Digital Villages

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Abstract

A notable phenomenon in the digital economy is the emergence of *digital villages*: e-commerce platforms support and subsidize suppliers in less developed rural areas to manufacture and sell products online. We argue that, despite platforms' philanthropic claims, these actions are strategically designed to enhance profitability. By providing early subsidies to young sellers, platforms incentivize entry and reduce learning costs, later recouping these investments as sellers gain experience and increase sales. Using a dynamic model of two-sided markets, we analyze the intertemporal and cross-side pricing strategies of a platform with market power. Our findings indicate that sellers' network externalities and learning-by-doing effects reinforce each other, motivating the platform to subsidize them. This study bridges two typically distinct areas of the economy: global online platforms and less developed rural regions.

Keywords: digital villages, two-sided markets, dynamic pricing, price discrimination

JEL Codes: D62, O12, L11, L81

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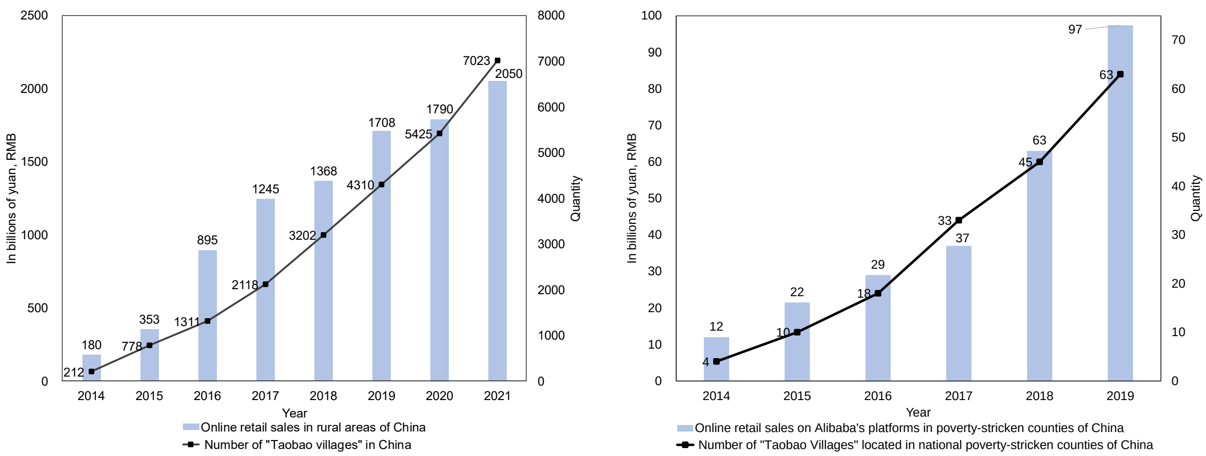
1 Introduction

Digital marketplaces have grown rapidly in recent years. E-commerce giants—such as Amazon, eBay, Mercado Libre, Alibaba, JD, and Temu—link buyers and sellers through their online trading services. A notable phenomenon in China is the rise of digital villages: rural villages with a high volume of transactions and a significant number or proportion of online stores (Luo and Niu, 2019). Defined by AliResearch (2015), a "Taobao village" on Alibaba's platforms (mainly including Taobao and Tmall) is an administrative village in which the number of active online shops exceeds 100 or the ratio of the active online shops to the local households exceeds 10%, and the annual turnover in e-commerce exceeds 10 million yuan (approximately US$1.52 million).[1] In the span of eight years from 2014 to 2021, online sales in rural China increased more than 11-fold from $27.36 billion to $311.55 billion, and the number of Taobao villages surged more than 33-fold from 212 to 7,023 (see Figure 1(a)).When considering only less developed areas (LDAs)—832 poverty-stricken counties—in China, online retail sales on Alibaba's platforms increased eight-fold from $1.82 billion in 2014 to $14.74 billion in 2019, and during the same period, the number of Taobao villages increased more than 15-fold from 4 to 63 (see Figure 1(b)).[2]



(a) All rural counties

(b) Poverty-stricken counties

Figure 1: Online retail sales and the number of Taobao Villages in rural China, 2014–2021

Notes: Data sources are provided in Online Appendix A. Alibaba Group's China Taobao Village Research Report is only updated through 2021; thus, the data in the left panel is reported up to 2021. The 832 poverty-stricken counties officially designated by the Chinese government were declared to have exited poverty in 2020, so the data in the right panel ends in 2019.

[1] Dollar to yuan nominal exchange rate used in this paper is 6.58, the average from 2014 to 2021.

[2] In 2014, the State Council Leading Group Office of Poverty Alleviation and Development identified 832 counties in China as poverty-stricken because of their extremely low income per capita. We refer to them as LDAs.

What role have e-commerce giants played in driving economic development in LDAs? Take Alibaba as an example. In October 2014, Alibaba launched the "1,000 Counties and 10,000 Villages" program (also known as the rural Taobao model). This program planned to invest 10 billion yuan ($1.52 billion) over three to five years and aimed to build an e-commerce service system with 100,000 administrative villages in 1,000 counties in rural areas of China.[3] In December 2017, Alibaba initiated the Poverty Alleviation Fund, investing 10 billion yuan ($1.52 billion) over five years. This fund aimed at poverty reduction and alleviation in five target areas: e-commerce, ecology, education, health, and women.[4] In 2018, Alibaba held a 58.2% share of China's e-commerce market, significantly surpassing other platforms.[5] Why would a platform with significant market power be incentivized to engage heavily in seemingly philanthropic public affairs? How does a profit-maximizing company like Alibaba benefit from investing in LDAs? The existing literature lacks a targeted theory to analyze these paramount questions related to two-sided markets and economic development. This paper aims to fill this void.

By incorporating sellers' learning-by-doing into a two-sided market framework, we develop an overlapping generations model in which a monopoly platform (or any platform with market power) adopts intertemporal and cross-side pricing strategies. The model proceeds as follows. The profit-maximizing platform sets membership fees—possibly negative—for buyers and sellers. Upon observing these fees, buyers and sellers simultaneously choose, in each period, between participating on the platform and taking an outside option. After each period, new generations of sellers and buyers are born; sellers live for two periods (young and old), whereas buyers live for one period. All agents are rational and perfectly anticipate others' responses in each period. The utility function for buyers and sellers participating in online sales on the platform follows

[3] According to AliResearch (2015) and World Bank and Alibaba (2018), the following three outcomes are noteworthy: (1) By the end of 2018, there were 30,000 village-level service stations and a rural service team at the village level of nearly 60,000 people (including part-time and full-time workers). This program incubated 160 regional agricultural brands; (2) By the end of 2017, Taobao University had built 11 e-commerce training bases that conducted 133 e-commerce courses to train entrepreneurs; (3) By the end of 2017, Ant Financial, a subsidiary of Alibaba Group, had provided $1.70 billion in loans to entrepreneurs in poor counties and underdeveloped areas.

[4] World Bank and Alibaba (2018) highlight three notable outcomes of this program. First, in 2019, poverty-stricken counties recorded sales revenues of $14.76 billion on Alibaba's platforms. Second, in 2018, Alibaba trained over 260,000 people, both employed and self-employed in e-commerce and cloud computing, and opened nine e-commerce training bases in impoverished counties. Third, the program trained 18,200 women and helped 10,600 women gain employment in e-commerce.

[5] In 2018, the second and third biggest e-commerce giants, JD and Pinduoduo, owned 16.3% and 5.2% of the retail e-commerce sales shares in China, respectively.

a simplified version of Weyl (2010): buyers and sellers have heterogeneous membership values drawn from uniform distributions; each side receives a homogeneous interaction value from online trade; and all buyers and sellers obtain zero utility from offline trade (the outside option).[6] More specifically, the membership values of buyers are always positive (we call them membership benefits), and the membership values of sellers are negative (we call them membership costs) when they are young. Due to learning-by-doing, sellers who participated in online sales early on have lower membership costs in the next period, while the costs of new entrants remain unchanged. This specification for sellers' membership costs is novel, and the heterogeneity of sellers' membership costs can be interpreted as the difference in online business ability.

In the baseline model, the platform charges different fees to young and old generations of sellers (third-degree price discrimination), whereas buyers pay a uniform fee each period.[7] In equilibrium, the platform subsidizes young sellers when either (i) the gain in business ability from learning-by-doing is sufficiently large, or (ii) a seller generates greater cross-side network externalities than a buyer. Furthermore, prices for buyers and old sellers exceed those for young sellers. Although sellers incur entry costs when young, the platform's intertemporal and cross-side pricing reflects their anticipated productivity gains and higher cross-side network externalities. This mechanism can explain why monopoly e-commerce platforms invest in training inexperienced merchants, establishing rural service stations, and extending credit in LDAs. The model also predicts equal numbers of buyers across periods and equal numbers of sellers across periods, as the platform fully internalizes learning-by-doing and cross-side network externalities through dynamic pricing.

We also examine the implications of our findings for economic development. Specifically, we characterize rich sellers in two ways and assess how the equilibrium results change. First, based on the baseline model, we consider the case in which rich sellers obtain lower learning-by-doing benefits than poor sellers due to the law of diminishing marginal returns. We find that: (i) lower learning-by-doing raises the platform's fee for young rich sellers; (ii) learning-by-doing and cross-side externalities are complementary, reinforcing each other in reducing the platform's charges—or increasing its subsidies—to young poor sellers; and (iii) lower learning-by-doing narrows the platform's scope to shift price pressure from young rich sellers to buyers. Second, in an extended model, rich sellers are distributed on an interval with higher business ability

[6]One could argue that the specification of unidimensional heterogeneity is similar to that of Armstrong (2006), but our framework allows for the outside option and does not involve the so-called "cost of distance" under the Hotelling-based framework. In this sense, our framework is closer to that of Weyl (2010).

[7]Price discrimination between entrants and incumbents—particularly among sellers—is common in practice; see Cabral (2019).

and have lower learning-by-doing benefits. We find that the condition under which it is more stringent for rich sellers to receive platform subsidies than for poor sellers in the baseline model. Second, we consider an extended model in which rich sellers are distributed over an interval with higher business ability and have lower learning-by-doing benefits. We find that the condition under which rich sellers receive the platform's subsidies is more stringent than that for poor sellers in the baseline model. Overall, these results imply that the platform tends to subsidize new and poor sellers.

We further extend the model in three directions: (i) generalizing the distribution functions of buyers and sellers; (ii) allowing intra-cohort price discrimination among old sellers based on prior participation when young; and (iii) modeling sellers' learning-by-doing as enhancing the cross-side externality from buyers to sellers. Across all three extensions, the equilibrium results remain consistent with those of the baseline model—namely, that the platform may subsidize young sellers because of their learning-by-doing—thereby confirming the robustness of our findings.

The rest of this paper is organized as follows. The remainder of this section reviews the related literature. Section 2 presents the baseline model, and Section 3 analyzes its equilibrium. Section 4 discusses implications for economic development. Section 5 provides several robustness checks and model extensions, and Section 6 concludes. All proofs are included in the Online Appendix.

Related literature

This paper enriches the theoretical literature on two-sided markets. The theory of two-sided markets is proposed in seminal works by Rochet and Tirole (2003, 2006), Caillaud and Jullien (2003), and Armstrong (2006).[8] More recent papers, such as Weyl (2010), White and Weyl (2010, 2016), Jullien and Pavan (2019), Karle et al. (2020), and Tan and Zhou (2021), further develop the literature on two-sided markets. Although these papers offer general frameworks for exploring conventional topics in industrial organization, such as pricing, competition, and platform entry in the presence of cross-group externalities, these models are static and cannot be used to analyze the dynamic pricing strategy of the monopoly platform driving e-commerce development in LDAs. Some theoretical articles have studied dynamic games with network effects, but they do not explicitly model the dynamic game among three rational parties—buyers, sellers, and the monopoly platform—in a two-sided market. For example, Doganoglu (2003), Cabral (2011), Radner et al. (2014), Biglaiser and Crémer (2020) and Halaburda et al. (2020) focus solely on one-sided network effects of consumers, rather than cross-side externalities in two-sided markets. Although Chen and Tse (2008) and Cabral

[8]Previously, network externalities in the information communication technology industry were studied in pioneer papers like Katz and Shapiro (1985, 1986) and Farrell and Saloner (1985, 1986).

(2019) consider dynamic pricing in two-sided markets, the agents in Cabral (2019) only take current payoffs into account, and the numbers of buyers and sellers in Chen and Tse (2008) only depend on the anticipated market segment in the subsequent period.[9] Using a two-period model of two-sided markets, Lam (2017) analyzes the impact of switching costs on price competition between two symmetric platforms.[10] In contrast, by incorporating overlapping generations of sellers into a two-sided market framework, we examine the cross-side and intertemporal pricing strategies of a monopoly platform, account for distinct business-ability distributions of buyers and sellers, and derive closed-form pricing solutions.[11]

Our paper links industrial organization and economic development. Specifically, it incorporates learning by doing among merchants in LDAs into a dynamic model of two-sided markets. Existing studies, theoretical or empirical, cover various industries, such as payment systems (Bedre-Defolie and Calvano, 2013; Dolfen et al., 2019; Li et al., 2020; Rochet and Tirole, 2003; Wright, 2012), informational intermediaries via the internet (Caillaud and Jullien, 2003), video games (Hagiu, 2006; Landsman and Stremersch, 2011; Lee, 2013; Zhou, 2017; Zhu and Iansiti, 2012), media markets (Anderson and Coate, 2005; Anderson et al., 2018; Athey et al., 2013; Ferrando et al., 2004), newspapers (Argentesi and Filistrucchi, 2007; Chandra and Collard-Wexler, 2009; Fan, 2013; Seamans and Zhu, 2014), magazines (Kaiser and Wright, 2006), sport card conventions (Jin and Rysman, 2015), labor matching markets (Lee and Schwarz, 2017), and online real estate trade (Karle et al., 2020). However, the incentives for a platform with market power to foster economic development in LDAs remain theoretically unexplored.

In recent years, some empirical papers have examined the development of e-commerce in rural China, for example, Luo and Niu (2019) and Couture et al. (2021). In particular, the rapid development of digital villages has attracted much academic and policy attention (Ding et al., 2018; Luo and Niu, 2019; Qi et al., 2019). For example, Fan et al. (2018) demonstrate that e-commerce can reduce spatial consumption inequality by lowering fixed market-entry costs and distance-based trade barriers; Luo and Niu (2019) highlight the role of e-commerce in fostering entrepreneurship. Moreover, several studies have explored how learning-by-doing

[9] Doganoglu (2003) and Radner et al. (2014) assume that consumers are myopic. Moreover, Chen and Tse (2008) and Cabral (2019) do not explicitly characterize the cross-side network externalities between buyers and sellers as in Rochet and Tirole (2003), Armstrong (2006), and Weyl (2010).

