TA Session Notes (Sketch)
October 4, 2013

First some logistics. TA sessions are usually in Stu 102 on Monday 5-6 pm. Raluca and Hanzhe will split, with Raluca responsible for a majority of the sessions. Usually we cover psets/lectures-related problems, open to any question. No new materials will be covered. They are somewhat optional - you can come and leave whenever you want.

Office hours are 4:30-5:30pm Thursdays in Stuart Cafe. Except for possible changes, including this week - Sunday 4:30-5:30pm.

Problem sets are due before TA sessions, partially because solutions are posted right after they are handed. Usually 3 or 4 problems will be graded from the set of ~10 problems. TAs and graders will be responsible for grading. It takes a small portion of your total grade, so request for regrading is discouraged.

Let's start from the beginning.

Economics in general studies human and firm behavior. Usually "to study" means to explain observed phenomena and to predict.

Economic theory usually uses models to abstract a phenomenon and try to identify the quintessential components and effects that cause the observed phenomenon, assuming that agents are "rational". A simple, effective language therefore commonly used for modeling is mathematics.

Game theory in particular is a branch of economic theory. It focuses on situations that involve strategic decision making (therefore, more than one agent). To quote Roger Myerson (via Wikipedia), game theory is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers".

Therefore, when we use game theoretical models to study a situation involving strategic decision making, we generally proceed in the following steps.

1. We define rational human behavior (and formulate into axioms, for example, WARP)
    - Decision theorists develop a self-consistent system.
    - Experimentalists (e.g. John List) test these assumptions.
    - Game theorists generally use them.
2. Based on the rational human behavior, we model the situation in hand, usually by putting it into the mathematical language
    - In undergraduate classes, most problems and situations are well-formulated.
    - In research, modeling to capture the essential components may be the hardest and the most important part.
    - But also in everyday life, we abstract situations and have hypotheses about why a friend didn't show for dinner, why stock market crashes.
3. We use (solve) the model to make predictions.
    - Majority of your tests and problem sets are about.
4. ~~We verify the predictions by testing them by data.~~

We can summarize and say the three steps are

1. Assume

2. Model
3. Solve

Today, I demonstrate each of the three relevant steps in this session by examples and problems.

1. Assume: WARP (and other rationality assumptions) and 1.3
2. Model: 1.8
3. Solve: Nash Equilibrium and 1.8

1. Assume

Most assumptions on rationality are intuitive and reasonable - they must be, because they are intended to capture how we, ordinary people, think and behave. Completeness requires us to be able to compare and rank any two alternatives. Together with transitivity, they require that there is a complete ordering of the alternatives, which we cal preference relation.

The preference relation is based on how you think you will choose, but more directly, we should be able to observe how people actually choose when they are faced with a set of goods. Thus, the choice function. And we call these choices revealed preferences. An important axiom that the choices should satisfy is WARP, the weak axiom of revealed preference, defined as follows.

Definition 1. A choice function satisfies WARP if

for all $x, y \in A \cap B$, $x \in c(A)$ and $y \in c(B)$, then $x \in c(B)$

Think of two food menus A and B. Suppose both fish (x) and pork (y) are available and are in both menus. If when you are presented with menu A, your order included fish, and when presented with B, your order included pork, it must be the case that you have ordered fish too.

This is a simple and intuitive axiom[1]. It is a powerful axiom too. Because as long as this axiom holds for the choice function c, then there is a preference relation R that generates the choice function (Proposition 1.2, Note 1). Also any choice function that is generated by a preference relation satisfies WARP (Proposition 1.1, Note 1).

Now we look at the first part of 1.3: $c(A \cap B) = c(A) \cap c(B)$ is in general false. Let's try to understand this from the menus interpretation. On the left hand side of the equation, we are restricted to pick from the common items of the two menus. On the right hand side of the equation, we pick some items from menu A and some items from menu B, then order those we have picked from both menus. (Venn diagram) We can see that this does not necessarily hold. You can (?) come up with the exact example - with just three alternatives.

Some emailed about WARP. Hope this helped.

2. Model

Now remember that if we have a preference relation, then it can be represented by a utility function. (half of Proposition 1.3, Note 1). We can use numbers and mathematics!

Now let's try to model a situation. 1.8 Hawk-Dove game.

> Two animals are fighting over some prey. Each can be passive or aggressive. Each prefers to be aggressive if its opponent is passive, and passive if its opponent is aggressive; given its own stance, it prefers the outcome in which its opponent is passive to that in which its opponent is aggressive. Formulate this situation as a strategic game and find its Nash equilibria.

[1]But it may be contested: A PhD student in the department is running experiments to test it.

We can put this into a payoff matrix. Remember that these numbers only represent ordinal information (so far in the course), and do not have comparison between agents, so we can put arbitrary, normalized utilities in one box. I choose (0,0) for actions (P,P).

	P	A
P	$0,0$	$\underline{-1},\underline{1}$
A	$\underline{1},\underline{-1}$	$-2,-2$

3. Solve

Now we have the model. We need to predict the outcome of such game. In general, it could be anything. But rationality constraints and Nash equilibrium notion help us make sharp and fairly reasonable predictions.

Nash equilibrium offers a particularly attractive prediction for a wide range of games, in fact, all games that can be put into a strategic form game.

The definition of NE given by Osborne is as follows.

> A Nash equilibrium is an action profile a^* with the property that no player i can do better by choosing an action different from a_i^*, given that every other player j adheres to a_j^*.

Let's put it mathematically for the Hawk-Dove game. Two players: H and D. And two actions: P and A. Let $u_i(a_H, a_D)$ be the utility of each player when H plays a_H and D plays a_D. Mathematically, by definition of Nash equilibrium, (a_H^*, a_D^*) satisfies that H does not want to change strategy and D does not want to change strategy:

$$u_H(a_H^*, a_D^*) \geq u_H\left(a_H', a_D^*\right)$$

$$u_D(a_H^*, a_D^*) \geq u_D\left(a_H^*, a_D'\right)$$

We can verify that (A,A) and (P,P) are Nash Equilibria.