Safeguarding and not disclosing the participation list would encourage borrowing.

Public Stop-Out Rate. In reality, the Fed announces the stop-out rate after each auction. However, whether or not the actual market-clearing borrowing rate is announced does not affect banks' bidding decisions ex ante. Banks rationally and correctly expect the distribution of stop-out rates in equilibrium and make appropriate borrowing and bidding decisions accordingly. The late DW borrowing decision may be affected by the disclosed stop-out rate, as opposed to an expected stop-out rate when it is not publicly announced. The actual borrowing from the post-auction DW may change due to the disclosure policy, but the expected aggregate borrowing is unaffected by the disclosure policy.

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3. Different Detection Probabilities

Suppose the probability of being detected borrowing in DW differs from in TAF. For example, the equilibrium probability of being detected can depend on the number of banks that actually participate in liquidity provision programs. It is straightforward to show that Theorem 2 continues to hold. Mathematically, the terms involving stigma costs all cancel out in the single-crossing conditions. Intuitively, heterogeneous detection probability does not affect the relative trade-off between using DW and TAF across banks with different financial strengths θ.

V. Empirical Relevance

This section discusses the empirical relevance of our model. We will summarize existing empirical evidence and very briefly describe empirical analysis conducted by ourselves. A full empirical test of our theory is beyond the scope of this paper. However, the evidence here offers partial support for some of the assumptions and implications of the theory. Detailed specifications and results are available in the Supplementary Material.

The issue of DW stigma has been documented since at least Peristiani (1998) and Furfine (2001), (2003), (2005), who offer evidence that banks prefer the federal funds market to DW. During the recent financial crisis, Armantier et al. (2015) use TAF as a laboratory to show the existence of DW stigma and estimate its magnitude. Armantier and Holt (2020) use laboratory experiments to test policies that have been proposed to mitigate the stigma.

Several empirical papers have studied how government intervention affects liquidity provision during a crisis. Acharya and Mora (2015) show government-sponsored facilities such as FHLB advances and Federal Reserve liquidity facilities enabled banks to continue to provide liquidity during the crisis. Acharya, Fleming, Hrung, and Sarkar (2017) further show that dealers with lower equity returns and greater leverage were more likely to participate in the Securities Lending Facility (TSLF) and bid higher (and thus borrow more) in the Primary Dealer Credit Facility (PDCF). Using data during the European Sovereign Debt Crisis, Drechsler, Drechsel, Marques-Ibanez, and Schnabl (2016) show that weakly capitalized banks borrowed more from LOLR and subsequently invested in risky assets. TAF was shown to be effective in reducing liquidity concerns (Wu (2011)), lowering LIBOR (McAndrews, Sarkar, and Wang (2017)), and conferring a benefit on the real economy (Berger, Black, Bouwman, and Dlugosz (2017), Moore (2017)).

A central prediction of our model is that financially weaker banks borrowed relatively more from DW than TAF, compared with their stronger peers. To test this hypothesis, we collect granular data on DW and TAF borrowing during the crisis and match them with the regulatory Y-9C data. We show that compared with TAF banks, DW banks have less core deposit, higher leverage, lower tier 1 capital ratio, and more unused loan commitment and rely more on short-term wholesale funding after controlling for bank size, profitability, and bank- and time-fixed effects. According to Cornett, McNutt, Strahan, and Tehranian (2011), banks that relied more on core deposits continued to lend relative to other banks during the financial crisis because core deposits are stable sources of financing. Therefore, these banks could be less affected by liquidity shortages. Moreover, they argue that unused commitments expose banks to liquidity risk and find that banks with higher levels of unused commitments hoarded more liquidity and cut more lending during the crisis. Given this, our result can be interpreted as DW banks were more exposed to liquidity risks compared with TAF banks. Leverage and tier 1 capital ratios capture banks' loss-absorbing capacities.[17] Finally, the subprime risks taken by banks were funded mostly by short-term market borrowing, and our result on short-term wholesale funding suggests that DW banks may be more exposed to subprime risks than TAF ones.

VI. Conclusion

In this paper, we investigate how the design of emergency lending facilities can mitigate the stigma associated with borrowing from the central bank's LOLR. We constructed an auction model with endogenous participation and showed that auction bidding strategies that internalized the stigma increased participation and consequently mitigated the borrowing stigma.