[10] There are only two types of agents—consumers and platforms—in Lam's paper, although the consumers are distributed on the different sides. To simplify the analysis, Lam also assumes that the interaction benefits each consumer obtains from any consumer on the other side are the same no matter which side this consumer belongs to.

[11] Our research topic and the model setup are quite different from those of Lam (2017). Lam's model focuses on the effects of switching costs on the first-period price competition and social welfare in two-sided markets.

fosters economic development in theory (Arrow, 1962; Lucas, 1988). More broadly, Nunn (2020) reviews the literature on economic development from a historical perspective. However, the formation of sellers' learning-by-doing and the role of monopoly platforms in economic development remain underexplored. In summary, the development of e-commerce in LDAs and two-sided markets have been examined separately in the literature. Our paper links them.

Our study also contributes to the literature on price regulation and price discrimination in two-sided markets.[12] Price discrimination between groups of agents on the two sides is well documented in early studies, for example, Caillaud and Jullien (2003), Armstrong (2006) and Weyl (2010). However, when the two sides consist of different types of agents (e.g., buyers and sellers), the price difference between the two sides does not qualify as price discrimination.[13] A few papers have recognized price discrimination on different types of agents on each side. For example, using a model of a two-sided monopoly platform, Jeon et al. (2022) document second-degree price discrimination on two types of agents on one side and analyze the impact of price regulation on social welfare.[14] Liu and Serfes (2013) explore perfect price discrimination within each group in two-sided markets using a static model. In contrast, our dynamic model allows for the characterization of third-degree price discrimination across generations of sellers as well as by their past endogenous homing choices.

2 Baseline Model

There is an infinitely lived monopoly platform that hosts buyers and overlapping generations of sellers over infinitely repeated discrete time indexed $t \in \{0, 1, 2, ...\}$. In each period, there is a unit mass of short-lived buyers. Let N_t^b denote the number of buyers that choose to join the platform at time t. In addition, a generation of measure 1/2 sellers is born each period, where every seller lives for two periods. Let $N_{1,t}^s$ and $N_{2,t+1}^s$ denote the mass of sellers of generation t who join the platform in the first and second periods of their

[12]Some papers have examined price discrimination in two-sided markets (Evans, 2003; Gomes and Pavan, 2016; Jeon et al., 2022; Liu and Serfes, 2013; Rysman, 2009; Wang and Wright, 2017; Zhang and Liu, 2016). Weisman and Kulick (2010), referring to the Notice of Proposed Rulemaking issued by the Federal Communications Commission in the United States, provide a detailed discussion of price discrimination and two-sided markets in the regulation circumstance of net neutrality.

[13]The price difference between different sides is called "cross-subsidization" in some papers, for example, Gomes and Pavan (2016), Cabral (2019) and Tan and Zhou (2021).

[14]Choi et al. (2015), Böhme (2016), and Lin (2020) also analyze second-degree price discrimination in two-sided markets.

lives (i.e., periods t and $t+1$), respectively. Also, let $N_t^s = N_{1,t}^s + N_{2,t}^s$ represent the total mass of sellers present on the platform at time t. We now provide more details for each player.

2.1 The Monopoly Platform

The platform has pricing power over both buyers and sellers in each period and aims to maximize its aggregate profit. The platform charges a uniform fee P_1^s and P_2^s in each seller's first and second period of life, respectively. For buyers, the platform charges a uniform fee P^b.

In other words, the platform's maximization problem at each time t is

$$\pi = \max_{\boldsymbol{P}} \left[P_1^s \cdot N_{1,t}^s(\boldsymbol{P}) + P^b \cdot N_t^b(\boldsymbol{P}) + \frac{1}{1+r} \cdot P_2^s \cdot N_{2,t}^s(\boldsymbol{P}) \right], \tag{1}$$

where $\boldsymbol{P} = \left\{P_1^s, P_1^b, P_2^s\right\}$ represents the list of platform prices and r is the interest rate on the platform's profit from period t to period $t+1$. The buyers and sellers affiliated with the platform will be endogenously determined by the list of prices.

2.2 Sellers

The target of each seller is to maximize the aggregate discounted utility of selling products in their two periods of life. For each seller in each period, there are two choices: selling online and offline. Importantly, for each generation born at time t, sellers initially differ in their business ability $B_{i1,t}^s$, which obeys a uniform distribution on the interval $[-1, 0]$. If sellers engage with the platform when they are young, they gain a positive increment $c \in (0, 1)$ in business ability in the second period due to learning-by-doing; otherwise, their business ability remains unchanged across the two periods.

Here, each seller's initial business ability is nonpositive. Intuitively, poor sellers face a relatively higher startup cost due to their lower degree of education and the lower internet penetration in rural areas compared to rich sellers.[15] With the deepening of the digital economy in rural regions, poor merchants who are affiliated with the platform when they are young obtain the learning-by-doing benefit that is available in the next period. In this scenario, the law of motion of business ability is

$$B_{i2,t+1}^s = B_{i1,t}^s + c, \tag{2}$$

where $B_{i2,t+1}^s$ represents the seller i's business ability when they are old. However, if the merchant i chooses

[15] This comparison implies that when a poor seller and a rich seller face the same entrepreneurship program, it is more difficult for the poor one to accomplish it.

not to sell products on the digital platform in period 1, the law of motion of the business ability becomes

$$B^s_{i2,t+1} = B^s_{i1,t}, \tag{3}$$

which means there is no enhancement in the business ability of the seller i. As documented in Spence (1981), the hypothesis of learning curve (or learning-by-doing) implies that the unit cost of making a product of a firm decreases with the accumulation of experience. Stokey (1988) studies the role of learning by doing in driving economic growth. Minniti and Bygrave (2001) also argue that the knowledge of entrepreneurs from past experiences determines the sequence of their choices. In contrast, all old (heterogeneous) sellers obtain the same level of cost reduction c from period 1 to period 2 in our model. This can be seen as a simplified version of the learning-by-doing setup in the literature.

For each generation born at time t, seller i's utilities from selling on the e-commerce platform when they are young and old are expressed as

$$U^s_{i1,t} = B^s_{i1,t} + \alpha N^b_t - P^s_1, \tag{4}$$

$$U^s_{i2,t+1} = \underbrace{B^s_{i1,t} + d^i_{1,t} c}_{\equiv B^s_{i2,t+1}} + \alpha N^b_t - P^s_2, \tag{5}$$

where $d^i_{1,t}$ is a binary variable that indicates whether the seller i sells online when they are young ($d^i_{1,t} = 1$ if yes, $d^i_{1,t} = 0$ if no); $\alpha \in (0,1)$ denotes the interaction benefit from each buyer on the other side; $\underline{U}^s$ is the utility of offline selling.

2.3 Buyers

A unit mass of heterogeneous short-lived buyers enters the economy in each period. Each buyer chooses between buying goods on the online platform and from offline stores. Buyers are distinguished by their ability to conduct business online. Specifically, $B^b_{j,t}$ is the level of business ability of the buyer j, which follows a uniform distribution on the interval [0,1]. There is no enhancement of buyers' online business abilities as there is for sellers; thus, the distribution of buyers does not change over time. We assume that the utility of buying offline is a constant value $\underline{U}^b$. Thus, buyer j's utility from buying online at time t is

$$U^b_{j,t} = B^b_{j,t} + \beta N^s_t - P^b, \tag{6}$$

where $\beta \in (0,1)$ denotes the buyer's interaction benefit from each seller.

2.4 Discussion, Timing, and Solution Concept

All prices in our model can be interpreted as *membership fees*, as discussed in Rochet and Tirole (2006). These fees measure the allocation of the gross surplus between buyers and sellers. For example, consider a one-shot trade on the monopoly platform. The highest willingness of a buyer to pay for an online transaction is W, and the seller's reserve price is R. The platform charges buyers and sellers fees P^b and P^s, respectively. This transaction occurs only when $P^b + P^s \leq W - R$. Then, prices P^b and P^s reflect the (latent) bargaining power of buyers and sellers, respectively. The higher the price or fee charged by the online platform, the weaker the agent's bargaining power. More importantly, the two prices are comparable, which will be useful for analyzing the price structure in equilibrium later.

Given the fees charged by the monopoly platform, each living buyer and seller chooses to join it or take the outside option (e.g., offline) in each period. All agents are rational and possess complete information. Therefore, at the outset of the game, the platform can establish its list of pricing, enabling all buyers and sellers to make participation decisions for each period.

Throughout the rest of the paper, we solve for the unique steady-state equilibrium. That is, we solve for the unique Nash equilibrium such that for all (sufficiently high) t, N_t^b and N_t^s are equal, which we will now conveniently denote N^b and N^s. Note that this implies that N_t^b, $N_{1,t}^s$, and $N_{2,t+1}^s$ are the same across generations in the steady-state equilibrium, because the platform pricing does not depend on t. As such, we will now refer to periods 1 and 2 synonymously with the time when an agent is young and old, respectively.

2.5 Buyers' and Sellers' Homing Choices

Next, we characterize the homing choices of sellers and buyers to solve for the unique steady-state equilibrium. First, consider the buyers' problem. Buyers only affiliate with the online platform if their net utility from joining is greater than their outside option. Thus, we have the numbers of buyers affiliated with the online platform as

$$N_t^b(P^b, N_t^s) = \Pr(B_{j,t}^b + \beta N_t^s - P^b \geq \underline{U}^b) = 1 + \beta N_t^s - P^b - \underline{U}^b. \tag{7}$$

Now, we characterize the sellers' optimal homing choice. Consider a single generation at time t of sellers and their optimal homing choice in both periods of life (periods 1 and 2). In general, given the prices implemented by the monopoly platform, each seller's homing choices in the two periods are concluded as a two-period game as in Figure 2. $\widehat{U}_{i2,t+1}^s$ denotes seller i's utility in period 2 when the seller did not choose to join the platform in period 1, and $\widetilde{U}_{i2,t+1}^s$ denotes seller i's utility in period 2 under the condition that this seller had chosen the platform in period 1. Specifically, according to equation (5), $\widehat{U}_{i2,t+1}^s$ and $\widetilde{U}_{i2,t+1}^s$ can be

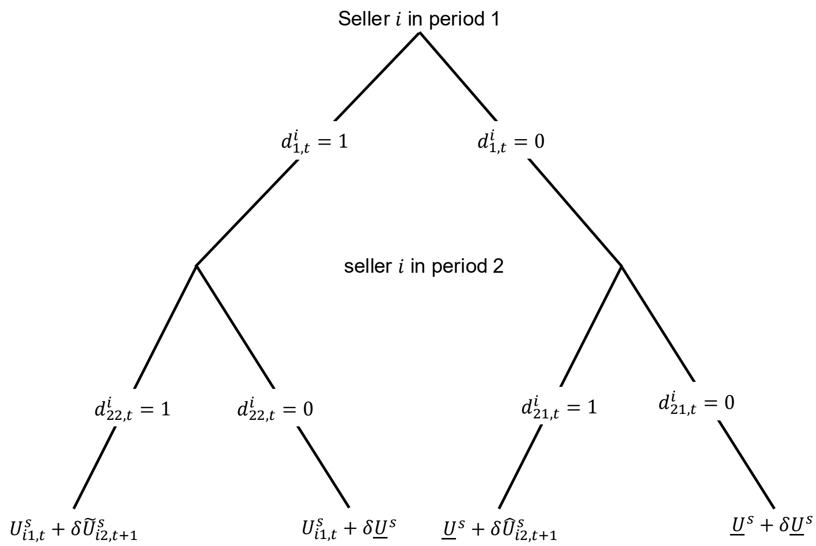


Figure 2: Two-Period Homing-Choice Game for Seller i of Generation t

expressed as $\widehat{U}^s_{i2,t+1} = B^s_{i1,t} + \alpha N^b_{t+1} - P^s_2$ and $\widetilde{U}^s_{i2,t+1} = B^s_{i1,t} + c + \alpha N^b_{t+1} - P^s_2$, respectively. Clearly, $\widetilde{U}^s_{i2,t+1} > \hat{U}^s_{i2,t+1}$ due to the enhancement of seller i's business ability.

Seller i's maximizing problem can be formulated as

$$\max_{d^i_{1,t}\in\{0,1\}} \Bigg\{ (1-d^i_{1,t}) \left[\underline{U}^s + \delta \max_{d^i_{21,t}\in\{0,1\}} \left\{ (1-d^i_{21,t})\underline{U}^s + d^i_{21,t}\widehat{U}^s_{i2,t+1} \right\} \right] + d^i_{1,t} \left[U^s_{i1,t} + \delta \max_{d^i_{22,t}\in\{0,1\}} \left\{ (1-d^i_{22,t})\underline{U}^s + d^i_{22,t}\widetilde{U}^s_{i2,t+1} \right\} \right] \Bigg\},$$

where $d^i_{21,t}$ is a binary variable indicating whether seller i, who did not sell on the platform in period 1 (i.e., $d^i_{1,t} = 0$), chooses to sell on the platform in period 2 ($d^i_{21,t} = 1$ if the seller does, and $d^i_{21,t} = 0$ otherwise); $d^i_{22,t}$ is a binary variable representing whether seller i, who did sell on the platform in period 1 (i.e., $d^i_{1,t} = 1$), continues to sell on the platform in period 2 ($d^i_{22,t} = 1$ if the seller does, and $d^i_{22,t} = 0$ otherwise); δ denotes the discount factor. For simplicity, we assume that the utility of both buyers and sellers is intertemporally additive with a zero discount rate (i.e., $\delta = 1$).

The seller's homing choice in the two periods can be characterized as shown in Figure 3. Although the figure only presents a specific scenario of sellers' homing choice, the logic of their best responses can be seen. Here, the seller k is the critical one in period 1, and the seller h is at the critical point in period 2. For the sake of analysis, let $\underline{B}^s$ denote the business ability of seller k in period 1.

Theoretically, given the prices proposed by the monopoly platform, there are three scenarios for the critical seller(s) in the two periods as follows.

