201 Notes: Tax Incidence and Midterm Quantitative Questions

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1 Midterm Breakdown by Topics

- Power Law (25.5): TFU5 (5), SAa (2.5), QLa (4), QLb (4)
  - Statistical Value of Life: QTb (10)
- Monopoly/Deadweight Loss (20): QTa (10), QTb (10)
- Incidence (19): SA1 (15), QLc (4)
- Durable/Storable, elastic/inelastic (16.5): TFU3 (5), TFU4 (5), SA2c (2.5), QLc (4)
- Miscellaneous
  - Homoegenous Production Function: TFU1 (5)
  - Externalities: TFU2 (5)
  - Contingent Valuation Survey: QLd (4)
  - Coase Theorem: QLe (4)
  - First/Second Welfare Theorems: SA2b (2.5)
  - Advantageous Selection in Insurance Market: SA2d (2.5)

2 Incidence

Define **pass-through** $\rho \equiv dp/dt$, that is, the increase in p in response to the increase in t; since p is born by the consumers, ρ is the burden on consumer, whereas, $1-\rho$ is the burden on producers. And $0 \leq \rho \leq 1$, and the ratio of the two is the **incidence** rate,

$$I = \frac{\rho}{1-\rho}$$

*Any error is solely mine. For questions, email hanzhe@uchicago.edu.

2.1 Perfect Competition

At the market equilibrium price,

$$D(p(t)) = S(p(t) - t)$$

where $p - t$ is the real cost. Differentiate both sides with respect to t,

$$\begin{aligned}
\frac{dD(p)}{dp} \cdot \frac{dp}{dt} &= \frac{dS(p-t)}{d(p-t)} \cdot \frac{d(p-t)}{dt} \\
\frac{dD(p)}{dp} \cdot \frac{p}{D(p)} \frac{dp}{dt} &= \frac{dS(p-t)}{dp} \cdot \frac{p}{S(p-t)} \cdot \frac{d(p-t)}{dt} \\
-\varepsilon_D \frac{dp}{dt} &= \varepsilon_S \left(\frac{dp}{dt} - 1 \right) \\
\left(1 + \frac{\varepsilon_D}{\varepsilon_S} \right) \frac{dp}{dt} &= 1 \Rightarrow \rho = 1 / \left(1 + \frac{\varepsilon_D}{\varepsilon_S} \right).
\end{aligned}$$

If $\varepsilon_D = \varepsilon_S$, then $\rho = 1/2$. As $\varepsilon_D \downarrow$, $\varepsilon_S \uparrow$, $\rho \uparrow$, therefore the inelastic side bears the larger burden.

As $t \to \bar{t}$, the equilibrium quantity of the market goes to zero, essentially killing the market, then at that point there is no CS or PS. Therefore, without tax, consumer and producer surpluses are

$$\begin{aligned}
CS &= \int_0^{\bar{t}} \rho(t) q(t) dt \\
PS &= \int_0^{\bar{t}} (1 - \rho(t)) q(t) dt
\end{aligned}$$

The average pass-through rate $\bar{\rho}$ is defined by

$$\bar{\rho} \equiv \frac{\int_0^{\bar{t}} \rho(t) q(t) dt}{\int_0^{\bar{t}} q(t) dt}$$

and the average incidence is

$$\bar{I} = \frac{CS}{PS} = \frac{\bar{\rho}}{1 - \bar{\rho}}$$

which is just the division/split of surplus.