We derive several theoretical predictions from the model for empirical tests. First, banks with strong financial health are reluctant to borrow from DW due to their reluctance to associate themselves with banks worse than them. Second, weaker banks borrow from DW, and stronger ones participate in TAF when both DW and TAF are available. Of those that lose in the auction, weaker ones borrow from DW. Third, we show that TAF alone may or may not expand the set of banks that obtain liquidity; it is the combination of TAF and DW that mitigates borrowing stigma and increases liquidity provision. Lastly, the stop-out rate of TAF may be higher or lower than the discount rate.

Our analysis provides a better understanding of the role a special monetary program, TAF, played during the financial crisis, and suggests how to better design LOLR programs in the future. Our results show that the Fed's design of DW and delayed-funds-release TAF achieved its intended goal of lowering the borrowing stigma by separating the banks into distinct groups, encouraging participation by stronger banks, and providing more liquidity to the economy. The improvement over the current design is a quantitative matter of setting the more appropriate discount rate, minimum bid, and number of days to delay the release of funds. We leave this important quantitative exercise to future research.

Appendix. Omitted Proofs

Proof of Theorem 1. Bank θ prefers borrowing from DW over not borrowing if and only if

[17] Demirguc-Kunt, Detragiache, and Merrouche (2013) document that during the crisis, banks with higher tier 1 capital ratios also performed better in the stock market.

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$$u_D(\theta) = (1-\theta)R - r_D - pk_D - (1-p)k_{\varnothing} \geq 0.$$

As we normalize $k_{\varnothing}$ to be 0, we can simplify the condition to

$$(1-\theta)R - r_D - pk_D \geq 0.$$

Clearly, the gain from borrowing from DW is strictly decreasing in θ. Therefore, for any given k_D, bank θ borrows from DW if and only if

$$\theta \leq 1 - \frac{r_D + pk_D}{R}.$$

Therefore, there exists a threshold—let us denote it by θ^{DW}—such that bank θ^{DW} is indifferent between borrowing from DW and not borrowing; banks worse than θ^{DW} borrow from DW; and banks better than θ^{DW} do not borrow. In equilibrium, k_D depends on θ^{DW}:

$$k_D = K - \kappa \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F(\theta^{DW})}.$$

Plugging equilibrium k_D into the equilibrium condition previously, we see that θ^{DW} is determined by

$$(1-\theta^{DW})R - r_D - p\left[K - \kappa \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F(\theta^{DW})}\right] = 0,$$

which is rearranged as

$$\text{(DW)} \qquad R - r_D - \theta^{DW} R + p\kappa \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F(\theta^{DW})} = 0.$$

The terms involving θ^{DW} can be rearranged as

$$-\theta^{DW}(R - p\kappa) - p\kappa\left[\theta^{DW} - \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F(\theta^{DW})}\right].$$

The first term, $-\theta^{DW}(R - p\kappa)$, is decreasing in θ^{DW}, because $R > 1 > p\kappa$. For the second term, $-p\kappa\left[\theta^{DW} - \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F(\theta^{DW})}\right]$, the expression in the square brackets is *mean advantage over inferiors*, as Bagnoli and Bergstrom (2005) name it. Because the distribution is assumed to be log-concave, by Bagnoli and Bergstrom (2005), Theorem 5, the term in the square brackets is weakly increasing in θ^{DW}, so the second term is weakly decreasing in θ^{DW}. In summary, the left-hand side of equation (DW) is strictly decreasing in θ^{DW}.

To show the existence of a unique solution to equation (DW), it remains to show that its left-hand side is positive for $\theta^{DW} = 0$ and negative for $\theta^{DW} = 1$. When $\theta^{DW} = 0$, the left-hand side is

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$$R - r_D - p\kappa \int_0^1 \theta dF(\theta) = R - r_D - pK > 0,$$

where the equality follows from the normalization of $K = \kappa \int_0^1 \theta dF(\theta)$, and the inequality comes from the assumption that $R > r_D + pK$. When $\theta^{DW} = 1$, the left-hand side is

$$-r_D + p\kappa \int_0^1 \theta dF(\theta) = -r_D + pK < 0,$$

where the inequality follows from $r_D > 1 > pK$. Hence, there is a unique equilibrium. □