Scenario A (the sellers on the platform are the same in both periods). The seller h and the seller k are

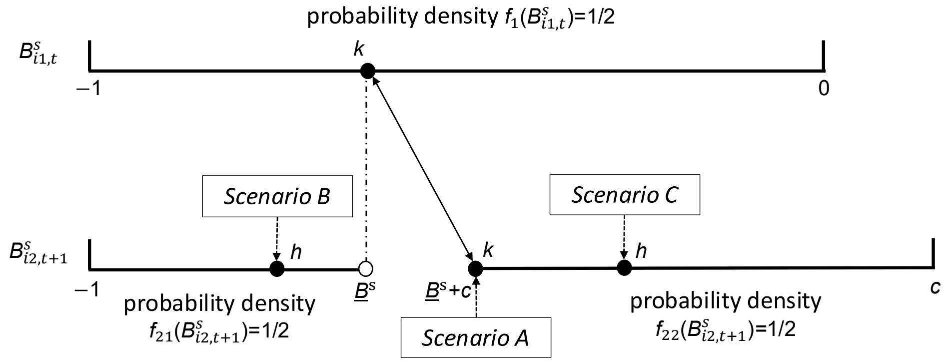


Figure 3: Distribution of Heterogeneous Sellers in Two Periods for a Given Generation

the same, which implies that three conditions, $U^s_{k1,t} = \underline{U}^s$, $\widetilde{U}^s_{h2,t+1} = \underline{U}^s$, and $B^s_{k1,t} = B^s_{h1,t}$, are satisfied. From these conditions, we have that the number of sellers on the platform in the two periods are

$$N^s_{1,t}(P^s_1, N^b_t) = \frac{1}{2}\Pr(B^s_{i1,t} + \alpha N^b_t - P^s_1 \geq \underline{U}^s) = \frac{1}{2}(\alpha N^b_t - P^s_1 - \underline{U}^s) \tag{8}$$

and

$$N^s_{2,t+1}(P^s_2, N^b_{t+1}) = \frac{1}{2}\Pr(B^s_{i1,t} + \alpha N^b_{t+1} + c - P^s_2 \geq \underline{U}^s) = \frac{1}{2}(\alpha N^b_{t+1} + c - P^s_2 - \underline{U}^s). \tag{9}$$

Because the sellers on the platform are the same regardless of age, we know $N^s_{2,t+1} = N^s_{1,t}$. Using this fact and by combining (8) and (9), we get that $N^b_{t+1} = N^b_t$ only if the following condition is satisfied:

$$P^s_2 = P^s_1 + c. \tag{10}$$

Note that $N^b_{t+1} = N^b_t$ also implies that $N^s_{1,t} = N^s_{2,t+1}$ is the same across time, i.e. (10) is necessary for a steady-state equilibrium to exist.

Scenario B (some sellers enter the platform in period 2). There is only one critical seller h in period 2, and this seller has weaker business ability compared with seller k, as shown in Figure 3. In this scenario, three conditions should be satisfied, $\widehat{U}^s_{h2,t+1} = \underline{U}^s$, $U^s_{k1,t} + \widetilde{U}^s_{k2,t+1} = \underline{U}^s + \widehat{U}^s_{k2,t+1}$, and $\widehat{U}^s_{h2,t+1} < \widehat{U}^s_{k2,t+1}$. From the first condition, we can obtain the number of sellers on the platform in period 2 as

$$N^s_{2,t+1}(P^s_2, N^b_{t+1}) = \frac{1}{2}\Pr(B^s_{i1,t} + \alpha N^b_{t+1} - P^s_2 \geq \underline{U}^s) = \frac{1}{2}(\alpha N^b_{t+1} - P^s_2 - \underline{U}^s) \tag{11}$$

and from the second condition, we can derive the number of sellers on the platform in period 1 as

$$N^s_{1,t}(P^s_1, N^b_t) = \frac{1}{2}\Pr(B^s_{i1,t} + \alpha N^b_t - P^s_1 + c \geq \underline{U}^s) = \frac{1}{2}(\alpha N^b_t - P^s_1 + c - \underline{U}^s). \tag{12}$$

Scenario C (some sellers leave the platform in period 2). There is only one critical seller h in period 2, and this seller has stronger business ability compared to seller k. The seller h's business ability is on the interval $(\underline{B}^s + c, c]$. In this scenario, three conditions, $U^s_{k1,t} = \underline{U}^s$, $\widetilde{U}^s_{h2,t+1} = \underline{U}^s$, and $\widetilde{U}^s_{k2,t+1} < \widetilde{U}^s_{h2,t+1}$ are satisfied. Then, from the first condition, we have the same expression for the number of sellers on the platform in period 1 as in equation (8). Based on the second condition, we obtain the number of sellers on the platform in period 2 as in equation (9).

3 Equilibrium Analysis

To simplify this dynamic problem, but not to jeopardize the core mechanics, we assume that $\underline{U}^s = 0, \underline{U}^b = 0$, and $r = 0$. Three possible homing choices in equilibrium are described in Subsection 2.5. The discussion of these three scenarios describes the results of combining the best responses of buyers and sellers under all the possible prices to which the platform has access. The final results after a two-stage game indicate that *Scenario A* is the Nash equilibrium as described in the following lemma.

Lemma 1. *The platform's optimal pricing strategy indicates that Scenario A, characterized in Subsection 2.5, is the only possible scenario under a steady-state equilibrium.*

The proof of Lemma 1 shows that *Scenarios B and C* tend to cross the discontinuity point of the sellers' distribution, and that *Scenario A* is the only Nash equilibrium after the optimization of the monopoly platform.[16] One notable feature in *Scenario A* is that sellers sell online or offline in both periods, which implies that the monopoly platform is more powerful in charging sellers a higher fee in period 2. This also means that the platform tends to set a higher cross-side price difference between buyers and sellers in period 1 and the change in this price difference completely depends on the increment in online sellers' skills, i.e. the intertemporal difference in cross-side price differences satisfies $(P^b - P_2^s) - (P^b - P_1^s) = -c$. More specifically, the source of this intertemporal cross-side change is only the increase in online sellers' prices from period 1 to 2, as discussed above. A key reason is that the increment in online sellers' ability can be anticipated at the beginning because of complete information, but it is realized in the next and last period.

Next, we present and discuss the profit-maximizing prices set by the platform, the homing choices of buyers and sellers, and the two-period profit the platform obtains in a steady-state equilibrium.

Proposition 1 (Steady-State Equilibrium)**.** *There exists a unique steady-state equilibrium. In equilibrium,*

1. *The prices on sellers in the two periods are*

$$P_1^s = \frac{2(\alpha - \beta) - c(2 - \alpha(\alpha + \beta))}{2(4 - (\alpha + \beta)^2)} \text{ and } P_2^s = P_1^s + c. \tag{13}$$

2. *The price on buyers is*

$$P^b = \frac{4 - 2\alpha(\alpha + \beta) - c(\alpha - \beta)}{2(4 - (\alpha + \beta)^2)}. \tag{14}$$

[16]All proofs of remarks, lemmas, and propositions are provided in the Online Appendix.

3. *The measures of sellers and buyers joining the platform are*

$$N_1^s = N_2^s = \frac{\alpha + \beta + c}{2(4 - (\alpha + \beta)^2)} \tag{15}$$

$$N^b = \frac{4 + c(\alpha + \beta)}{2(4 - (\alpha + \beta)^2)}. \tag{16}$$

4. *The two-period profit of the monopoly platform for each generation is*

$$\pi = \frac{4 + c^2 + 2c(\alpha + \beta)}{4(4 - (\alpha + \beta)^2)}. \tag{17}$$

The proposition implies that, according to learning-by-doing, the enhancement in business ability of one type of agent in two-sided markets will affect the cross-side and intertemporal price structures and the platform enterprise's profit. Although these agents only have relatively low initial skills in running a business, the platform includes them in online transactions through intertemporal and/or cross-side price adjustments. The intertemporal and cross-side pricing strategy implemented by the monopoly platform also serves its goal of maximizing aggregate two-period profit, and hence the platform has an incentive to do this.

Before we analyze the equilibria, we need to provide a feasible set of parameters as shown in the following lemma.

Lemma 2. *To establish* $0 < N_1^s, N_2^s < 1/2,\ 0 < N^b < 1,$ *and* $\pi > 0$*, we have a feasible set of parameters as* $0 < \alpha, \beta < 1,\ 0 < \alpha + \beta < \sqrt{2}$*, and* $0 < c < \min\{2, \frac{4-2(\alpha+\beta)^2}{\alpha+\beta}\}$.

Intuitively, if the network externality parameters α and β and the enhancement of business ability c are too large, the outside options for buyers and sellers, such as offline selling and buying, will disappear in the economy. Throughout the following analysis in this subsection, Lemma 2 always holds.[17] First of all, we explore the first-period price that the platform charges (poor) sellers.

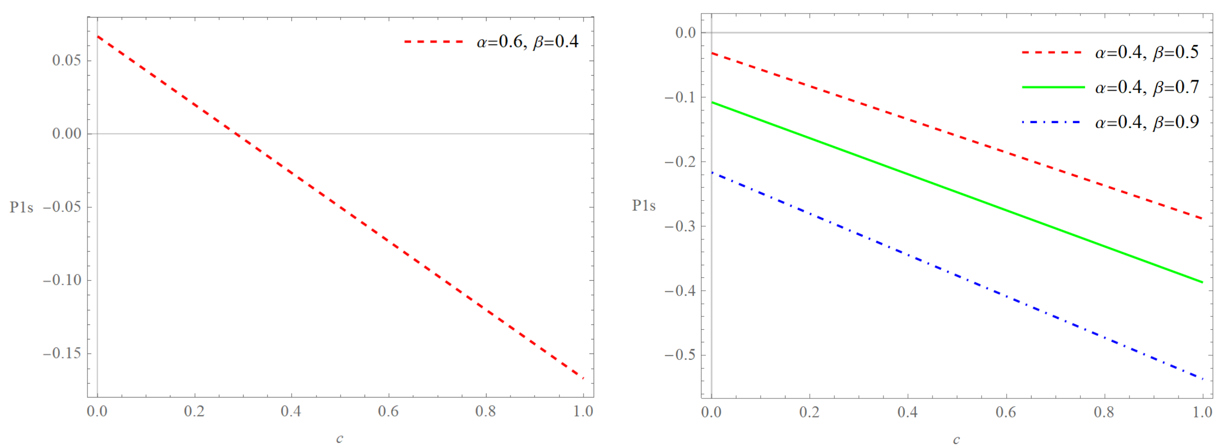
Proposition 2 (Platform Subsidizes Young Sellers or Not)**.** *In equilibrium, if either (i)* $\beta > \alpha$ *or (ii)* $\alpha > \beta$ *and* $c > \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)}$*, the price for young sellers is negative (that is,* $P_1^s < 0$*). If* $\alpha > \beta$ *and* $c < \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)}$*, the opposite result applies (that is,* $P_1^s > 0$*).*

Proposition 2 demonstrates that the platform has incentives to subsidize sellers in the initial period when their growth in business ability or network externality is sufficiently high. First, although each seller generates a lower network externality compared to each buyer, if the improvement in sellers' business ability

[17] In the two-sided markets literature, the parameters are usually limited to a certain range. For example, Armstrong (2006) provides an assumption on the relationship between the network externality parameters and the differentiation parameters, to avoid a corner solution. Choi (2010) also imposes some assumptions on parameters to obtain certain homing choices.

is sufficiently large (i.e., $c > \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)}$), the platform subsidizes sellers in period 1. The rationale is that the platform anticipates greater future utility gains from these sellers' enhanced business abilities in period 2, which provides a foundation for charging higher fees subsequently. The effect of learning by doing on pricing subsidies will be examined in detail later. Second, when $\beta > \alpha$, the price P_1^s becomes negative. The condition $\beta > \alpha$ indicates sellers possess greater market power than buyers, as buyers derive more value from each seller than vice versa. Consequently, the platform prioritizes retaining valuable sellers and thus subsidizes them to encourage increased participation in online transactions. This result highlights that the platform adjusts cross-side pricing based on the relative market power of cross-side users.

Figure 4 presents several numerical examples illustrating the conditions under which subsidies are granted to online sellers in period 1. Specifically, Figure 4(a) shows the case where $\alpha > \beta$, while Figure 4(b) depicts the case where $\beta > \alpha$.



(a) $\alpha = 0.6$, $\beta = 0.4$ (b) $\alpha = 0.4$, $\beta = 0.5, 0.7, 0.9$

Figure 4: Relationship between the Price for Young Sellers and the Increment in Online Business Ability c
Notes: Panel (a) plots the relationship between P_1^s and c when $\alpha = 0.4$ and $\beta = 0.6$ with $0 < c < 1$. Panel (b) plots the relationships between P_1^s and c in three numerical examples, (1) $\alpha = 0.4$ and $\beta = 0.5$; (2) $\alpha = 0.4$ and $\beta = 0.7$; and (3) $\alpha = 0.4$ and $\beta = 0.9$, respectively, with $0 < c < 1$.

Subsidization is a common pricing strategy not only in traditional one-sided markets but also in two-sided markets. In static multi-sided markets, cross-subsidization is attributed to cross-side externalities (Tan and Zhou, 2021). Subsidies (that is, negative prices) are also discussed in dynamic models (Cabral, 2011, 2019; Halaburda et al., 2020). Moreover, Cabral (2019) points out that under a dynamic framework, cross-side and intertemporal externalities are addressed when a platform determines the optimal pricing strategy.

Although Cabral (2019) points out that subsidizing participants at an early stage may be optimal in dynamic pricing theory with learning-by-doing or network effects, the coordinated effect between the learning-by-doing benefit and the cross-sided network externalities has not attracted much attention. In the model of Cabral (2019), the cross-sided network externalities are not explicitly characterized, and agents are assumed to be myopic. However, in our model, all agents have rational expectations, and the cross-sided network externalities and the learning-by-doing benefit are simultaneously explicitly featured within a unified framework. Our model shows a closed-form solution of prices for buyers and sellers in the two periods and thus provides the concrete conditions under which the platform will subsidize early sellers.[18]

Proposition 3 (Cross-Side Price Discrimination)**.** *In equilibrium, (i) in period 1, buyers are charged a higher fee than sellers, that is, $P^b > P_1^s$; (ii) if c is sufficiently low, the price for buyers is higher than that for old sellers, that is, $P^b > P_2^s$ (otherwise, the opposite result applies).*

Proposition 3 highlights the cross-side price structures in the two periods. In period 1, the platform always charges a higher fee to buyers than to sellers. This occurs for the same reason as explained earlier—buyers have an absolute advantage in obtaining utility from their own ability compared to sellers. In period 2, the cross-side price discrimination may reverse. Here, two main features are noteworthy. One is that period 2 is the last period, and the game ends on this date. It also implies that there is no further change in behavior, which the platform needs to consider. Another feature is that sellers who sell online in period 1 have achieved enhanced business ability, which gives them an increase in utility. Then, as described in the proposition, when the business ability increment c of sellers is low enough or the buyers' network externality parameter β is high enough, the platform charges buyers a higher fee than sellers.