2.2 Monopoly

Physical and economic incidence are independent: p is still final price paid by consumers and ρ is still pass-through rate. A firm is maximizing profit π

$$\pi(q) = [p(q) - t] q - C(q)$$

FOC is

$$p(q) - t + p'(q) q - C'(q) = 0,$$

or

$$p(q) + p'(q) q \equiv MR(q) = MC(q) + t.$$

Implicitly differentiate,

$$\begin{aligned}
\frac{dMR(q)}{dq} \frac{dq}{dt} &= \frac{dMC(q)}{dq} \frac{dq}{dt} + 1 \\
\frac{dq}{dt} &= 1 / \left(MR'(q) - MC'(q) \right)
\end{aligned}$$

Pass-through is

$$\rho = \frac{dp(q)}{dt} = \frac{dp}{dq} \cdot \frac{dq}{dt} = \frac{p'(q)}{MR'(q) - MC'(q)}$$

Let $MR(q) = p(q) - MS(q)$. Recall that marginal cost curve is the supply curve, so

$$\frac{1}{\varepsilon_S(q^*)} = \frac{dMC(q^*)}{dq} \cdot \frac{q^*}{MC(q^*)} = MC'(q^*)\frac{q^*}{p} \Rightarrow MC'(q^*) = \frac{p}{q\varepsilon_S}.$$

Similarly for $p(q)$, it is the inverse demand function, so

$$\begin{aligned}
\frac{1}{-\varepsilon_D} &= p'(q)\frac{q}{p} \Rightarrow p'(q) = -\frac{p}{q\varepsilon_D}, \\
\frac{1}{\varepsilon_{MS}} &= -MS'(q)\frac{q}{MS(q)} = -\frac{MS'(q)}{p'(q)} \Rightarrow MS'(q) = \frac{1}{\varepsilon_{MS}}\frac{p}{q\varepsilon_D}
\end{aligned}$$

Plugging these in,

$$\rho = \frac{-\frac{p}{q\varepsilon_D}}{-MS'(q) - \frac{p}{q\varepsilon_D} - \frac{p}{q\varepsilon_S}} = \frac{1}{1 + \frac{\varepsilon_D}{\varepsilon_S} + \frac{1}{\varepsilon_{MS}}}.$$

To get a sense of how this is dependent of curvature of the demand curve, we see that

$$MS'(q) = -p''(q)q - p'(q),$$

which measures how much valuation is in the tail. Approximately intuitively, we have elasticity measuring the variance of valuations and curvature of the demand curve measuring the kurtosis, or the tail distribution.

2.3 Comparisons

Similarities

- In both PC and monopoly, pass-through rate is defined as dp/dt, the change in price with respect to change in tax.
- Elasticities of demand and of supply affect the pass-through rate the same way in both markets.

Differences

- There is deadweight loss in monopoly but the effect is minimal in PC.
- Pass-through under monopoly, as derived, depends on curvature of the demand curve.

3 Midterm Quantitative Problems

a

What is the size of the deadweight loss of a monopolist facing a demand with a constant elasticity $\varepsilon > 1$, with a constant marginal cost $c > 0$ whose demand when she prices at cost is 1.

Solution. *The deadweight loss (DWL) is going to be the area demonstrated, that is, the reduction in consumer and producer surplus with reduced optimal quantity supplied by the monopoly. It is determined by the following integrals, depending on which way you take the integral*

$$\begin{aligned} DWL(p^*) &= \int_c^{p^*} [Q(p) - Q(p^*)]\, dp \\ &= \int_{Q(p^*)}^{1} \left[p^{-1}(Q) - c\right] dQ \end{aligned} \tag{1}$$

Therefore, we need to determine $Q(p)$ and optimal price and quantity chosen by the monopoly, p^* and $Q^* = Q(p^*)$.

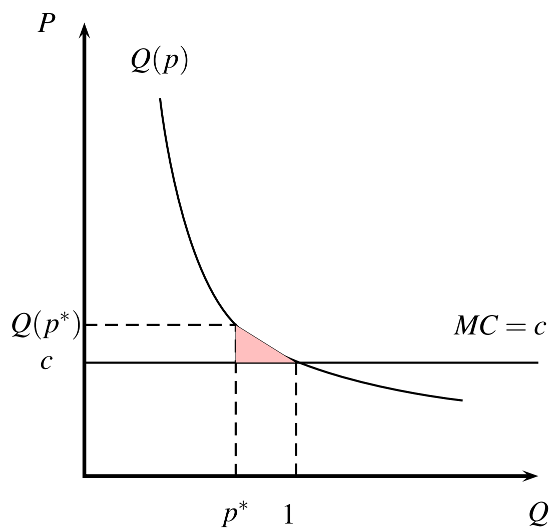

Figure 1: Deadweight Loss

First, we derive the demand curve $Q(p)$. Given that the constant elasticity is ε,