Proof of Proposition 1. The left-hand side of equation (DW) strictly shifts up when (i) R increases, (ii) r_D decreases, (iii) p increases, or (iv) κ increases. As the left-hand side of equation (DW) is strictly decreasing in θ^{DW}, the equilibrium θ^{DW} increases as a result of any of the changes (i)-(iv). □

Proof of Proposition 2. The left-hand side of equation (DW) strictly shifts down when F for $\theta < \theta^{DW}$ shifts in a first-order stochastically dominated way, because the only term affected by the change, $\int_0^{\theta_{DW}} \frac{\theta dF(\theta)}{F(\theta^{DW})}$, strictly decreases. Hence, the new threshold $\tilde{\theta}^{DW}$ is strictly smaller than θ^{DW}. Total liquidity expected to be provided, $\tilde{L}^{DW} = nF\left(\tilde{\theta}^{DW}\right)$, is also smaller than $L^{DW} = nF\left(\theta^{DW}\right)$. □

Proof of Proposition 3. From the previous proof, we see that the equilibrium condition for the banks that borrow from DW in the DW-and-TAF setting is

$$\left(1 - \theta_D^*\right)R - r_D - pk_D^* = 0.$$

Compare this condition to the equilibrium condition for banks that borrow from DW in the DW-only setting:

$$\left(1 - \theta^{DW}\right)R - r_D - pk^{DW} = 0.$$

As long as $k^{DW} < k_D^*$, fewer banks are willing to borrow from DW in the DW-and-TAF setting. This condition indeed holds, because the strongest banks of the banks worse than θ_D win in the auction.

For total liquidity, consider the expected marginal borrower. The expected marginal borrower is better than θ^{DW}, because they borrow from the auction, and in the DW-and-TAF setting, the type distribution of banks winning in TAF first-order stochastically dominates that of banks borrowing form DW.

Proof of Proposition 4. Bank θ bids (gross) interest rate $\beta(\theta)$ such that its payoff from winning in the auction with this rate is the same as the payoff from not borrowing,

$$\delta(1 - \theta)R - \beta(\theta) - pk_A = 0.$$

In other words, the bid is the bank's maximum willingness to pay (WTP) for the loan:

$$\beta(\theta)=\delta(1-\theta)R-(pk_A).$$

Note that the bid is strictly decreasing in θ. Therefore, worse banks are willing to bid higher interest rates. Consequently, given any stigma cost k_A, there exists a threshold bank θ^{TAF} such that banks worse than θ^{TAF} are willing to bid more than the minimum bid r_A, and all banks better than θ^{TAF} are not willing to bid more than r_A. Bank θ^{TAF} bids exactly the prespecified minimum bid r_A:

$$\beta\left(\theta^{TAF}\right)=r_A\Rightarrow\theta^{TAF}=1-\frac{pk_A+r_A}{\delta R}.$$

Now, consider the equilibrium stigma cost:

$$k_A\left(\theta^{TAF}\right)=K-\kappa\int_0^{\theta^{TAF}}\int_0^{\theta_s}\frac{\theta dF(\theta)}{F(\theta_s)}dH(\theta_s)-\kappa\int_{\theta^{TAF}}^1\int_0^{\theta^{TAF}}\frac{\theta dF(\theta)}{F\left(\theta^{TAF}\right)}dH(\theta_s),$$

where $H(\theta_s)$ is the distribution of the m^{th} weakest bank of all; i.e., $H(\theta_s)=\int_0^{\theta_s}h(\theta)d\theta$, where

$$h(\theta)=\binom{n}{m}F^{m-1}(\theta)f(\theta)(1-F(\theta))^{n-m}.$$

Rearranging the expression for θ^{TAF}, we have

$$\text{(TAF)}\qquad [\delta R-r_A]-\left[\delta R\theta^{TAF}+pk_A\left(\theta^{TAF}\right)\right]=0.$$