4 Implications for Economic Development

In this paper, we emphasize that the dynamic effect of poor merchants' learning-by-doing is a critical force that shapes monopolistic platforms' decision to provide early-stage subsidies, beyond the inherent characteristics of two-sided markets. We demonstrate that this dynamic effect interacts with cross-side network externalities, resulting in a reinforcing incentive for platform's early-stage subsidies. These findings offer new insights into the economic characteristics of low-skilled participants in LDAs. Their relevance is particularly pronounced in an era dominated by platform economies. Our model characterizes poor sellers along two dimensions. First, poor sellers on the platform gain learning-by-doing benefits, which may exceed those of richer sellers due to the law of diminishing marginal returns. Second, the initial distribution of business

[18] Halaburda et al. (2020) present the dynamic competition between the two platforms with one-sided network effects and homogeneous consumers, while they focus on the quality and focal status of the two competing platforms.

ability—captured by fixed utility—is lower for poor sellers than for their richer counterparts. Next, we analyze how variations in learning-by-doing benefits and shifts in the distribution of seller business ability toward that of richer sellers influence the platform's pricing strategy. This analysis sheds light on the mechanisms through which platform's pricing strategies can shape market structure and economic development.

4.1 The Effects of Learning-by-Doing

Compared to rich sellers, poor sellers may obtain more learning-by-doing benefits, meaning that c in the model could be larger. We next examine how changes in c and its interaction with merchants' cross-side externalities affect the platform's early-stage subsidies.

Proposition 4. *In equilibrium, P_1^s is decreasing in c and β. Furthermore, if $\beta > \frac{2-\alpha^2-2\sqrt{1-\alpha^2}}{\alpha}$, we have $\frac{\partial^2 P_1^s}{\partial c \partial \beta} < 0$ (otherwise, the opposite result applies).*

Proposition 4 shows that a higher increment in business ability, c, leads to a lower price for sellers in period 1. This result follows directly from the platform's intertemporal pricing strategy. The greater the learning-by-doing benefit from early participation, the stronger the platform's ability to charge higher fees later. Consequently, the platform is more likely to set lower initial fees or even provide subsidies.[19] If sellers do not participate, the platform earns nothing in either period. Intuitively, for rich sellers with low intertemporal learning-by-doing benefits, the platform has weaker incentives to charge lower initial fees or provide subsidies in the early stage. The proposition further shows that when the seller's cross-side externality parameter (β) is sufficiently large and the buyer's cross-side externality parameter (α) is not too high (since $(2 - \alpha^2 - 2\sqrt{1-\alpha^2})/\alpha$ increases with α), β and c (learning-by-doing) are complementary in reducing the platform's charges or increasing its subsidies to young sellers. Under these conditions, the platform has stronger incentives to leverage learning-by-doing benefits to attract sellers, thereby offering greater price concessions to young sellers. Figure 4 above also illustrates the findings of Proposition 4 in an intuitive manner.

Next, we examine the dynamics of cross-side price structure and the impact of learning by doing on these dynamics.

Proposition 5 (Cross-Side Price Structure)**.** *In equilibrium, (i) the cross-side price difference $P^b - P_1^s$ increases with c and β and decreases with α; (ii) the cross-side price difference $P^b - P_2^s$ increases with*

[19]This result highlights the mechanism underlying the platform's subsidy decision. A distinctive feature of our model is that the price charged to sellers by the monopolistic platform depends on their own externality parameter α, which differs from the result in Armstrong (2006).

β and decreases with c and α; (iii) β and c are complements in determining both price differences; (iv) α and c are substitutes in determining both price differences.

Proposition 5 shows that the cross-side price structure depends on the improvement in business ability c and the two externality parameters, α and β. Four findings are particularly noteworthy. First, because the platform anticipates a greater improvement in sellers' business ability, it tends to raise the relative price between buyers and young sellers. As shown in Proposition 4, P_1^s decreases as c increases. However, the response of P^b to an increase in c is ambiguous, as it depends on the relative magnitudes of α and β.[20] Even if P^b decreases as c rises, the relative price $P^b - P_1^s$ still increases with c, as shown in Online Appendix B.5, because an increase in c alters the relative market power between buyers and sellers.

Second, the price gap between buyers and old sellers decreases with the business ability increment c. Based on the equilibrium results in Proposition 1, we have $\frac{\partial P_2^s}{\partial c} > 0$. Intuitively, the greater the utility sellers derive from improved ability, the higher the fee the platform charges them. Even when P^b increases with c under $\beta > \alpha$, the price difference $P^b - P_2^s$ declines as c rises because P_2^s grows faster. This implies that as less experienced sellers become more capable through learning by doing, the platform tends to impose a disproportionately higher fee on them relative to buyers.

Third, the cross-side price difference captures the (latent) relative bargaining power between buyers and sellers in negotiations with the monopoly platform. Recall that α and β represent the network externality values of each buyer and seller, respectively, in two-sided markets. It is evident that both $P^b - P_1^s$ and $P^b - P_2^s$ rise with β and fall with α.[21]

Fourth, both externality parameters, α and β, interact with the learning-by-doing benefit c in determining the platform's price structure. Specifically, the seller's externality β amplifies the impact of learning by doing on cross-side price discrimination between buyers and sellers, whereas the buyer's externality α mitigates it. This interaction illustrates how learning by doing shapes platform pricing strategies in two-sided markets and, more broadly, economic development.

4.2 Do High-Ability Sellers Receive Platform Subsidies?

As noted above, in our model poor sellers are characterized by two features: strong learning-by-doing and a distribution of business ability confined to the lower range. This subsection models the case of rich sellers,

[20] From equality (14), the derivative of P^b with respect to c is $\frac{\beta-\alpha}{8-2(\alpha+\beta)^2}$. Hence, if $\beta > \alpha$, P^b increases with c; if $\beta < \alpha$, the opposite holds.

[21] These findings are consistent with the literature. For example, Hagiu (2009) shows that the platform extracts more surplus from the side with greater bargaining power.

which differ from poor sellers in these two aspects. We then compare this scenario with the baseline model of poor sellers and analyze its implications for economic development. Assume that $B^s_{i1,t}$ follows a uniform distribution on the interval $[-\xi, 0]$, where $0 < \xi < 1$. The learning-by-doing benefit is given by $d > 0$, satisfying $d < c$. All other assumptions remain the same as in the baseline model. Accordingly, we can derive the equilibrium results as follows.

Remark 1. Given $\max\{[(\alpha+\beta)(\alpha+\beta+1)+d]/4, (\alpha+\beta)(\alpha+\beta+d/2)/2\} < \xi < 1$ to ensure $0 < N^s_1, N^s_2 < 1/2$, $0 < N^b < 1$, and $\pi > 0$, there exists a unique equilibrium:

1. The sellers' prices in the two periods are
$$P^s_1 = \frac{2\xi(\alpha-\beta) - d\,(2\xi - \alpha(\alpha+\beta))}{2\,(4\xi - (\alpha+\beta)^2)} \text{ and } P^s_2 = P^s_1 + c.$$
2. The buyers price is
$$P^b = \frac{2\,(2\xi - \alpha(\alpha+\beta)) + d(\beta-\alpha)}{2\,(4\xi - (\alpha+\beta)^2)}.$$
3. The numbers of sellers and buyers joining the platform are
$$N^s_1 = N^s_2 = \frac{\alpha+\beta+d}{2\,(4\xi - (\alpha+\beta)^2)} \text{ and } N^b = \frac{d(\alpha+\beta)+4\xi}{2\,(4\xi - (\alpha+\beta)^2)}.$$
4. The two-period profit of the monopoly platform for each generation is
$$\pi = \frac{d^2 + 2d(\alpha+\beta) + 4\xi}{4\,(4\xi - (\alpha+\beta)^2)}.$$

The equilibrium results in Remark 1 lead to the following proposition.

Proposition 6. *If either (i) $\beta > \alpha$ or (ii) $\alpha > \beta$ and $d > \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)/\xi}$, the price for young sellers is negative (that is, $P^s_1 < 0$). Otherwise, the opposite result applies (that is, $P^s_1 > 0$).*

Proposition 6 corresponds to Proposition 2 for poor sellers. The difference lies in an additional condition for the platform's implicit subsidy to young sellers when $\alpha > \beta$. In comparison, as shown below:
$$d > \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)/\xi} \text{ for rich sellers versus } c > \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)} \text{ for poor sellers.}$$
Recall the two parameter restrictions: $0 < \xi < 1$ and $0 < d < c$. The condition for rich sellers to receive subsidies is clearly more stringent. The reason is that, relative to poor sellers, rich sellers have higher initial business ability (which justifies higher fees or smaller subsidies) and weaker learning-by-doing effects (due to the law of diminishing marginal returns). This also explains why the platform is more inclined to offer subsidies to low-skill young sellers in underdeveloped areas.

4.3 Further Discussion

As mentioned in Section 2, our model focuses on poor sellers in rural areas to analyze the emergence of digital villages. Compared to the literature on the learning-by-doing hypothesis that emphasizes productivity improvement or cost reduction due to increased production volume within a firm, our model focuses on the improvement of business ability of more microscopic individuals (sellers) on the digital platform. In this sense, these individuals have the characteristics of entrepreneurs. Thus, the learning-by-doing benefit of old sellers over time is very similar to human capital accumulation, permanently improving business ability. Although there are only two periods in our model, we can expect continued benefits for sellers from learning-by-doing in the future. Including high-skill sellers from metropolitan areas in the model will complicate the analysis, while the core mechanics have not been changed (and thus the main results of this paper may still hold). Indeed, the logic in our model can be widely applied to any dynamic case with cross-side network externalities in imperfectly competitive markets.

Educational investment is widely recognized as a key determinant of economic development. Acquiring knowledge and skills through various channels such as preschool programs, schools, and formal training programs has been shown to improve productivity (Behrman, 2010). In rural areas, education investments are predominantly funded by the government, while enterprise-based skills training rarely reaches impoverished populations. In industries that lack network effects, providing skills training to the poor becomes profitable only when learning-by-doing is sufficiently strong. Hypothetically, if the financial market functioned perfectly, the poor would have incentives to invest in their own education and skills. However, in industries characterized by one-sided network effects, such as the telephone and electronic payment industries, the exchange of physical goods is minimal, limiting the positive network externalities that the rural poor can generate. Digital technology, which facilitates the establishment of two-sided markets, has disrupted this landscape. When cross-side network effects become sufficiently strong, digital platform enterprises are incentivized to provide skills training to the rural poor, particularly during the early stages of economic development. As a result, high-quality agricultural products from rural areas gain access to online markets, significantly broadening consumer choices and enhancing social welfare.

5 Model Extensions

This section extends the baseline model in three ways. First, it allows for a more general distribution of buyers' and sellers' business ability. Second, it introduces discriminatory pricing for old sellers based on their earlier participation. Third, it models learning-by-doing as an enhancement of sellers' ability to match with buyers in generating cross-side utility, replacing the assumption of fixed utility. These extensions demonstrate the robustness of the main results derived from the baseline model.

5.1 Generalized Model

We generalize many of the assumptions in our baseline model and show that our key results remain robust. Specifically, we now allow $\underline{U}^s, \underline{U}^b > 0$ and $0 < \delta < 1$.[22] In addition, we assume $-B^s_{i1,t}$ and $B^b_{j,t}$ are independently and identically distributed according to distribution function F and density f over the interval $[0, 1]$. We assume that F is log-concave, that is, $-f/(1-F)$ is decreasing. All other aspects of the baseline model remain the same.

The logic in solving for the equilibrium outcome follows similarly to before. First, the number of buyers joining the platform in each period is

$$\begin{aligned} N^b_t(P^b, N^s_t) &= \Pr(B^b_{j,t} - P^b + \beta N^s_t \geq \underline{U}^b) = 1 - F(P^b - \beta N^s_t + \underline{U}^b) \\ &= 1 - F(-u^b_t) \end{aligned} \tag{18}$$

where $u^b_t \equiv \beta N^s_t - P^b - \underline{U}^b$ represents the net utility gained from joining the platform excluding the seller's business ability. Next, we characterize the seller homing choices under each of the three scenarios. For each scenario, the conditions that must be satisfied are the exact same.

Scenario A (the sellers on the platform are the same in both periods). From our previous conditions, we have that the number of sellers on the platform in the two periods are

$$\begin{aligned} N^s_{1,t}(P^s_1, N^b_t) &= \frac{1}{2}\Pr(B^s_{i1,t} + \alpha N^b_t - P^s_1 \geq \underline{U}^s) = \frac{1}{2}F(\alpha N^b_t - P^s_1 - \underline{U}^s) \\ &= \frac{1}{2}F(u^s_{1,t}) \end{aligned} \tag{19}$$

and

$$\begin{aligned} N^s_{2,t+1}(P^s_2, N^b_{t+1}) &= \frac{1}{2}\Pr(B^s_{i1,t} + \alpha N^b_{t+1} + c - P^s_2 \geq \underline{U}^s) = \frac{1}{2}F(\alpha N^b_{t+1} + c - P^s_2 - \underline{U}^s) \\ &= \frac{1}{2}F(u^s_{2,t+1}) \end{aligned} \tag{20}$$

where we define $u^s_{1,t} \equiv \alpha N^b_t - P^s_1 - \underline{U}^s$ and $u^s_{2,t+1} \equiv \alpha N^b_{t+1} - P^s_2 - c - \underline{U}^s$. It is straightforward to derive the same relationship between the seller prices as $P^s_2 = c + P^s_1$ given a steady-state equilibrium. Note that because our critical sellers k and h are the same, we have $u^s_{1,t} = u^s_{2,t}$.

