ECON244 Midterm

November 6, 2012

In solving these problems, I ask you to use the utility function u as explicitly as possible and to rely on little as possible on the general proofs that tell you some of the relationships hold, you may wish, however, adapt these proofs for the special utility function at hand.

Let $u(\mathbf{x}) = \ln(x_1+2) + 2\ln(x_2+3) + 4\ln(x_3+2)$ and $p_1^* = 2$, $p_2^* = 3$, $p_3^* = 1$ and $\omega^* = 27$.

1. Compute demand $\mathbf{x}(\mathbf{p}^*, \omega^*)$ at this $\mathbf{p}^*$ and ω^*.

2. Compute the terms $\frac{\partial x_h}{\partial p_k} + x_k \frac{\partial x_h}{\partial \omega}$ evaluated at this $(\mathbf{p}^*, \omega^*)$.

3. Verify that the matrix of these terms is symmetric and negative semi-definite.

4. Compute $v(\mathbf{p}^*, \omega^*)$ at this $(\mathbf{p}^*, \omega^*)$.

5. Derive $H_h(\mathbf{p}^*, v(\mathbf{p}^*, \omega^*))$, the Hicksian compensated demand for the h^{th} commodity associated with the utility function u when the utility level is $v(\mathbf{p}^*, \omega^*)$. Verify that $\partial H_h / \partial p_h \leq 0$.

6. Derive the function $e(\mathbf{p}, v(\mathbf{p}^*, \omega^*))$ and verify that it satisfies the monotonicity and concavity properties of an expenditure function.

7. Verify that $\nabla e(\mathbf{p}^*, u) = \mathbf{H}$, where $u = v(\mathbf{p}^*, \omega^*)$.

8. Derive $S_h(\mathbf{p}, x(\mathbf{p}^*, \omega^*))$, the Slutsky compensated demand for the h^{th} commodity associated with the demand function $x(\cdot)$ generated by the utility function u and verify that $\partial S_h / \partial p_h \leq 0$.

9. Verify that $\frac{\partial^2 e}{\partial p_h \partial p_k}(\mathbf{p}^*, v(\mathbf{p}^*, \omega^*))$ equals terms derived in 2.