The terms in the first pair of square brackets do not depend on θ^{TAF}. The terms in the second pair of square brackets can be expanded and rearranged as

$$(\delta R-p\kappa)\theta^{TAF}+pK+p\kappa\int_0^{\theta^{TAF}}\int_{\theta_s}^{\theta^{TAF}}\frac{\theta dF(\theta)}{F(\theta_s)}dH(\theta_s)$$
$$+p\kappa\left[\theta^{TAF}-\int_0^{\theta^{TAF}}\frac{\theta dF(\theta)}{F\left(\theta^{TAF}\right)}dH(\theta_s)\right].$$

The square bracket in the integral is increasing in θ^{TAF}, and the second term is also increasing in θ because each term in the integral Bagnoli and Bergstrom (2005), mean advantage over inferiors) is positive, as long as $\delta R>p\kappa$. The term in the third pair of square brackets in equation (TAF) is decreasing in θ^{TAF}. Therefore, the left-hand side of equation (TAF) is strictly decreasing in θ^{TAF}.

To show the existence of a unique solution to equation (TAF), it remains to show that its left-hand side is positive for $\theta^{TAF}=0$ and negative for $\theta^{TAF}=1$. When $\theta^{TAF}=0$, its left-hand side is $\delta R-r_A-pk(0)>0$, and when $\theta^{TAF}=1$, its left-hand side equals $-r_A<0$. Hence, there is a unique equilibrium. □

Proof of Lemma 1. Bank θ would borrow in DW if and only if $(1-\theta)R-r_D-pk_D\geq 0$, which simplifies to $\theta\geq\theta_D\equiv 1-(r_D+pk_D)/R$. □

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Proof of Lemma 2. Banks that could still get a positive payoff from borrowing in DW if they lose in the auction are willing to pay up to $\beta^D(\theta)$:

$$R(1-\theta)-r_D-pk_D=\delta R(1-\theta)-c-\beta^D(\theta)-pk_A.$$

Rearrange:

$$\beta^D(\theta)=r_D+pk_D-pk_A-(1-\delta)R(1-\theta)-c.$$

Note that the bid is increasing in θ, for $\theta<\theta_D$. However, for banks that could not get a positive payoff from borrowing in DW, they are willing to pay up to $\beta^N(\theta)$:

$$0=\delta R(1-\theta)-c-\beta^N(\theta)-pk_A.$$

Rearrange:

$$\beta^N(\theta)=\delta R(1-\theta)-c-pk_A.$$

Note that the bid is decreasing in θ, for $\theta>\theta_D$.

Altogether, the maximum WTP in the auction is

$$\beta(\theta)=\begin{cases}\beta^D(\theta)=r_D+pk_D-pk_A-(1-\delta)R(1-\theta)-c & \text{if } \theta<\theta_D,\\ \beta^N(\theta)=\delta R(1-\theta)-c-pk_A & \text{if } \theta\geq\theta_D.\end{cases}$$

Bank θ participates in the auction if its maximum WTP in the auction is greater than the minimum required bid r_A (i.e., if the bank's type is between θ_1 and θ_A, where $\beta^D(\theta_1)=r_A$ and $\beta^N(\theta_A)=r_A$). Solving for those conditions and simplifying, we get

$$\theta_1=1-\frac{r_D-r_A+pk_D-pk_A-c}{(1-\delta)R}, \quad \text{and} \quad \theta_A=1-\frac{r_A+c+pk_A}{\delta R}.$$

□

Proof of Lemma 3. By Lemma 1, banks borrow from DW if and only if

$$\theta\leq\theta_D=1-\frac{r_D+pk_D}{R}.$$

Of these banks, some are willing to wait for the auction if and only if

$$\theta>\theta_1=1-\frac{r_D-r_A+pk_D-pk_A}{(1-\delta)R}.$$

Banks that borrow from DW would not participate in the auction if and only if $\theta_1\geq\theta_D$, which is

$$1-\frac{r_D-r_A+pk_D-pk_A}{(1-\delta)R}\geq 1-\frac{r_D+pk_D}{R}.$$

The inequality can be simplified to

$$r_D+pk_D\geq\frac{r_D-r_A+pk_D-pk_A}{1-\delta},$$

which further simplifies to

$$r_D + pk_D - \delta(r_D + pk_D) \geq r_D + pk_D - r_A - pk_A,$$

which can be further simplified to $\delta \leq (r_A + pk_A)/(r_D + pk_D)$. Hence, in equilibrium, if $\delta \leq r_A/\left(r_D + pk_D^*\right)$, banks that would borrow from DW if they lost in the auction would not participate in the auction in the first place.