Scenario B (some sellers enter the platform in period 2). From our conditions, the number of sellers on

[22] Note that we still assume $r = 0$, but our results would remain robust given r is sufficiently small. If r is too large, then the platform's optimal strategy would be to focus on extracting profits when agents are young.

the platform in period 2 as

$$\begin{aligned} N^s_{2,t+1}(P^s_2, N^b_{t+1}) &= \frac{1}{2}\Pr\left(B^s_{i1,t} + \alpha N^b_{t+1} - P^s_2 \geq \underline{U}^s\right) = \frac{1}{2}F(\alpha N^b_{t+1} - P^s_2 - \underline{U}^s) \\ &= \frac{1}{2}F(u^s_{2,t+1}) \end{aligned} \tag{21}$$

and rom the second condition, we can derive the number of sellers on the platform in period 1 as

$$\begin{aligned} N^s_{1,t}(P^s_1, N^b_t) &= \frac{1}{2}\Pr\left(B^s_{i1,t} + \alpha N^b_t - P^s_1 + \delta c \geq \underline{U}^s\right) = \frac{1}{2}F(\alpha N^b_t - P^s_1 + \delta c - \underline{U}^s) \\ &= \frac{1}{2}F(u^s_{1,t}) \end{aligned} \tag{22}$$

where $u^s_{1,t} \equiv \alpha N^b_t - P^s_1 + \delta c - \underline{U}^s$ and $u^s_{2,t+1} \equiv \alpha N^b_{t+1} - P^s_2 - \underline{U}^s$.

Scenario C (some sellers leave the platform in period 2). Similarly to the baseline, we have the same expression for the number of sellers on the platform in period 1 as in equation (19) and the number of sellers on the platform in period 2 as in equation (20).

Lemma 3. *In the generalized model, the platform's optimal pricing strategy still indicates that Scenario A, where the sellers on the platform are the same for each generation, is the only possible scenario under a steady-state equilibrium.*

Our finding that sellers either join or leave the platform in both stages of life remains the same even when generalizing the distribution of agents' business abilities. The intuition remains largely the same. If sellers are expected to join or leave in latter periods, then the platform maximizes profit by adjusting its fee for old sellers. However, as the platform must commit to its pricing structure, the sellers will anticipate the difference in pricing and optimize accordingly when young. Next, we check whether the platform may still be incentivized to subsidize sellers when their business ability is initially weak.

Proposition 7 (Generalized Intertemporal Price Structure)**.** *In a steady-state equilibrium, let*

$$\varepsilon^b = -\frac{P^b f(-u^b)}{1 - F(-u^b)} \quad \textit{and} \quad \varepsilon^s = -\frac{P^s_1 f(u^s_1)}{F(u^s_1)}$$

be the buyers' and young sellers' price elasticity of demand for a given level of the other group's participation on the platform. Then, in a steady-state equilibrium:

1. *The price on buyers is:*

$$P^b = -\frac{\alpha N^s}{1 + 1/\varepsilon^b}. \tag{23}$$

2. *The prices on sellers in the two periods satisfy $P^s_2 = P^s_1 + c$ where*

$$P^s_1 = -\frac{\beta N^b + c/2}{1 + 1/\varepsilon^s}, \tag{24}$$

implying that

$$P^s_2 = \frac{-\beta N^b + c(1/2 + 1/\varepsilon^s)}{1 + 1/\varepsilon^s}. \tag{25}$$

Proposition 7 reiterates several insights on the overall pricing structure set by the platform. First, when ε^b is elastic, the platform will opt to subsidize buyers, i.e. $P^b < 0$ if $|\varepsilon^b| > 1$. Similarly, when ε^s is elastic, the platform will subsidize sellers when they are young, i.e. $P_1^s < 0$ if $|\varepsilon^s| > 1$. Intuitively, if buyers or young sellers are more price elastic, then the platform chooses to subsidize them to attract participation from the other group through indirect network effects. However, such a condition is no longer sufficient for P_2^s. Even when $|\varepsilon^s| > 1$, P_2^s may still be positive if the learning-by-doing effect is strong enough. Thus, there may potentially exist an outcome where sellers are initially subsidized and then the platform charges non-negative prices when they become old.

5.2 Third-Degree Price Discrimination

We now consider the case where the platform can enforce third-degree price discrimination for old sellers based on their previous homing choice. Specifically, the platform can now charge a fee P_{22}^s if the seller was affiliated with the platform when they were young and P_{21}^s otherwise. This means that our monopoly platform's two-period profit maximization problem for each generation t is now

$$\pi = \max_{\widetilde{\boldsymbol{P}}} \left[P_1^s N_{1,t}^s + P^b N_t^b + \frac{1}{1+r} (P_{21}^s N_{21,t+1}^s + P_{22}^s N_{22,t+1}^s) \right], \tag{26}$$

where $\widetilde{\boldsymbol{P}} = \left\{ P_1^s, P_1^b, P_{21}^s, P_{22}^s \right\}$ represents the new list of platform prices; $N_{21,t+1}^s$ denotes the number of sellers who did not join the platform when young but now joins when they are old; and $N_{22,t+1}^s$ denotes the number of sellers who joins the platform in both periods. We characterize the conditions required and the steady-state number of buyers and sellers affiliated on the platform for each scenario in Online Appendix B.12.

As before, when solving the equilibrium, we must consider the various scenarios for the critical seller(s) in each period. With third-degree price discrimination, we have a fourth possible scenario where some sellers leave and some join when they get old. However, the following lemma shows that *Scenario A*, where the sellers on the platform are the same in each period, is the only possible outcome in equilibrium.

Lemma 4. *When third-degree price discrimination is allowed, the platform's optimal pricing strategy indicates that Scenario A, in which sellers on the platform are the same regardless of age, is the only possible outcome in a steady-state equilibrium.*

Lemma 4 implies that in a steady-state equilibrium, the platform's maximization problem is simplified to

$$\max_{P_1^s, P_{22}^s, P^b} \left[P_1^s \cdot N_{1,t}^s(P_1^s, P_{22}^s, P^b) + P^b \cdot N_t^b(P_1^s, P_{22}^s, P^b) + \frac{1}{1+r} \cdot P_{22}^s \cdot N_{2,t+1}^s(P_1^s, P_{22}^s, P^b) \right],$$

as $N^s_{21,t+1} = 0$ and $N^s_{22,t+1} = N^s_{2,t+1}$ under *Scenario A*. This is the same exact maximization problem as the one without third-degree price discrimination. Therefore, the optimal price for young sellers will be the same and the optimal P^s_{22} will be the same as the old price for sellers without price discrimination. Therefore, sellers who end up joining the platform without price discrimination effectively face the same prices and will still be the ones to join under price discrimination. Naturally, this also implies our analysis in Section 3 can be extended here.

Our robustness result here is not immediately intuitive. With the addition of third-degree price discrimination, the platform can, in theory, extract additional surplus from sellers even if they were not initially affiliated. However, because these sellers never received a learning-by-doing benefit and face a negative business ability, such a possibility cannot occur, at least in a steady-state. Therefore, the platform's problem is merely to focus on capturing sellers who join when they are young and extract additional surplus from buyers and older sellers.

5.3 Enhanced Network Effects

As in Weyl (2010), platform users are heterogeneous along two dimensions: the member benefit or cost (denoted by c in our previous setup) and the interaction benefit or cost, which determines cross-side network externalities. In this section, we consider an alternative learning-by-doing scenario: if a seller joins the platform when young, then in the old period he experiences an enhanced cross-side network effect. In practice, early entrants often develop stronger capabilities to leverage relevant technologies and strategies, thereby improving their ability to match with buyers over time on the platform. Relative to the baseline model, the only modification is that if seller i joins the platform when young, his utility in the old period becomes

$$\widetilde{U}^s_{i,2t+1} = B^s_{i1,t} + \lambda\alpha N^b_{t+1} - P^s_2 \tag{27}$$

where $\lambda > 1$ measures the increase in the cross-side network externality.

Remark 2. Given $0 < \alpha < 2(\sqrt{2} - \beta)/(\lambda + 1)$, there exists a unique equilibrium:

1. The sellers' prices in the two periods are
$$P^s_1 = \frac{2\alpha(3-\lambda) - 4\beta}{4\,(4-\beta^2) - \alpha^2(\lambda+1)^2 - 4\alpha\beta(\lambda+1)} \text{ and } P^s_2 = \frac{2(3\alpha\lambda - \alpha - 2\beta)}{4\,(4-\beta^2) - \alpha^2(\lambda+1)^2 - 4\alpha\beta(\lambda+1)}.$$
2. The buyers' price is
$$P^b = \frac{8 - \alpha^2(\lambda+1)^2 - 2\alpha\beta(\lambda+1)}{4\,(4-\beta^2) - \alpha^2(\lambda+1)^2 - 4\alpha\beta(\lambda+1)}.$$

3. The numbers of sellers and buyers joining the platform are

$$N_1^s = N_2^s = \frac{\alpha\lambda + \alpha + 2\beta}{4\,(4-\beta^2) - \alpha^2(\lambda+1)^2 - 4\alpha\beta(\lambda+1)}$$

$$N^b = \frac{8}{4\,(4-\beta^2) - \alpha^2(\lambda+1)^2 - 4\alpha\beta(\lambda+1)}.$$

4. The two-period profit of the monopoly platform for each generation is

$$\pi = \frac{4}{4\,(4-\beta^2) - \alpha^2(\lambda+1)^2 - 4\alpha\beta(\lambda+1)}.$$

The equilibrium results in Remark 2 yield the following proposition.

Proposition 8. *If $0 < \alpha < \sqrt{2}/2$ and $3 - 2\beta/\alpha < \lambda < 2(\sqrt{2} - \beta)/\alpha - 1$, the price for young sellers is negative ($P_1^s < 0$). Otherwise, the opposite result holds ($P_1^s > 0$). Moreover, the price for old sellers always exceeds that for young sellers ($P_2^s > P_1^s$).*

The proposition states that a monopolistic platform subsidizes young sellers only when the increase in learning-by-doing, λ, falls within an intermediate range. When $\lambda < 3 - 2\beta/\alpha$, the growth from learning-by-doing is too weak to ensure that the platform can charge higher fees once sellers mature, thus the platform has no incentive to subsidize young sellers. Conversely, when $\lambda \geq 2(\sqrt{2} - \beta)/\alpha - 1$, the entire mass of buyers (normalized to one) is already homed to the platform, eliminating any incentive to subsidize young sellers because the platform cannot sufficiently recoup costs through higher buyer prices.

This section broadens the model along three dimensions: distributional generalization, platform pricing flexibility, and the specification of learning-by-doing. Across these extensions, the platform retains the incentive to subsidize young sellers, with learning-by-doing as a central driving force. This result aligns with the baseline model and its underlying mechanism, reinforcing the robustness of the theoretical framework.

6 Conclusion

By incorporating the learning-by-doing of sellers into the theory of two-sided markets with heterogeneous sellers and buyers and a monopoly platform, we provide a novel dynamic model to analyze a new platform-based phenomenon of economic development—emerging digital villages in China. The most important contribution of our paper is that we fill the void in understanding the interaction between two fields, industrial organization and economic development. Our paper also provides a tractable framework to explore the dynamic pricing strategy of a monopoly platform in a situation where price discrimination occurs within one group (on the sellers' side), along with the enhancement of the business ability of online sellers.

The key result is that due to the anticipated growth of the business ability of online sellers and cross-side network externalities, the rational and profit-maximizing platform tends to charge sellers a lower fee in period 1 by manipulating cross-side and intertemporal price structure. In particular, if (i) the network externality that each online seller produces is higher than that which each online buyer produces or (ii) the growth in the business ability of online sellers is high enough, the platform will subsidize sellers in period 1. When the externality of each seller is high enough and the externality of each buyer is not too high, the cross-side externality and learning-by-doing of each online seller can mutually reinforce each other in reducing the platform's charges (or increasing its subsidies) to sellers in period 1. With the regulation on price discrimination, homing choices and social welfare are unaffected. Therefore, we conclude that the usage of third-degree price discrimination for sellers stems from the platform's inability to commit to uniform pricing.

Our dynamic model of two-sided markets can be used to analyze other aspects of economic development. For example, two-sided platforms tend to invest substantial funds or resources during the early stages of development to cultivate users' stickiness, habits, and cross-side networks. Moreover, our theory can be extended to address at least the following two fundamental theoretical problems: dynamic development on the buyers' side and on both sides of the market. Other important issues, such as endogenous investment decisions (made by platforms, buyers, or sellers) and income (or wealth) inequality caused by heterogeneous abilities in the digital economy, are left for future research.

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Online Appendix

A Data Sources of Online Retail Sales

1. Data on the number of "Taobao villages" in China and the number of "Taobao villages" located in national poverty-stricken counties of China are from the China Taobao Village Research Report (2020) (Chinese title: 中国淘宝村研究报告 2020) and AliResearch: http://www.aliresearch.com/ch/information/informationdetails?articleCode=256317657652006912.
2. Data on online retail sales in rural areas of China come from the E-commerce in China 2019 (Chinese title: 中国电子商务报告 2019) and the E-commerce in China 2021 (Chinese title: 中国电子商务报告 2021).
3. Data sources of online retail sales on Alibaba's platforms in poverty-stricken counties of China:
   (a) The datum for 2014 is from 2014 county and rural e-commerce data (Chinese title: 2014 年县域暨农村电商数据).
   (b) The data for 2015 is from the 2015 China County E-Commerce Report (Chinese title: 2015 年中国县域电子商务报告).
   (c) The datum for 2016 is from E-commerce Development: Experience from China.
   (d) The datum for 2017 is from New Country, New Consumption, New Business: Rural Business Research Report (Chinese title: 新乡村新消费新商业——农村商业研究报告).
   (e) The datum for 2018 is from Alibaba Poverty Alleviation Work Report 2018 (Chinese title: 阿里巴巴脱贫工作报告 2018).
   (f) The datum for 2019 is from Alibaba's 2020 Fiscal Year Public Welfare "Financial Report" (Chinese title: 阿里巴巴 2020 财年公益“财报”).