$$\frac{d\ln Q(p)}{d\ln p} = -\varepsilon < 0 \Rightarrow d\ln Q(p) = -\varepsilon d\ln p \Rightarrow \ln Q(p) = -\varepsilon \ln p + k \Rightarrow Q(p) = \exp(k)\, p^{-\varepsilon} \equiv \alpha p^{-\varepsilon}.$$

We can calculate α because we know that $Q(c) = 1$, that is, $\alpha c^{-\varepsilon} = 1 \Rightarrow \alpha = c^{\varepsilon}$. Therefore, the demand curve is given to be

$$Q(p) = \left(\frac{p}{c}\right)^{-\varepsilon}.$$

Next, we can use ***Lerner's mark-up formula****,*

$$\frac{p^* - c}{p^*} = \frac{1}{\varepsilon} \Rightarrow p^* = \frac{\varepsilon}{\varepsilon - 1} c$$

or derive the monopolist's profit maximization problem explicitly[1]*,*

$$\max_p Q(p)(p - c)$$

[1] or the maximization problem is

$$\max_Q Q \cdot \left(p^{-1}(Q) - c\right)$$

which yields the FOC,

$$\begin{aligned} Q'(p^*)(p^*-c)+Q(p^*) &= 0 \\ \left[\frac{Q'(p^*)}{Q(p^*)}\frac{1}{p^*}\right]\frac{p^*-c}{p^*}+1 &= 0 \\ -\varepsilon\frac{p^*-c}{p^*}+1 &= 0. \end{aligned}$$

The optimal quantity is

$$Q^* = Q(p^*) = \left(\frac{p^*}{c}\right)^{-\varepsilon} = \left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}$$

Now use the integral formula for DWL,

$$\begin{aligned} DWL &= \int_c^{p^*}\left[\left(\frac{p}{c}\right)^{-\varepsilon}-\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}\right] = \frac{c}{\varepsilon-1}\left(\frac{p}{c}\right)^{-(\varepsilon-1)}\Bigg|_c^{\frac{\varepsilon}{\varepsilon-1}c}-\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}(p^*-c) \\ &= \frac{c}{\varepsilon-1}\left[1-\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-(\varepsilon-1)}-\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}\right]. \end{aligned}$$

b

Suppose that the elasticity of the statistical value of life with respect to income is .7 and the distribution of income among financiers is given by a power law (Pareto distribution) with $\alpha = 1.5$ and minimum income $\underline{I}$ of \$200k, while among professors it is given by a power law with $\alpha = 3$ and $\underline{I} = \$100k$. How much more valuable is the average financier's life compared to the average professor?

Solution. *Just like in the previous question, with constant elasticity ε of value with respect to income, the value of life is in the form of*

$$v(I) = \beta I^{\varepsilon}, \varepsilon = 0.7.$$

Power laws have density

$$f(I) = \alpha\frac{\underline{I}^{\alpha}}{I^{\alpha+1}},$$

which is derived from differentiating the CDF,

$$F(I) = 1-\left(\frac{\underline{I}}{I}\right)^{\alpha}$$

so the average value of a human life is

$$\begin{aligned} V \equiv \mathbb{E}[v] &= \int_{-\infty}^{\infty} v(I)f(I)\,dI = \int_{\underline{I}}^{\infty} v(I)f(I)\,dI = \int_{\underline{I}}^{\infty}\beta I^{\varepsilon}\alpha\frac{\underline{I}^{\alpha}}{I^{\alpha+1}}dI = \alpha\beta\underline{I}^{\alpha}\int_{\underline{I}}^{\infty} I^{\varepsilon-\alpha-1}dI \\ &= \alpha\beta\underline{I}^{\alpha}\frac{I^{\varepsilon-\alpha}}{\varepsilon-\alpha}\Bigg|_{\underline{I}}^{\infty} = -\alpha\beta\underline{I}^{\alpha}\frac{\underline{I}^{\varepsilon-\alpha}}{\varepsilon-\alpha} = \frac{\alpha\beta}{\alpha-\varepsilon}\underline{I}^{\varepsilon} \end{aligned}$$