Knowing the condition derived previously, we can directly verify that banks $\theta \in \left[0, \theta^{DW}\right]$ borrowing from DW immediately are part of an equilibrium. When banks $\theta \in \left[0, \theta^{DW}\right]$ borrow from DW, the equilibrium DW stigma is $k_D^* = k_D^{DW}\left(\theta^{DW}\right)$, and as we have the assumption $\delta \leq r_A/\left[r_D + pk_D^{DW}\left(\theta^{DW}\right)\right]$, by the condition derived previously, we have that no DW bank would be willing to participate in the auction. Furthermore, as bank θ^{DW}, which should have the highest WTP in the auction, is not willing to participate in the auction, no bank will participate in the auction.

Proof of Theorem 2. An equilibrium is determined by three thresholds, θ_1, θ_D, and θ_A, where

$$\theta_D = 1 - \frac{r_D + pk_D}{R},$$

$$\theta_1 = 1 - \frac{r_D + pk_D - r_A - pk_A}{(1-\delta)R},$$

$$\theta_A = 1 - \frac{r_A + pk_A}{\delta R}.$$

Rearranging the three equations, we have

(DW2) $$(1-\theta_D)R - r_D - pk_D = 0,$$

(DW1) $$(1-\theta_1)(1-\delta)R - r_D - pk_D = r_A + pk_A,$$

(A) $$(1-\theta_A)\delta R = r_A + pk_A.$$

The stigma costs are

$$k_D(\theta_D, \theta_1, \theta_A) = K - \kappa \frac{\int_0^{\theta_1} \theta dF(\theta) + \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{\theta dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A)}{F(\theta_1) + \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A)},$$

and

$$k_A(\theta_1, \theta_A) = K - \kappa \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_{s2}(s)} \frac{\theta dF(\theta)}{F(\theta_{s2}(s)) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A),$$

where $[\theta_{s1}(s), \theta_{s2}(s)]$ is the interval of types of banks winning the auction when s is the stop-out rate, and $H(s|\theta_1, \theta_2)$ is the distribution of the stop-out rate.

Plugging $k_A(\theta_1, \theta_A)$ into equation (A), we have

$$\delta R - r_A - pK - (\delta R - p\kappa)\theta_A - p\kappa\left[\theta_A - \int_{r_A}^{\infty}\int_{\theta_{s1}(s)}^{\theta_{s2}(s)} \frac{\theta dF(\theta)}{F(\theta_{s2}(s)) - F(\theta_{s1}(s))} dH(s|\theta_1,\theta_A)\right] = 0.$$

The expression in the square brackets is mean advantage over inferiors for an order statistics distribution. Then, by Chen, Xie, and Hu (2009), the order statistics distribution is log-concave. Hence, by Bagnoli and Bergstrom (2005), Theorem 5, the expression in the square brackets is increasing in θ_A. If $\delta R > p\kappa$, then the left-hand side of the equation previously is strictly decreasing in θ_A. For each fixed θ_1, there is a unique θ_A that satisfies the equation. Let $\tilde{\theta}_A(\theta_1)$ represent this function, and note that $\tilde{\theta}_A(\theta_1)$ is strictly increasing in θ_1.

Plugging k_D into equation (DW2) and rearranging, we have

$$R - r_D - pK - \theta_D R + p\kappa\frac{1}{\Delta}\int_0^{\theta_1}\theta dF(\theta) - p\kappa\frac{1}{\Delta}\int_{r_A}^{\infty}\int_{\theta_{s1}(s)}^{\theta_D}\frac{\theta dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH\left(s|\theta_1,\tilde{\theta}_A(\theta_1)\right) = 0,$$

where $\Delta = F(\theta_1) + \int_{r_A}^{\infty}\int_{\theta_{s1}(s)}^{\theta_D}\frac{dF(\theta)}{F(\theta_D)-F(\theta_{s1}(s))} dH(s|\theta_1,\theta_A)$ represents the denominator in the fractional part of the expression of k_D. The terms that include θ_D can be rearranged as