B Ommitted Proofs

B.1 Proof of Lemma 1

Proof. All possible scenarios are described above (i.e., *Scenarios A, B*, and *C* in subsection 2.5). Let N^s and N^b denote the number of sellers and buyers on the platform in a steady-state, respectively. First, we check whether *Scenario B* is the optimal choice of both buyers and sellers after the monopoly platform sets the optimal prices. Combining (7), (11), and (12), we have that in a steady-state equilibrium, the total number of buyers and sellers on the platform in each period are

$$N^b = \frac{2 - 2P^b + \beta c - \beta(P_1^s + P_2^s)}{2(1-\alpha\beta)} \text{ and } N^s = \frac{2\alpha(1-P^b) + c - (P^s + P_2^s)}{2(1-\alpha\beta)}. \tag{A1}$$

By making decisions on P^b, P_1^s, and P_2^s, the platform's problem is to maximize profit (1) subject to equalities (7), (11), and (12). The maximization problem is reduced to

$$\max_{\boldsymbol{P}} \frac{P_1^s}{2}(\alpha N^b - P_1^s + c) + \frac{P_2^s}{2}(\alpha N^b - P_2^s) + P^b(1 - P^b + \beta N^s)$$

where N^b and N^s are characterized by (A1). Equating the F.O.C.s with respect to P_1^s and P_2^s gives us $P_1^s = P_2^s + 0.5c$. But, this implies $N_{1,t}^s > N_{2,t+1}^s$ which contradicts this scenario that some sellers enter the platform when old.

Now, we consider *Scenario C*. Combining (7), (8), and (9), we have that in a steady-state equilibrium, the total number of buyers and sellers on the platform in each period are still that in (A1). By choosing P^b, P_1^s and P_2^s, the monopoly platform maximizes profit (1) subject to equalities (7), (8), and (9). The profit maximization problem can be reduced to

$$\max_{\boldsymbol{P}} \frac{P_1^s}{2}(\alpha N^b - P_1^s) + \frac{P_2^s}{2}(\alpha N^b - P_2^s + c) + P^b(1 - P^b + \beta N^s).$$

Equating the F.O.C.s with respect to P_1^s and P_2^s gives us $P_1^s = P_2^s - 0.5c$. But, this implies that $N_{2,t+1}^s > N_{1,t}^s$ which contradicts this scenario that some sellers leave the platform when they get old.

In summary, *Scenario B* and *Scenario C* cannot form a final equilibrium. Thus, only *Scenario A* is a final equilibrium. □

B.2 Proof of Proposition 1

Proof. By choosing P^b, P_1^s and P_2^s, the monopoly platform maximizes profit (1) subject to equalities (7), (8), (9), and (10). The profit maximization problem can be reduced to

$$\max_{P_1^s, P^b} \frac{2P_1^s + c}{2} N^s + P^b N^b$$

where N^s and N^b are characterized by (A1) (see Proof of Lemma 1 in Appendix B.1). Solving the F.O.C.s easily gives us items 1 and 2. Item 3 is derived by plugging (13) and (14) into (A1). Item 4 is derived by substituting (13), (14), (15) and (16) into (1) while recalling that $P_2^s = P_1^s + c$. □

B.3 Proof of Lemma 2

Proof. Consider the final equilibria (15), (16), and (17) with $\alpha, \beta, c > 0$. We have that $\pi > 0$ if $0 < \alpha + \beta < \sqrt{2}$ and $0 < \alpha, \beta < 1$. (Note the latter is sufficient but not necessary.) Now, we need $N_t^b < 1$ and $N_{1,t}^s = N_{2,t}^s < 1/2 \iff N_t^s < 1$. Note that N_t^s and N_t^b are increasing and equal to each other when

$c = 2$. Moreover, it is easy to verify that for $c \le 2$, $N_t^b \ge N_t^s$ and for $c \ge 2$, $N_t^b \le N_t^s$. When $\alpha + \beta < 1$, $N_t^s < N_t^b < 1$ for $c \in (0, 2)$. When $1 \le \alpha + \beta < \sqrt{2}$, both measures are greater than 1 for $c > 2$, so our bound on c is such that $N_t^b < 1$. This occurs for $c < \frac{4-2(\alpha+\beta)^2}{\alpha+\beta}$. All these conditions can be expressed as $0 < \alpha, \beta < 1, 0 < \alpha + \beta < \sqrt{2}$, and $0 < c < \min\{2, \frac{4-2(\alpha+\beta)^2}{\alpha+\beta}\}$. □

B.4 Proof of Proposition 2

Proof. Recall P_1^s in equality (13) as

$$P_1^s = \frac{2(\alpha - \beta) - c(2 - \alpha(\alpha + \beta))}{2(4 - (\alpha + \beta)^2)}.$$

Under our parameter restrictions in Lemma 2, the denominator and the term $2 - \alpha(\alpha + \beta)$ in the numerator are positive, as $\alpha, \beta \in (0, 1)$. Thus, $P_1^s < 0 \iff c > \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)}$ which gives the desired result. □

B.5 Proof of Proposition 3

Proof. From equalities (13) and (14), we have

$$P^b - P_1^s = \frac{(2 + c)(1 - \alpha)}{2(2 - (\alpha + \beta))} > 0$$

given that $\alpha, \beta \in (0, 1)$ and $c > 0$. From equalities (10), (13) and (14), $P^b - P_2^s$ can be expressed as

$$P^b - P_2^s = \frac{(2 + c)(1 - \alpha)}{2(2 - (\alpha + \beta))} - c = \frac{2(1 - \alpha) - c(3 - \alpha - 2\beta)}{2(2 - \alpha - \beta)}.$$

Thus, we can easily obtain $P^b > P_2^s \iff c < \frac{2(1-\alpha)}{3-\alpha-2\beta}$. □

B.6 Proof of Proposition 4

Proof. The derivatives of P_1^s with respect to c and β satisfy

$$\frac{\partial P_1^s}{\partial c} = \frac{-(2 - \alpha(\alpha + \beta))}{2(4 - (\alpha + \beta)^2)} < 0,$$

$$\frac{\partial P_1^s}{\partial \beta} = -\frac{1}{4}\left[\frac{(\alpha + 1)(2 - c)}{(2 + \alpha + \beta)^2} + \frac{(1 - \alpha)(c + 2)}{(2 - \alpha - \beta)^2}\right] < 0.$$

Meanwhile, we have the cross derivative $\frac{\partial^2 P_1^s}{\partial c \partial \beta}$ as

$$\frac{\partial^2 P_1^s}{\partial c \partial \beta} = \frac{\alpha^3 + \alpha\beta^2 - (4 - 2\alpha^2)\beta}{2(4 - (\alpha + \beta)^2)^2} < 0 \iff \alpha(\alpha + \beta)^2 < 4\beta.$$

This condition is satisfied if $\beta > \frac{2-\alpha^2-2\sqrt{1-\alpha^2}}{\alpha}$ (note that $\alpha < 1$ in Lemma 2 has been taken into account).

□

B.7 Proof of Proposition 5

Proof. Note that all parameters are subject to the constraints specified in Lemma 2. By taking derivatives of $P^b - P_1^s$ with respect to c, β, and α, we have the following results:

$$\frac{\partial(P^b - P_1^s)}{\partial c} = \frac{1-\alpha}{2(2-(\alpha+\beta))} > 0,$$
$$\frac{\partial(P^b - P_1^s)}{\partial \beta} = \frac{(c+2)(1-\alpha)}{2(2-(\alpha+\beta))^2} > 0,$$
$$\frac{\partial(P^b - P_1^s)}{\partial \alpha} = -\frac{(c+2)(1-\beta)}{2(2-(\alpha+\beta))^2} < 0.$$

Taking derivatives of $P^b - P_2^s$ with respect to c, α, and β, we have the following results:

$$\frac{\partial(P^b - P_2^s)}{\partial c} = \frac{\alpha+2\beta-3}{2(2-(\alpha+\beta))} < 0,$$
$$\frac{\partial(P^b - P_2^s)}{\partial \alpha} = \frac{\partial(P^b - P_1^s)}{\partial \alpha} < 0,$$
$$\frac{\partial(P^b - P_2^s)}{\partial \beta} = \frac{\partial(P^b - P_1^s)}{\partial \beta} > 0,$$

where the latter two inequalities are shown above. Meanwhile, the cross partial derivatives are given by:

$$\frac{\partial^2(P^b - P_1^s)}{\partial \beta \partial c} = \frac{\partial^2(P^b - P_2^s)}{\partial \beta \partial c} = \frac{1-\alpha}{2(2-(\alpha+\beta))^2} > 0,$$
$$\frac{\partial^2(P^b - P_1^s)}{\partial \alpha \partial c} = \frac{\partial^2(P^b - P_2^s)}{\partial \alpha \partial c} = -\frac{1-\beta}{2(2-(\alpha+\beta))^2} < 0.$$

Therefore, the proposition follows. □

B.8 Proof of Remark 1

Proof. In this case, there are also three scenarios.

Scenario A. Equations (8) and (9) become (taking $\underline{U}^s = 0$ into account):

$$N_{1,t}^s(P_1^s, N_t^b) = \frac{1}{2\xi}(\alpha N_t^b - P_1^s),$$
$$N_{2,t+1}^s(P_2^s, N_{t+1}^b) = \frac{1}{2\xi}(\alpha N_{t+1}^b + c - P_2^s).$$

Therefore, the total number of sellers at time t is

$$N_t^s = \frac{1}{2\xi}[\alpha(N_t^b + N_t^b) + c - P_1^s - P_2^s].$$

All other components remain identical to those in the baseline model. Accordingly, the equilibrium results follow as in Remark 1. To ensure that $0 < N_1^s, N_2^s < 1/2$, $0 < N^b < 1$, and $\pi > 0$, the condition $\max\left\{\frac{(\alpha+\beta)(\alpha+\beta+1)+d}{4}, \frac{(\alpha+\beta)(\alpha+\beta+d/2)}{2}\right\} < \xi < 1$ must hold. Note that $0 < \alpha, \beta, c, \xi < 1$ is assumed.

Scenario B. Equations (11) and (12) become (taking $\underline{U}^s = 0$ into account):

$$N^s_{2,t+1}(P^s_2, N^b_{t+1}) = \frac{1}{2\xi}(\alpha N^b_{t+1} - P^s_2),$$
$$N^s_{1,t}(P^s_1, N^b_t) = \frac{1}{2\xi}(\alpha N^b_t + c - P^s_1).$$

Similarly, the results can be derived under the assumption that an equilibrium exists:

$$N^s_1 = \frac{c(\alpha^2 + 2\alpha\beta + \beta^2 - 8\xi) - 4\xi(\alpha+\beta)}{8\xi(\alpha^2 + 2\alpha\beta + \beta^2 - 4\xi)}, \; N^s_2 = -\frac{(\alpha+\beta)(c(\alpha+\beta) + 4\xi)}{8\xi(\alpha^2 + 2\alpha\beta + \beta^2 - 4\xi)}.$$

Hence, we obtain $N^s_2 - N^s_1 = -\frac{c}{4\xi} < 0$, which contradicts the fundamental assumption of *Scenario B*. Therefore, this scenario does not hold.

Scenario C. The expressions for $N^s_{1,t}(P^s_1, N^b_t)$ and $N^s_{2,t+1}(P^s_2, N^b_{t+1})$ are the same as in Scenario A, except that the condition $P^s_2 = P^s_1 + c$ no longer holds. Thus, the results can be derived under the assumption that an equilibrium exists:

$$N^s_1 = -\frac{(\alpha+\beta)(c(\alpha+\beta) + 4\xi)}{8\xi(\alpha^2 + 2\alpha\beta + \beta^2 - 4\xi)},$$
$$N^s_2 = \frac{c(\alpha^2 + 2\alpha\beta + \beta^2 - 8\xi) - 4\xi(\alpha+\beta)}{8\xi(\alpha^2 + 2\alpha\beta + \beta^2 - 4\xi)}.$$

Hence, we obtain $N^s_1 - N^s_2 = -\frac{c}{4\xi} < 0$, which contradicts the fundamental assumption of Scenario C. Therefore, this scenario does not hold.

Overall, *Scenario A* is the only scenario in which an equilibrium exists. □

B.9 Proof of Proposition 6

Proof. Given the feasible set $\max\left\{\frac{(\alpha+\beta)(\alpha+\beta+1)+d}{4}, \frac{(\alpha+\beta)(\alpha+\beta+d/2)}{2}\right\} < \xi < 1$, it follows that the denominator of the equilibrium expression for P^s_1 in Remark 1 is always strictly positive. Therefore, the sign of P^s_1 is determined solely by the sign of the numerator. Accordingly, we have

$$P^s_1 < 0 \Longleftrightarrow 2\xi(\alpha - \beta) < d\,(2\xi - \alpha(\alpha+\beta))\,.$$

Given the feasible set, it holds that $2\xi - \alpha(\alpha+\beta) > 0$. Thus, the condition can be rewritten as

$$d > \frac{2(\alpha-\beta)}{2 - \alpha(\alpha+\beta)/\xi},$$

regardless of whether $\alpha > \beta$ or $\alpha < \beta$. If $\alpha < \beta$, the inequality is clearly satisfied since $d > 0$. Proposition 6 follows. □

B.10 Proof of Lemma 3

Let N^s and N^b denote the number of sellers and buyers on the platform in a steady-state, respectively. We can then drop the subscript t for $u_t^b, u_{1,t}^s$, and $u_{2,t}^s$ for each scenario. First, we check whether *Scenario B* is the optimal choice of both buyers and sellers after the monopoly platform sets the optimal prices. By making decisions on P^b, P_1^s, and P_2^s, the platform's problem is to maximize profit (1) subject to equalities (18), (21), and (22). The maximization problem is reduced to

$$\max_{\{u_1^s,u_2^s,u^b\}} [1-F(-u^b)]\left[\beta\left(\frac{1}{2}F(u_1^s)+\frac{1}{2}F(u_2^s)\right)-u^b-\underline{U}^b\right]$$
$$+\frac{1}{2}F(u_1^s)\left[\alpha(1-F(-u^b))+\delta c-u_1^s-\underline{U}^s\right]+\frac{1}{2}F(u_2^s)\left[\alpha(1-F(-u^b))-u_2^s-\underline{U}^s\right].$$

Solving the F.O.C.s gives us the optimal seller prices in a steady-state as

$$P_1^s=-\beta N^b+\frac{F(u_1^s)}{f(u_1^s)} \text{ and } P_2^s=-\beta N^b+\frac{F(u_2^s)}{f(u_2^s)}.$$

By log-concavity of F, we know that $P_2^s > P_1^s \implies u_2^s > u_1^s \iff \alpha N^b - P_2^s > \alpha N^b - P_1^s + c \iff P_1^s > P_2^s + c \implies P_1^s > P_2^s$ which is a contradiction. Then, it must be that $P_1^s \geq P_2^s \iff u_1^s \geq u_2^s$ (again, by log-concavity). However, this implies that $N_{2,t+1}^s \leq N_{1,t}^s$ which contradicts this scenario where sellers enter the platform in period.