The two professions differ by $\underline{I}_f = 200k, \alpha_f = 1.5$ and $\underline{I}_p = 100k, \alpha_p = 3$.

$$\frac{V_f}{V_p} = \frac{\frac{\alpha_f\beta}{\alpha_f-\varepsilon}\underline{I}_f^{\varepsilon}}{\frac{\alpha_p\beta}{\alpha_p-\varepsilon}\underline{I}_p^{\varepsilon}} = \frac{\frac{\alpha_f}{\alpha_f-\varepsilon}}{\frac{\alpha_p}{\alpha_p-\varepsilon}}\left(\frac{\underline{I}_f}{\underline{I}_p}\right)^{\varepsilon} = \frac{1.5/(1.5-0.7)}{3/(3-0.7)}2^{0.7} \approx 2.33.$$

c

Suppose that the inverse demand for a product is $a - bq$ and average cost is $c - dq$ where $a > c > 0$ and $b > 2d > 0$.

Solution. *Average cost pricing would choose quantity such that*

$$a - bq^* = c - dq^* \Rightarrow q^* = \frac{a-c}{b-d}, p^* = c - d\frac{a-c}{b-d}.$$

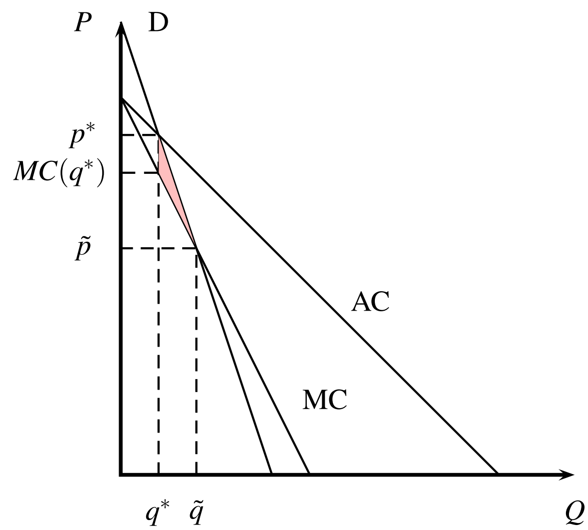

Figure 2: Deadweight Loss

Marginal cost pricing would yield better outcome. The total cost function is

$$q(c - dq) = cq - dq^2,$$

so the marginal cost curve is $c - 2dq$.

$$a - b\tilde{q} = c - 2d\tilde{q} \Rightarrow \tilde{q} = \frac{a-c}{b-2d}, \tilde{p} = c - 2d\frac{a-c}{b-2d}.$$

The DWL is the area with the height to be the quantity reduced which is

$$\tilde{q} - q^* = \frac{a-c}{b-2d} - \frac{a-c}{b-d} = \frac{(a-c)d}{(b-d)(b-2d)}$$

and the base to be

$$p^* - MC(q^*) = a - b\frac{a-c}{b-d} - \left(a - 2d\frac{a-c}{b-d}\right) = d\frac{a-c}{b-d}.$$

The area is then

$$\frac{1}{2} \cdot \frac{(a-c)d}{(b-d)(b-2d)} \cdot \frac{(a-c)d}{b-d} = \frac{1}{2(b-2d)}\left[\frac{(a-c)d}{b-d}\right]^2.$$

Alternatively, we can get the area by differentiation,

$$\int_{q^*}^{\tilde{q}} [D(q) - MC(q)]\,dq = \int_{q^*}^{\tilde{q}} [(a - bq) - (c - 2dq)]\,dq.$$

References