$$-\theta_D(R - p\kappa) - p\kappa\left[\theta_D - \frac{\int_0^{\theta_1}\theta dF(\theta) + \int_{r_A}^{\infty}\int_{\theta_{s1}(s)}^{\theta_D}\frac{\theta dF(\theta)}{F(\theta_D)-F(\theta_{s1}(s))} dH\left(s|\theta_1,\tilde{\theta}_A(\theta_1)\right)}{F(\theta_1) + \int_{r_A}^{\infty}\int_{\theta_{s1}(s)}^{\theta_D}\frac{dF(\theta)}{F(\theta_D)-F(\theta_{s1}(s))} dH\left(s|\theta_1,\tilde{\theta}_A(\theta_1)\right)}\right].$$

Again, the expression in the square brackets is mean advantage over inferiors for a truncated order statistics distribution, which continues to be log-concave, so it is increasing in θ_D. Therefore, for each θ_1, there is a unique θ_D that satisfies equation (DW2). Let $\tilde{\theta}_D(\theta_1)$ represent this function.

Plugging $\tilde{\theta}_D(\theta_1)$, $\tilde{\theta}_A(\theta_1)$, k_D, and k_A into equation (DW1), we have

$$-r_D - r_A + (1-\delta)R - \theta_1(1-\delta)R - pk_D\left(\theta_1,\tilde{\theta}_D(\theta_1),\tilde{\theta}_A(\theta_1)\right) - pk_A\left(\theta_1,\tilde{\theta}_A(\theta_1)\right) = 0.$$

Using the same trick as before, we extract and rearrange all the terms that include θ_1:

$$-\theta_1[(1-\delta)R - p\kappa] - pk_A\left(\theta_1,\tilde{\theta}_A(\theta_1)\right)$$
$$-p\kappa\left[\theta_1 - \frac{\int_0^{\theta_1}\theta dF(\theta) + \int_{r_A}^{\infty}\int_{\theta_{s1}(s)}^{\theta_D}\frac{\theta dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1,\theta_A)}{F(\theta_1) + \int_{r_A}^{\infty}\int_{\theta_{s1}(s)}^{\theta_D}\frac{dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1,\theta_A)}\right].$$

The expression is strictly decreasing for the same reason as in the previous argument, as long as $(1-\delta)R > p\kappa$. Therefore, there is a unique θ_1. □

Proof of Corollary 2. By (DW), threshold θ^{DW} satisfies

$$\left(1 - \theta^{DW}\right)R - r_D - pk^{DW}\left(\theta^{DW}\right) = 0 \Rightarrow \theta^{DW} = 1 - \left[r_D + pk^{DW}\left(\theta^{DW}\right)\right]/R.$$

By Lemma 1,

$$\theta_D = 1 - [r_D + pk_D]/R.$$

Because θ^{DW} has a strict incentive to bid in the auction, $\theta^{DW} > \theta_D$. □

Proof of Proposition 5. Banks $\theta \leq \theta_D$ prefer borrowing from DW to not borrowing, where $\theta_D = 1 - (r_D + pk_D)/R$, as characterized in the proof of Proposition 1. Banks $\theta \leq \theta_D$ bid $\beta(\theta) = r_D + pk_D - pk_A$, which follows from $(1-\theta)R - r_D - pk_D = (1-\theta)R - \beta(\theta) - pk_A$. If they participate in the auction, banks $\theta > \theta_D$ would bid $\beta(\theta) = (1-\theta)R - pk_A$, which follows from $(1-\theta)R - \beta(\theta) - pk_A = 0$. Only banks θ such that $\beta(\theta) \geq r_A$ participate in the auction. That is, only banks $\theta \leq \theta_A$ participate in the auction, where $\theta_A = 1 - (r_A + pk_A)/R$ is derived from$(1-\theta_A)R - pk_A = r_A$.