Now, we consider *Scenario C*. By choosing P^b, P_1^s and P_2^s, the monopoly platform maximizes profit (1) subject to equalities (18), (19), and (20). The profit maximization problem can be reduced to

$$\max_{\{u_1^s,u_2^s,u^b\}} [1-F(-u^b)]\left[\beta\left(\frac{1}{2}F(u_1^s)+\frac{1}{2}F(u_2^s)\right)-u^b-\underline{U}^b\right]$$
$$+\frac{1}{2}F(u_1^s)\left[\alpha(1-F(-u^b))-u_1^s-\underline{U}^s\right]+\frac{1}{2}F(u_2^s)\left[\alpha(1-F(-u^b))+c-u_2^s-\underline{U}^s\right],$$

Solving the F.O.C.s gives us

$$P_1^s=-\beta N^b+\frac{F(u_1^s)}{f(u_1^s)} \text{ and } P_2^s=-\beta N^b+\frac{F(u_2^s)}{f(u_2^s)}.$$

By log-concavity of F, we know that $P_1^s > P_2^s \iff u_1^s > u_2^s \iff \alpha N_t^b - P_1^s > \alpha N_t^b - P_2^s + c \iff P_2^s > P_1^s + c \implies P_2^s > P_1^s$ which is a contradiction. Then, it must be that $P_2^s \geq P_1^s \iff u_2^s \geq u_1^s$ (again, by log-concavity). However, this implies that $N_{2,t+1}^s \geq N_{1,t}^s$ which contradicts this scenario where sellers leave the platform in period 2.

In summary, *Scenario B* and *Scenario C* cannot form a final equilibrium. Thus, only *Scenario A* is a final equilibrium.

B.11 Proof of Proposition 7

Proof. Again, we can drop the subscript t for u_t^b, $u_{1,t}^s$, and $u_{2,t}^s$ assuming we are in a steady-state equilibrium. By choosing P^b, P_1^s and P_2^s, the monopoly platform maximizes profit (1) subject to equalities (10), (18), (19), and (20). The profit maximization problem can be reduced to

$$\begin{aligned}
&\max_{\{u^b,u_1^s,u_2^s\}}[1-F(-u^b)]\{\beta[\frac{1}{2}F(u_1^s)+\frac{1}{2}F(u_2^s)]-u^b-\underline{U}^b\} \\
&\qquad\qquad +\frac{1}{2}F(u_1^s)[\alpha(1-F(-u^b))-u_1^s-\underline{U}^s]+\frac{1}{2}F(u_2^s)[\alpha(1-F(-u^b))+c-u_2^s-\underline{U}^s] \\
=\ &\max_{\{u^b,u_1^s\}}[1-F(-u^b)][\beta F(u_1^s)-u^b-\underline{U}^b]+F(u_1^s)[\alpha(1-F(-u^b))-u_1^s+\frac{1}{2}c-\underline{U}^s].
\end{aligned}$$

Solving the F.O.C.s with respect to u^b and u_1^s gives us

$$P_1^s=-\beta N^b-\frac{c}{2}+\frac{F(u_1^s)}{f(u_1^s)}\text{ and }P^b=-\alpha N^s+\frac{1-F(-u^b)}{f(-u^b)}.$$

Note that the buyer's price elasticity of demand is

$$\begin{aligned}
\varepsilon^b &= \frac{d(1-F(-u^b))/dP^b}{(1-F(-u^b))/P^b}=\frac{d(1-F(-u^b))}{du^b}\frac{du^b}{dP^b}\frac{P^b}{1-F(-u^b)} \\
&= \frac{f(-u^b)}{1-F(-u^b)}(-P^b)<0
\end{aligned}$$

and when sellers are young, their price elasticity of demand is

$$\varepsilon^s=\frac{dF(u_1^s)/F(u_1^s)}{dP_1^s/P_1^s}=\frac{dF(u_1^s)}{du_1^s}\frac{du_1^s}{dP_1^s}\frac{P_1^s}{F(u_1^s)}=\frac{f(u^s)}{1-F(u^s)}(-P_1^s)<0.$$

Substituting these elasticities into the solved equilibrium prices gives us the result. □

B.12 Scenario Conditions with Third-Degree Price Discrimination

Because there may now be two critical sellers when they get old (because there will be two different prices based on previous homing choice), we denote $\widetilde{h}$ as the critical seller who joined when young and h as the critical seller who did not join when young.

Scenario A (the sellers on the platform are the same in both periods). The seller h and the seller k are the same, which implies that three conditions, $U_{k1,t}^s=\underline{U}^s$, $\widetilde{U}_{k2,t+1}^s=\underline{U}^s$, and $U_{k1,t}^s+\widetilde{U}_{k2,t+1}^s=\underline{U}^s+\widehat{U}_{k2,t+1}^s$, are satisfied. In period 1, given price P_1^s, the critical condition $U_{k1,t}^s=\underline{U}^s$ is satisfied. The reason is that in the second (last) period, the platform will charge a high enough fee such that the critical seller's utility is indifferent between online and offline sales. From this, we can obtain the number of young sellers choosing the platform as

$$N_{1,t}^s(P_1^s,N_t^b)=\frac{1}{2}\Pr\left(B_{i1,t}^s-P_1^s+\alpha N_t^b\geq\underline{U}^s\right)=\frac{1}{2}(\alpha N_t^b-P_1^s-\underline{U}^s).\tag{A2}$$

From the second condition and from the fact that h and k are the same, we have that the number of old sellers affiliated with the platform at time $t+1$ is

$$N^s_{2,t+1} = N^s_{22,t+1} = \frac{1}{2}(\alpha N^b_{t+1} + c - P^s_{22} - \underline{U}^s) = N^s_{1,t}$$

which implies that $P^s_{22} = c + P^s_1$. Finally, the last condition implies that the critical seller k is indifferent between joining and leaving the platform which is only satisfied if

$$P^s_{21} = P^s_{22} - c. \tag{A3}$$

Recall that this also means that $P^s_{21} = P^s_1$. Combining the characterizations of $N^s_{1,t}$ and $N^s_{2,t+1}$ with (7) and (A3), we can easily derive that the total number of buyers and sellers on the platform in a steady-state are:

$$N^b = \frac{1 - \underline{U}^b - \beta \underline{U}^s - \beta P^s_1 - P^b}{1 - \alpha\beta}, \tag{A4}$$

$$N^s = \frac{\alpha - \alpha \underline{U}^b - \underline{U}^s - P^s_1 - \alpha P^b}{1 - \alpha\beta}. \tag{A5}$$

Scenario B (some sellers enter the platform in period 2). There is only one critical seller h in period 2, and this seller has weaker business ability compared with seller k, as shown in Figure 3; that is, the seller h's business ability is on the interval $[-1, \underline{B}^s)$. In this scenario, four conditions, $\widetilde{U}^s_{k2,t+1} = \underline{U}^s$, $\widehat{U}^s_{h2,t+1} = \underline{U}^s$, $U^s_{k1,t} + \widetilde{U}^s_{k2,t+1} = \underline{U}^s + \widehat{U}^s_{k2,t+1}$ and $\widehat{U}^s_{h2,t+1} < \widehat{U}^s_{k2,t+1}$, should be satisfied. From the first condition, we can indirectly obtain the number of young sellers on the platform:

$$N^s_{1,t}(P^s_{22}, N^b_{t+1}) = \frac{1}{2}\Pr(B^s_{i1,t} - P^s_{22} + \alpha N^b_{t+1} + c \geq \underline{U}^s) = \frac{1}{2}(\alpha N^b_2 + c - P^s_{22} - \underline{U}^s). \tag{A6}$$

From the second condition, we can derive the number of old sellers on the platform in the next period as:

$$N^s_{2,t+1}(P^s_{21}, N^b_{t+1}) = \frac{1}{2}\Pr\left(B^s_{i2,t+1} \geq B^s_{h2,t+1}\right) = \alpha N^b_{t+1} - P^s_{21} - \underline{U}^s \tag{A7}$$

because $B^s_{h2,t+1}$ is such that $B^s_{h2,t+1} - P^s_{21} + \alpha N^b_{t+1} = \underline{U}^s$. Moreover, the first and third conditions imply that $U^s_{k1,t} = \widehat{U}^s_{k2,t+1}$; that is,

$$P^s_1 = P^s_{21}. \tag{A8}$$

Combining the characterizations of $N^s_{1,t}$ and $N^s_{2,t+1}$ with (7), we can easily derive that the total number of buyers and sellers on the platform in a steady-state are:

$$N^b = \frac{1 - \underline{U}^b - \beta \underline{U}^s - P^b - 0.5\beta(P^s_{22} + P^s_{21}) + 0.5\beta c}{1 - \alpha\beta}, \tag{A9}$$

$$N^s = \frac{0.5(c - P^s_{21} - P^s_{22}) - \underline{U}^s + \alpha(1 - P^b - \underline{U}^b)}{1 - \alpha\beta}. \tag{A10}$$

Scenario C (some sellers leave the platform in period 2). There is only one critical seller $\widetilde{h}$ in period 2, and this seller has stronger business ability compared to seller k. The seller $\widetilde{h}$'s business ability is on the interval $(\underline{B}^s + c, c]$. In this scenario, four conditions, $U^s_{k1,t} = \underline{U}^s$, $\widetilde{U}^s_{\widetilde{h},2} = \underline{U}^s$, $\widetilde{U}^s_{k2,t+1} < \widetilde{U}^s_{\widetilde{h},2}$ and $\widehat{U}^s_{k2,t+1} \leq \underline{U}^s$, are satisfied. Then, from the first condition, we have the same expression for the number of young sellers on the platform in period 1 as in *Scenario A*. That is, we have $N^s_{1,t} = \frac{1}{2}(\alpha N^b_t - P^s_1 - \underline{U}^s)$. Based on the second condition, we obtain the number of old sellers on the platform as

$$N^s_{2,t+1}(P^s_{22}, N^b_{t+1}) = \frac{1}{2}\Pr\left(B^s_{i2,t+1} \geq B^s_{\widetilde{h},2}\right) = \frac{1}{2}(\alpha N^b_{t+1} + c - P^s_{22} - \underline{U}^s) \tag{A11}$$

because $B^s_{\widetilde{h},2}$ is such that $B^s_{\widetilde{h},2} + \alpha N^b_{t+1} - P^s_{22} + c = \underline{U}^s$

When the last condition binds, we get the same condition as (A3) that $P^s_{21} = P^s_{22} - c$. Putting together the characterizations of $N^s_{1,t}$ and $N^s_{2,t+1}$ with (7), we can easily derive that the total number of buyers and sellers on the platform in a steady-state are the same as in (A9) and (A10). Note that we can skip the derivation because in *Scenario B*, $P^s_{21} = P^s_1$ must hold.

Scenario D (some sellers leave the platform, but some sellers enter the platform in period 2). In this scenario, the following five conditions, $\widehat{U}^s_{h2,t+1} = \underline{U}^s$, $\widetilde{U}^s_{\widetilde{h},2} = \underline{U}^s$, $\underline{U}^s + \widehat{U}^s_{k2,t+1} = U^s_{k1,t} + \underline{U}^s$, $\widehat{U}^s_{h2,t+1} < \widehat{U}^s_{k2,t+1}$, and $\widetilde{U}^s_{k2,t+1} < \widetilde{U}^s_{\widetilde{h},2}$, are satisfied. From the first condition, we have the number of sellers who have higher business ability compared with seller h:

$$\widetilde{N}^s_{2,t+1} = \frac{1}{2}(\alpha N^b_{t+1} - P^s_{21} - \underline{U}^s).$$

Then, we can derive the number of sellers who are only affiliated with the platform in period 2 as

$$N^s_{21,t+1} = \widetilde{N}^s_{2,t+1} - N^s_{1,t} = \frac{1}{2}(\alpha N^b_{t+1} - P^s_{21} - \underline{U}^s) - N^s_{1,t}.$$

From the second condition, we obtain the number of sellers who have higher business ability than seller $\widetilde{h}$

$$N^s_{22,t+1} = \frac{1}{2}(\alpha N^b_{t+1} + c - P^s_{22} - \underline{U}^s).$$

We then obtain the number of old sellers affiliated with the platform from the above equations; that is,

$$N^s_{2,t+1} = N^s_{21,t+1} + N^s_{22,t+1} = \frac{1}{2}(2\alpha N^b_{t+1} + c - P^s_{21} - P^s_{22} - 2\underline{U}^s) - N^s_{1,t}. \tag{A12}$$

The third condition implies the following relationship between P^s_1 and P^s_{21} follows that of (A8), i.e. $P^s_{21} = P^s_1$. Combining (7) with (A8) and (A12), gives us that the total number of buyers and sellers on the platform in a steady-state follows exactly like that in *Scenario B* again.

B.13 Proof of Lemma 4

Proof. All possible scenarios are described in Online Appendix B.12. First, we check whether *Scenario B* is the optimal choice of both buyers and sellers after the monopoly platform sets the optimal prices. It is noteworthy that $N^s_{22,t+1} = N^s_{1,t}$ and $N^s_{21,t+1} = N^s_{2,t+1} - N^s_{22,t+1}$ in this scenario. By making decisions on P^b, P^s_{21}, and P^s_{22}, the platform's problem is to maximize profit (26) subject to equalities (7), (A6), (A7), (A8), (A9), and (A10). The maximization problem is reduced to

$$\max_{\widetilde{\boldsymbol{P}}}(P^s_{21}+P^s_{22})\frac{\alpha N^b+c-P^s_{22}}{2}+P^bN^b+P^s_{21}\frac{P^s_{22}-P^s_{21}-c}{2}.$$

Equating the F.O.C.s with respect to P^s_{21} and P^s_{22} gives us $P^s_{21}=P^s_{22}0.5c$. But, putting this into (A6) and (A7) implies that $N^s_{1,t}>N^s_{2,t+1}$ which is a contradiction that some sellers enter the platform when they get old. Thus, *Scenario B* does not exist in a steady-state.

Second, we consider *Scenario C*. Under this scenario, $N^s_{21,t}=0$ and $N^s_{22,t}=N^s_{2,t}$. By choosing P^b, P^s_1, and P^s_{22}, the monopoly platform maximizes profit (26) subject to equalities (7), (A2), (A3), (A9), (A10), and (A11). The profit maximization problem can be reduced to

$$\max_{\widetilde{\boldsymbol{P}}}\frac{P^s_1}{2}(\alpha N^b-P^s_1)+P^bN^b+\frac{P^s_{22}}{2}(\alpha N^b+c-P^s_{22}).$$

Equating the F.O.C.s with respect to P^s_1 and P^s_{22} gives us $P^s_1=P^s_{22}-0.5c$. But, putting this into (A2) and (A11), implies that $N^s_{2,t+1}>N^s_{1,t}$ which is a contradiction that some sellers leave the platform when they get old. Thus, *Scenario C* does not exist.

Third, we need to verify whether *Scenario D* is an equilibrium result. The platform's problem is to maximize (26) subject to equalities (7), (A8), (A9), (A10), and (A12), according to the platform's decisions on P^b, P^s_1, P^s_{21}, and P^s_{22}. Again, we reduce the platform's profit maximization problem to:

$$\max_{\widetilde{\boldsymbol{P}}}\frac{P^s_1}{2}(\alpha N^b-P^s_1)+P^bN^b+\frac{P^s_{22}}{2}(\alpha N^b+c-P^s_{22}).$$

Equating the F.O.C.s with respect to P^s_1 and P^s_{22} gives us $P^s_1=P^s_{22}-0.5c$ again. But this implies that $N^s_{22,t+1}>N^s_{21,t+1}+N^s_{1,t}$ which is a contradiction that some sellers leave the platform when they get old. Thus, *Scenario D* does not exist.

In summary, Scenarios B, C, and D cannot form a final equilibrium. Thus, we conclude that only *Scenario A* is a final equilibrium result. □

B.14 Proof of Remark 2

Proof. Given this setup, there are three possible scenarios of choice for one generation in the equilibrium.

Scenario A (the sellers on the platform are the same in both periods). This scenario requires that $\widetilde{U}^s_{k2,t+1}=\underline{U}^s$, $U^s_{k1,t}+\widetilde{U}^s_{k2,t+1}=2\underline{U}^s$, $U^s_{k1,t}+\widetilde{U}^s_{k2,t+1}>\underline{U}^s+\hat{U}^s_{k2,t+1}$, and $U^s_{k1,t}+\widetilde{U}^s_{k2,t+1}>U^s_{k1,t}+\underline{U}^s$. The first two conditions imply $U^s_{k1,t}=\underline{U}^s$. The last two conditions, together with the presence of the same critical seller k in both periods, must be verified in equilibrium.

For concreteness, putting equations $U^s_{k1,t} = \underline{U}^s$ and $\widetilde{U}^s_{k2,t+1} = \underline{U}^s$ together yields

$$N^s_{1,t} = (\alpha N^b_t - P^s_q - \underline{U}^s)/2,$$

$$N^s_{2,t+1} = (\lambda\alpha N^b_{t+1} - P^s_2 - \underline{U}^s)/2.$$

We can derive $N^s_{2,t} = (\lambda\alpha N^b_t - P^s_2 - \underline{U}^s)/2$ from the second equation. Thus, we have

$$N^s_t = N^s_{1,t} + N^s_{2,t} = [(\lambda+1)\alpha N^b_t - P^s_1 - P^s_2 - 2\underline{U}^s]/2. \tag{28}$$

Putting this equation together with equation (7) yields

$$N^b = \frac{\beta\left(P^s_1 + P^s_2\right) + 2P^b + 2\underline{U}^b + 2\beta\underline{U}^s - 2}{\alpha\beta(\lambda+1) - 2}, \tag{29}$$

$$N^s = \frac{P^s_1 + P^s_2 + \alpha(\lambda+1)P^b + \alpha(\lambda+1)\underline{U}^b + 2\underline{U}^s - \alpha(\lambda+1)}{\alpha\beta(\lambda+1) - 2}. \tag{30}$$

Thus, we have

$$N^s_1 = \frac{P^s_1 + \alpha(P^b + \underline{U}^b - 1) + \underline{U}^s}{2\alpha\beta - 2},$$

$$N^s_2 = \frac{P^s_1 + \alpha(P^b + \underline{U}^b - 1) + \underline{U}^s}{2\alpha\beta - 2}.$$

The presence of the same critical seller k implies $N^s_1 = N^s_2$, requiring

$$P^s_2 = \frac{P^s_1(\alpha\beta\lambda - 1) + \alpha(\lambda-1)(P^b + \underline{U}^b + \beta\underline{U}^s - 1)}{\alpha\beta - 1}.$$

So, given $\underline{U}^s = 0$, $\underline{U}^b = 0$, and $r = 0$, the monopoly platform's maximization problem for each generation at each time is

$$\pi = \max_{P^s_1, P^b} P^s_1 N^s_1 + P^s_2 N^s_2 + P^b N^b,$$

yielding equilibrium prices as

$$P^s_1 = \frac{2\alpha(\lambda-3) + 4\beta}{\alpha^2(\lambda+1)^2 + 4\alpha\beta(\lambda+1) - 4\left(4-\beta^2\right)},$$

$$P^b = \frac{\alpha^2(\lambda+1)^2 + 2\alpha\beta(\lambda+1) - 8}{\alpha^2(\lambda+1)^2 + 4\alpha\beta(\lambda+1) - 4\left(4-\beta^2\right)}.$$

These price results also imply

$$P^s_2 = \frac{2(3\alpha\lambda - \alpha - 2\beta)}{4\left(4-\beta^2\right) - \alpha^2(\lambda+1)^2 - 4\alpha\beta(\lambda+1)},$$

$$N^s_1 = N^s_2 = \frac{\alpha\lambda + \alpha + 2\beta}{4\left(4-\beta^2\right) - \alpha^2(\lambda+1)^2 - 4\alpha\beta(\lambda+1)},$$

$$N^s = \frac{2(\alpha\lambda + \alpha + 2\beta)}{4\left(4-\beta^2\right) - \alpha^2(\lambda+1)^2 - 4\alpha\beta(\lambda+1)},$$

$$N^b = \frac{8}{4\left(4-\beta^2\right) - \alpha^2(\lambda+1)^2 - 4\alpha\beta(\lambda+1)}$$

$$\pi = \frac{4}{4\left(4-\beta^2\right) - \alpha^2(\lambda+1)^2 - 4\alpha\beta(\lambda+1)}.$$

Recall that $\lambda > 1$. To ensure $0 < N_1^s, N_2^s < 1/2$, $0 < N^b < 1$, and $\pi > 0$, a feasible parameter set is given by $0 < \alpha < \frac{2(\sqrt{2}-\beta)}{\lambda+1}$, where $0 < \alpha, \beta < 1$ are assumed.

Scenario B (some sellers enter the platform in old period). This scenario requires that $\widetilde{U}^s_{\tilde{h},2t+1} = \underline{U}^s$, $U^s_{k1,t} + \underline{U}^s = 2\underline{U}^s$, $U^s_{k1,t} + \underline{U}^s > U^s_{k1,t} + \widetilde{U}^s_{k2,t+1}$, and $U^s_{k1,t} + \underline{U}^s > \underline{U}^s + \widehat{U}^s_{k2,t+1}$.These conditions imply that the number of sellers in the old period is lower than in the young period. The first two conditions yield:

$$N^s_{1,t} = (\alpha N^b_t - P^s_1 - \underline{U}^s)/2,$$
$$N^s_{2,t+1} = (\lambda\alpha N^b_{t+1} - P^s_2 - \underline{U}^s)/2.$$

The last equation implies

$$N^s_{2,t} = (\lambda\alpha N^b_t - P^s_2 - \underline{U}^s)/2.$$

Thus, we have

$$N^s_t = N^s_{1,t} + N^s_{2,t} = [(\lambda+1)\alpha N^b_t - P^s_1 - P^s_2 - 2\underline{U}^s]/2. \tag{31}$$

Putting this equation together with equation (7) yields

$$N^b = \frac{\beta\left(P^s_1 + P^s_2\right) + 2P^b + 2\underline{U}^b + 2\beta\underline{U}^s - 2}{\alpha\beta(\lambda+1) - 2}, \tag{32}$$

$$N^s = \frac{P^s_1 + P^s_2 + \alpha(\lambda+1)P^b + \alpha(\lambda+1)\underline{U}^b + 2\underline{U}^s - \alpha(\lambda+1)}{\alpha\beta(\lambda+1) - 2}. \tag{33}$$

Given $\underline{U}^s = 0$, $\underline{U}^b = 0$, and $r = 0$, the monopoly platform's maximization problem for the generation born at time t is:

$$\pi = \max_{P^s_1, P^s_2, P^b} P^s_1 N^s_1 + P^s_2 N^s_2 + P^b N^b,$$

yielding equilibrium prices as

$$P^s_1 = \frac{2(\beta-\alpha)}{\alpha^2\left(\lambda^2+1\right) + 2\alpha\beta(\lambda+1) + 2\left(\beta^2-4\right)},$$
$$P^s_2 = \frac{2(\beta-\alpha\lambda)}{\alpha^2\left(\lambda^2+1\right) + 2\alpha\beta(\lambda+1) + 2\left(\beta^2-4\right)},$$
$$P^b = \frac{\alpha^2\left(\lambda^2+1\right) + \alpha\beta(\lambda+1) - 4}{\alpha^2\left(\lambda^2+1\right) + 2\alpha\beta(\lambda+1) + 2\left(\beta^2-4\right)}.$$

Thus, we obtain

$$N^s_1 = \frac{\alpha+\beta}{2\left(4-\beta^2\right) - \alpha^2\left(\lambda^2+1\right) - 2\alpha\beta(\lambda+1)},$$
$$N^s_2 = \frac{\alpha\lambda+\beta}{2\left(4-\beta^2\right) - \alpha^2\left(\lambda^2+1\right) - 2\alpha\beta(\lambda+1)}.$$

Recall that $\lambda > 1$. It is then evident that $N^s_2 > N^s_1$, which contradicts the premise of *Scenario B*.

Scenario C (some sellers leave the platform in old period). This scenario requires that $\hat{U}^s_{h,2t+1} = \underline{U}^s$, $U^s_{k1,t} + \widetilde{U}_{k2,t+1} = \underline{U}^s + \hat{U}_{k2,t+1}$, $U^s_{k1,t} + \widetilde{U}_{k2,t+1} > 2\underline{U}^s$, and $U^s_{k1,t} + \widetilde{U}_{k2,t+1} > U^s_{k1,t} + \underline{U}^s$. These conditions imply that the number of sellers in the old period is greater that in the young period. The first two conditions imply

$$N^s_{2,t+1} = (\alpha N^b_{t+1} - P^s_2 - \underline{U}^s)/2,$$
$$N^s_{1,t} = [\alpha N^b_t + (\lambda - 1)\alpha N^b_{t+1} - P^s_1 - \underline{U}^s]/2.$$

The first equation implies $N^s_{2,t} = (\alpha N^b_t - P^s_2 - \underline{U}^s)/2$. Thus, we have

$$N^s_t = N^s_{1,t} + N^s_{2,t} = [2\alpha N^b_t + (\lambda - 1)\alpha N^b_{t+1} - P^s_1 - P^s_2 - 2\underline{U}^s]/2.$$

We consider the equilibrium case with $N^b_t = N^b_{t+1} = N^b$, and solve the optimal problem of the monopoly platform

$$\pi = \max_{P^s_1, P^s_2, P^b} P^s_1 N^s_1 + P^s_2 N^s_2 + P^b N^b.$$

Thus, we have equilibrium prices:

$$P^s_1 = \frac{2(\beta - \alpha\lambda)}{\alpha^2(\lambda^2+1) + 2\alpha\beta(\lambda+1) + 2(\beta^2-4)},$$
$$P^s_2 = \frac{2\beta - 2\alpha}{\alpha^2(\lambda^2+1) + 2\alpha\beta(\lambda+1) + 2(\beta^2-4)},$$
$$P^b = \frac{\alpha^2\left(\lambda^2+1\right) + \alpha\beta(\lambda+1) - 4}{\alpha^2(\lambda^2+1) + 2\alpha\beta(\lambda+1) + 2(\beta^2-4)}.$$

These results mean that

$$N^s_1 = \frac{\alpha\lambda + \beta}{2(4-\beta^2) - \alpha^2(\lambda^2+1) - 2\alpha\beta(\lambda+1)},$$
$$N^s_2 = \frac{\alpha + \beta}{2(4-\beta^2) - \alpha^2(\lambda^2+1) - 2\alpha\beta(\lambda+1)}.$$

Recall that $\lambda > 1$. Obviously, there is $N^s_1 > N^s_2$, which contradicts the premise of *Scenario C*.

Overall, of the three scenarios, only *Scenario A* is feasible, implying that its outcome is the unique equilibrium. □

B.15 Proof of Proposition 8

Proof. The feasible set $0 < \alpha < \frac{2(\sqrt{2}-\beta)}{\lambda+1}$ in Remark 2 can also be written as $\lambda < \frac{2(\sqrt{2}-\beta)}{\alpha} - 1$. Recall that $P^s_1 = \frac{2(\beta-\alpha\lambda)}{\alpha^2(\lambda^2+1)+2\alpha\beta(\lambda+1)+2(\beta^2-4)}$. Within the feasible set, the denominator is always positive, so the sign of P^s_1 depends only on the numerator. A sufficient condition for the numerator of P^s_1 to be positive is $\lambda > 3 - \frac{2\beta}{\alpha}$. For this range of λ to be non-empty, it is necessary that $0 < \alpha < \frac{\sqrt{2}}{2}$. Therefore, $P^s_1 < 0$ if and only if $0 < \alpha < \frac{\sqrt{2}}{2}$ and $3 - \frac{2\beta}{\alpha} < \lambda < \frac{2(\sqrt{2}-\beta)}{\alpha} - 1$ hold. Otherwise, $P^s_1 > 0$. Moreover, the difference between P_2^s and P_1^s is

$$P_2^s - P_1^s = \frac{8\alpha(\lambda - 1)}{4\left(4 - \beta^2\right) - \alpha^2(\lambda + 1)^2 - 4\alpha\beta(\lambda + 1)} > 0$$

since the denominator is strictly positive and $\lambda > 1$. The proposition then follows. $\square$