Fix cutoffs θ_D and θ_A. The stigma cost of borrowing from DW is

$$k_D(\theta_D) = K - p\kappa \int_0^{\theta_D} \theta \frac{dF(\theta)}{F(\theta_D)}.$$

The stigma cost $k_A(\theta_D, \theta_A)$ of borrowing from TAF is lower, as some banks stronger than θ_D may obtain liquidity from TAF:

$$K - p\kappa \left[\int_0^{\theta_D} \int_0^{\theta_D} \frac{\theta dF(\theta)}{F(\theta_D)} dH(\theta_s) + \int_{\theta_D}^{\theta_A} \int_0^{\theta'} \frac{\theta dF(\theta)}{F(\theta')} dH(\theta_s) + \int_{\theta_A}^{1} \int_0^{\theta_A} \frac{\theta dF(\theta)}{F(\theta_A)} dH(\theta_s) \right],$$

where $H(\theta_s)$ is the distribution of the m^{th} weakest bank, i.e., $H(\theta_s) = \int_0^{\theta_s} h(\theta) d\theta$, where

$$h(\theta_s) = \binom{n}{m} F^{m-1}(\theta_s) f(\theta_s) [1 - F(\theta_s)]^{n-m}.$$

In equilibrium, $\theta_{D'}$ is uniquely pinned down by $R(1-\theta) - r_D - pk_D(\theta) = 0$, and $\theta_{A'}$ is uniquely pinned down by $R(1-\theta) - r_A - pk_A(\theta, \theta_{D'}) = 0$. The uniqueness follows from the monotonicity of the left-hand side of the two equations, which is argued in previous proofs.

Proof of Proposition 6. A type θ bank who would participate in the auction would bid $\beta(\theta) = (1-\theta)R - pk_A$, which is a decreasing function of θ (i.e., worse banks would bid higher). Hence, the probability of winning, $w(\theta)$, is decreasing in θ (i.e., worse banks are more likely to win in the auction). The payoff of bank θ in the auction would be $u_A(\theta) = \int_s^{\beta(\theta)} ((1-\theta)R - s - pk_A) h(s) ds$, where s is the realized stop-out rate and $h(s)$ is the probability density of s. Alternatively, bank θ would get a payoff of $u_D(\theta) = (1-\theta)R - r_D - pk_D$ from borrowing in DW. The slope of $u_D(\theta)$ with respect to θ is $-R$, and the slope of $u_A(\theta)$ is $-R\int_s^{\beta(\theta)} h(s) ds$, negative but greater than $-R$. Hence, there is a single crossing in $u_D(\theta)$ and $u_A(\theta)$ such that there exists θ''_D such that for any $\theta \leq \theta''_D$, $u_D(\theta) \geq u_A(\theta)$, and for any $\theta > \theta''_D$, $u_D(\theta) < u_A(\theta)$. Banks $\theta < \theta''_A$ would be willing to participate in the auction, where $(1-\theta''_A)R - pk_A = 0$, which simplifies to $\theta''_A = 1 - pk_A/R$. □

Proof of Proposition 7. Bank θ, by borrowing in DW D, gets $u_D(\theta) = (1-\theta)R - r_D - pk_D$ and by borrowing in DW D' gets $u_{D'}(\theta) = \delta(1-\theta)R - r_{D'} - pk_{D'}$. Therefore, bank θ prefers borrowing from D to borrowing from D' if and only if

$$u_D(\theta) = (1-\theta)R - r_D - pk_D \geq u_{D'}(\theta) = \delta(1-\theta)R - r_{D'} - pk_{D'},$$

which is rearranged as

$$(1-\delta)(1-\theta)R - (r_D - r_{D'}) - (pk_D - pk_{D'}) \geq 0.$$

Hence, banks $\theta \leq \theta_1$ borrow from DW D, where

$$\theta_1 = 1 - \frac{(r_D - r_{D'}) + (pk_D - pk_{D'})}{(1-\delta)R}.$$

Furthermore, bank θ prefers borrowing from DW D' to not borrowing if and only if

$$u_{D'}(\theta) = \delta(1-\theta)R - r_{D'} - pk_{D'} \geq 0,$$

which is rearranged as

$$\theta \leq \theta_2 \equiv 1 - \frac{r_{D'} + pk_{D'}}{\delta R}.$$

To have banks borrowing from DW D', we must have $\theta_2 > \theta_1$, i.e.,

$$1 - \frac{r_{D'} + pk_{D'}}{\delta R} > 1 - \frac{(r_D - r_{D'}) + (pk_D - pk_{D'})}{(1-\delta)R},$$
$$\frac{r_{D'} + pk_{D'}}{\delta} < \frac{(r_D - r_{D'}) + (pk_D - pk_{D'})}{(1-\delta)},$$

which is rearranged as

$$\delta(r_D + pk_D) > r_{D'} + pk_{D'}.$$

As banks $\theta \in [0, \theta_1]$ borrow from DW D and banks $\theta \in (\theta_1, \theta_2]$ borrow from DW D', the stigma costs are

$$k_D(\theta_1) = K - \kappa \int_0^{\theta_1} \frac{\theta dF(\theta)}{F(\theta_1)} \quad \text{and} \quad k_{D'}(\theta_1, \theta_2) = K - \kappa \int_{\theta_1}^{\theta_2} \frac{\theta dF(\theta)}{F(\theta_2) - F(\theta_1)}.$$

Equilibria θ_1 and θ_2 satisfy

(D1) $$(1-\delta)(1-\theta_1)R - (r_D - r_{D'}) - pk_D(\theta_1) + pk_{D'}(\theta_1, \theta_2) = 0 \quad \text{and}$$

(D2) $$\delta(1-\theta_2)R - r_{D'} - pk_{D'}(\theta_1, \theta_2) = 0.$$

Plug $k_D(\theta_1)$ into and rearrange the left-hand side of equation (D1):

$$(1-\delta)R - (r_D - r_{D'}) - pK - [(1-\delta)R - p\kappa]\theta_1 - p\kappa\left[\theta_1 - \int_0^{\theta_1} \frac{\theta dF(\theta)}{F(\theta_1)}\right] + pk_{D'}(\theta_1, \theta_2).$$

The expression is strictly decreasing in θ_1 as long as $(1-\delta R) > p\kappa$. In addition, the expression is strictly decreasing in θ_2. Therefore, given any θ_2, there is a unique

$\theta_1(\theta_2)$ that satisfies equation (D1), and $\theta_1(\theta_2)$ is strictly decreasing in θ_2. Plug $k_{D'}(\theta_1,\theta_2)$ into and rearrange equation (D2):

$$\text{(D2')} \qquad \delta R - r_{D'} - pK - (\delta R - p\kappa)\theta_2 - p\kappa\left[\theta_2 - \int_{\theta_1(\theta_2)}^{\theta_2} \frac{\theta dF(\theta)}{F(\theta_2) - F(\theta_1(\theta_2))}\right] = 0.$$

Consider the derivative of $\theta_2 - \int_{\theta_1(\theta_2)}^{\theta_2} \frac{\theta dF(\theta)}{F(\theta_2)-F(\theta_1(\theta_2))}$ with respect to θ_2. Fixing $\theta_1(\theta_2)$, the derivative is positive, because the expression is a mean advantage over inferiors for the truncated cdf $F(\theta)$ between $\theta_1(\theta_2)$ and θ_2. The derivative with respect to $\theta_1(\theta_2)$ is decreasing, but $\theta_{1'}(\theta_2) < 0$. Hence, the derivative overall is increasing. Therefore, the left-hand side of equation (D2) is strictly decreasing in θ_2, as long as $\delta R > p\kappa$, and there is a unique θ_2 that satisfies equation (D2'). □

Proof of Proposition 8. Bank θ gets $(1-\theta)R - r_D - pk_D$ from D and gets $(1-\theta)R - r_{D'} - pk_{D'}$ from D'. All banks are indifferent between the two facilities if $r_D + pk_D^* = r_{D'} + pk_{D'}^*$. Therefore, the average bank borrowing from D is worse than the average bank borrowing from D', and consequently, the average bank of all borrowing banks is better than the average bank borrowing from D. The marginal bank θ^* satisfies $(1-\theta^*)R - r_D - pk_D^* = 0$. However, if the average bank of all banks $\theta \in [0,\theta^*]$ is better than the average bank borrowing from D, $(1-\theta^*)R - r_D - p\left[K - \kappa\int_0^{\theta^*} \frac{\theta dF(\theta)}{F(\theta^*)}\right] > 0$. Some banks $\theta > \theta^*$ would have borrowed if only D with interest rate $r_D < r_{D'}$ were offered.

Supplementary Material

To view supplementary material for this article, please visit http://doi.org/10.1017/S0022109023001187.

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